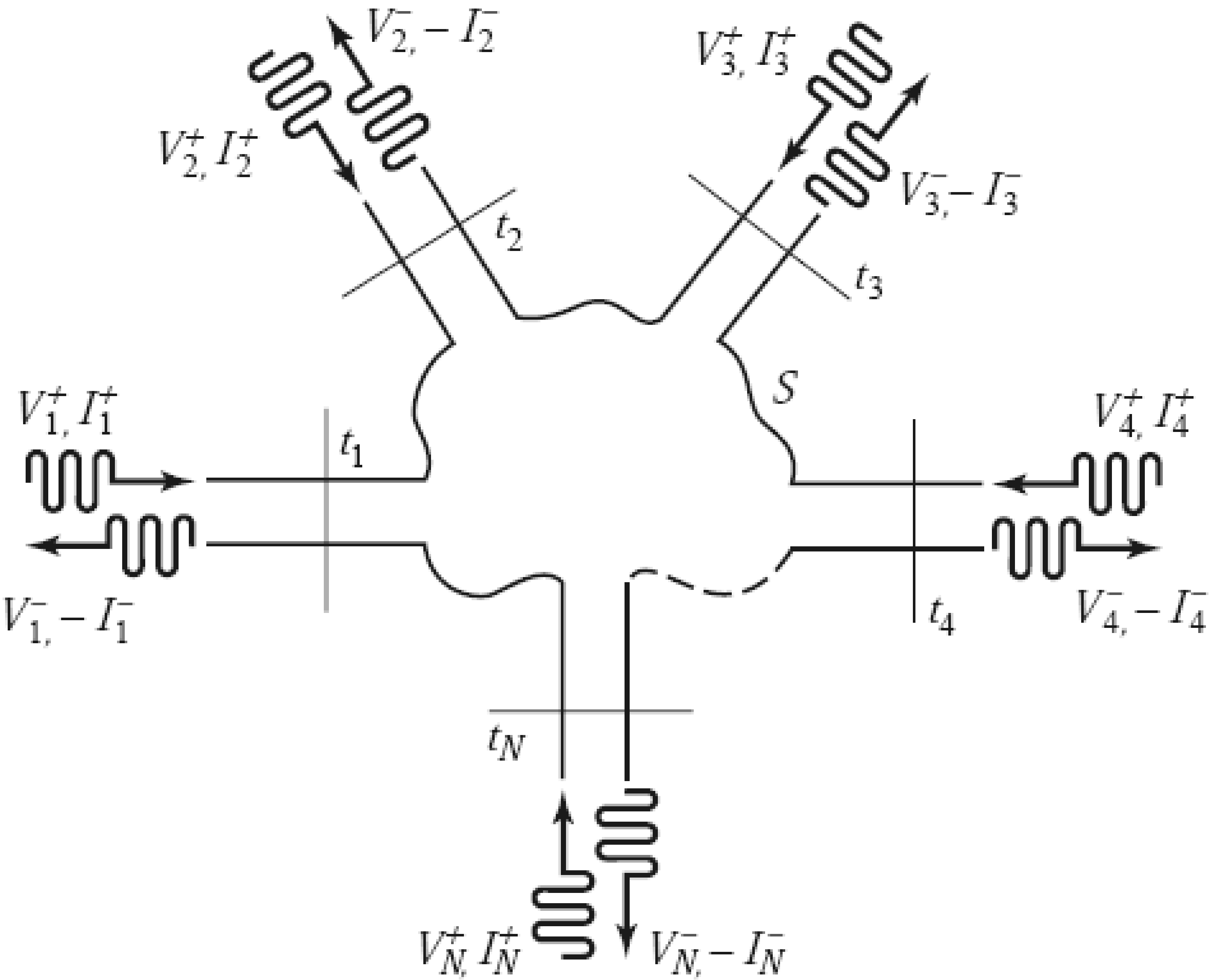
Lec11 Microwave Network Analysis （III）



$V_2^-, -I_2^-$
V_3^+, I_3^+
V_2^+, I_2^+
$V_3^-, -I_3^-$
t_2
t_3
S
V_1^+, I_1^+
t_1
V_4^+, I_4^+
$V_1^-, -I_1^-$
t_4
$V_4^-, -I_4^-$
t_N
V_N^+, I_N^+
$V_N^-, -I_N^-$

4.3 THE SCATTERING MATRIX

Reciprocal Networks and Lossless Networks

The scattering matrix for a reciprocal network is symmetric.
The scattering matrix for a lossless network is unitary (幺正).

Define
$$V_n = V_n^+ + V_n^-,$$
$$I_n = I_n^+ - I_n^- = V_n^+ - V_n^-.$$
with $Z_{0n} = 1.$

Then
$$V_n^+ = \frac{1}{2}(V_n + I_n),$$ or $$[V^+] = \frac{1}{2}([Z] + [U])[I].$$
$$V_n^- = \frac{1}{2}(V_n - I_n),$$ or $$[V^-] = \frac{1}{2}([Z] - [U])[I].$$

Eliminating [I] gives $[V^-] = ([Z] - [U])([Z] + [U])^{-1}[V^+],$

So $[S] = ([Z] - [U])([Z] + [U])^{-1}.$

Taking the transpose (转置) of [S] gives

$$[S]^t = \{([Z] + [U])^{-1}\}^t([Z] - [U])^t.$$

If the network is reciprocal, [Z] is symmetric. So that $[Z]^t = [Z]$.

$$[S]^t = ([Z] + [U])^{-1}([Z] - [U]),$$

is compared with $[S] = ([Z] + [U])^{-1}([Z] - [U])$,

We have thus shown that $[S] = [S]^t$,

If the network is lossless, no real power can be delivered to the network.

$$\begin{aligned} P_{avg} &= \frac{1}{2}\mathrm{Re}\{[V]^t[I]^*\} = \frac{1}{2}\mathrm{Re}\{([V^+]^t + [V^-]^t)([V^+]^* - [V^-]^*)\} \\ &= \frac{1}{2}\mathrm{Re}\{[V^+]^t[V^+]^* - [V^+]^t[V^-]^* + [V^-]^t[V^+]^* - [V^-]^t[V^-]^*\} \\ &= \frac{1}{2}[V^+]^t[V^+]^* - \frac{1}{2}[V^-]^t[V^-]^* = 0, \end{aligned}$$

the incident and reflected powers are equal:

$$[V^+]^t[V^+]^* = [V^-]^t[V^-]^*.$$

Using [V−] = [S][V+] gives

$$[V^+]^t[V^+]^* = [V^+]^t[S]^t[S]^*[V^+]^*,$$

so that, for nonzero [V+]

$$[S]^t[S]^* = [U],$$

$$\text{or} \quad [S]^* = \{[S]^t\}^{-1}.$$

The scattering matrix is a unitary matrix.

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij}, \text{ for all } i,\ j,$$

where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$,

EXAMPLE 4.5 APPLICATION OF SCATTERING PARAMETERS

端口反射系数定义？
端口匹配定义？

A two-port network is known to have the following scattering matrix:

$$[S] = \begin{bmatrix} 0.15\angle 0^\circ & 0.85\angle -45^\circ \\ 0.85\angle 45^\circ & 0.2\angle 0^\circ \end{bmatrix}$$

Determine if the network is reciprocal and lossless. If port 2 is terminated with a matched load, what is the return loss seen at port 1? If port 2 is terminated with a short circuit, what is the return loss seen at port 1

and what is the transmission coefficient between the port 1 and the port 2 ?

Solution

Because [S] is not symmetric, the network is not reciprocal.
To be lossless, the scattering parameters must be unitary. Taking the first column gives

$$|S_{11}|^2 + |S_{21}|^2 = (0.15)^2 + (0.85)^2 = 0.745 \neq 1,$$

When port 2 is terminated with a matched load, the reflection coefficient seen at port 1 is $S_{11} = 0.15$. So the return loss is

$$\text{RL} = -20 \log |\Gamma| = -20 \log(0.15) = 16.5 \text{ dB}.$$

When port 2 is terminated with a short circuit, $V_2^+ = -V_2^-$

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+ = S_{11} V_1^+ - S_{12} V_2^-,$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+ = S_{21} V_1^+ - S_{22} V_2^-.$$

The second equation gives $V_2^- = \frac{S_{21}}{1 + S_{22}} V_1^+.$

Dividing the first equation by V_1^+

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{12} \frac{V_2^-}{V_1^+} = S_{11} - \frac{S_{12} S_{21}}{1 + S_{22}}$$

$$= 0.15 - \frac{(0.85\angle -45°)(0.85\angle 45°)}{1 + 0.2} = -0.452.$$

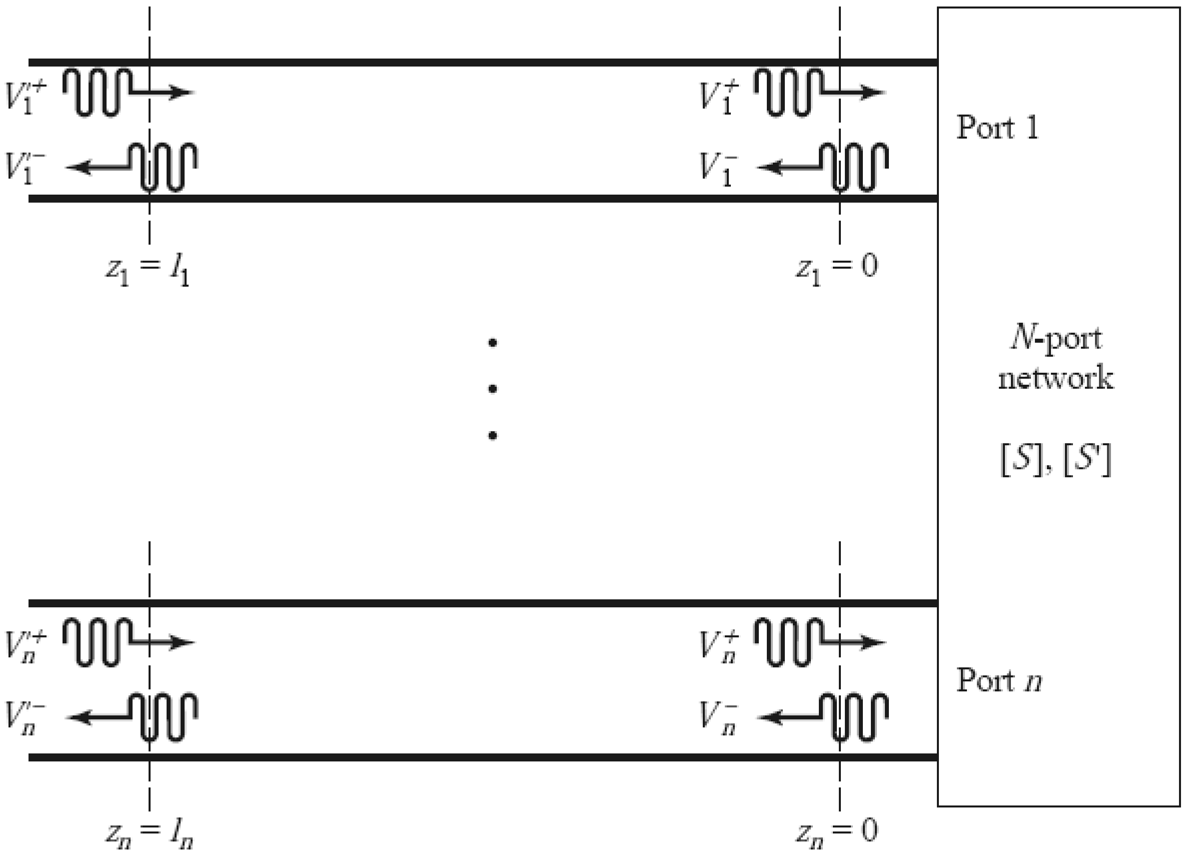
So the return loss is $\text{RL} = -20 \log |\Gamma| = -20 \log(0.452) = 6.9 \text{ dB}.$

Understanding about scattering parameters:

- The reflection coefficient looking into port *n* is not equal to *Snn* unless all other ports are matched .
- the transmission coefficient from port *m* to port *n* is not equal to *Snm* unless all other ports are matched.

- Changing the terminations or excitations of a network does not change its scattering parameters, but may change the reflection coefficient seen at a given port, or the transmission coefficient between two ports.

A Shift in Reference Planes



Consider the N-port microwave network, where the original terminal planes are assumed to be located at $z_n = 0$ for the nth port, where z_n is an arbitrary coordinate measured along the transmission line feeding the nth port.

The scattering matrix for the network with this set of terminal planes is denoted by [S]. Now consider a new set of reference planes defined at $z_n = l_n$ for the nth port, and let the new scattering matrix be denoted as [S'].

$$[V^-] = [S][V^+],$$

$$[V'^-] = [S'][V'^+],$$

From the theory of traveling waves on lossless transmission lines

$$V_n'^+ = V_n^+ e^{j\theta_n},$$

$$V_n'^- = V_n^- e^{-j\theta_n},$$

where $\theta n = \beta ln$ is the electrical length of the outward shift of the reference plane of port *n*.

Since $[V^-] = [S][V^+]$.

$$\begin{bmatrix} e^{j\theta_1} & & & 0 \\ & e^{j\theta_2} & & \\ & & \ddots & \\ 0 & & & e^{j\theta_N} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \ddots & \\ 0 & & & e^{-j\theta_N} \end{bmatrix} [V'^+].$$

Then

$$[V'^-] = \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \ddots & \\ 0 & & & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \ddots & \\ 0 & & & e^{-j\theta_N} \end{bmatrix} [V'^+].$$

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix},$$

Note that $S'_{nn} = e^{-2j\theta n} S_{nn}$, meaning that the phase of S_{nn} is shifted by twice the electrical length of the shift in terminal plane *n* because the wave travels twice over this length upon incidence and reflection.

This result is consistent with

$$\Gamma(\ell) = \frac{V_o^- e^{-j\beta\ell}}{V_o^+ e^{j\beta\ell}} = \Gamma(0) e^{-2j\beta\ell},$$

Generalized scattering matrix (广义散射矩阵)

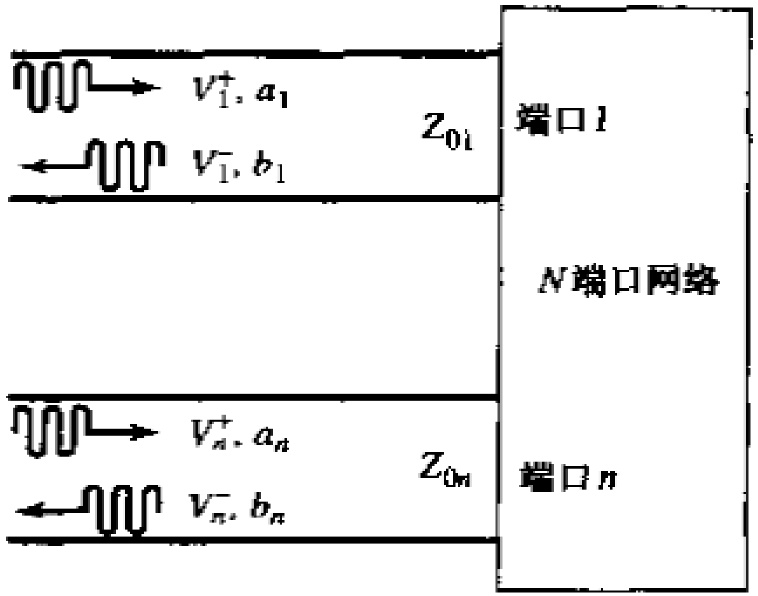


图 4.10　有不同特征阻抗的 N 端口网络

通常，端口的特性阻抗均为50欧姆，然而有些情况下，多端口网络的阻抗彼此不同。

考虑示于图 4.10 中的 N 端口网络，其中 Z_{0n} 是第 n 个端口的(实)特征阻抗，V_n^+ 和 V_n^- 分别代表第 n 个端口的入射和反射电压波。为了得到用波振幅表示的有物理意义的功率关系，有必要定义新的一组波振幅如下：

归一化的端口电压

$$a_n = V_n^+ / \sqrt{Z_{0n}}$$

$$b_n = V_n^- / \sqrt{Z_{0n}}$$

因此有

$$V_n = V_n^+ + V_n^- = \sqrt{Z_{0n}}(a_n + b_n)$$

$$I_n = \frac{1}{Z_{0n}}(V_n^+ - V_n^-) = \frac{1}{\sqrt{Z_{0n}}}(a_n - b_n)$$

传送到第n个端口的传输功率为

$$P_n = \frac{1}{2}\mathrm{Re}\{V_n I_n^*\} = \frac{1}{2}\mathrm{Re}\{|a_n|^2 - |b_n|^2 + (b_n a_n^* - b_n^* a_n)\} = \frac{1}{2}|a_n|^2 - \frac{1}{2}|b_n|^2$$

即为入射功率减去反射功率。

散射系数定义将归一化入射波与发射波联系起来

$$[b] = [S][a]$$

其中散射参数 $S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0,\quad k\neq j}$

用电压波和端口阻抗表示为： $S_{ij} = \left.\frac{V_i^-\sqrt{Z_{0j}}}{V_j^+\sqrt{Z_{0i}}}\right|_{V_k^+=0,\quad k\neq j}$

这说明了具有相同特征阻抗的网络的 S 参量（V_i^- / V_j^+，并有 $V_k^+ = 0, k\neq j$），是如何转换到连接有不同特征阻抗的传输线的网络上去的。

Homework

4.14 A four-port network has the scattering matrix shown as follows. (a) Is this network lossless? (b) Is this network reciprocal? (c) What is the return loss at port 1 when all other ports are terminated with matched loads? (d) What is the insertion loss and phase delay between ports 2 and 4 when all other ports are terminated with matched loads? (e) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are terminated with matched loads?

$$[S] = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix}.$$