Lec15 Impedance Matching and Tuning (I)
阻抗匹配和调谐

A lossless network matching an arbitrary load impedance to a transmission line.

The **matching network（匹配网络）** is ideally **lossless（无耗）**, to avoid unnecessary loss of power, and is usually designed so that the impedance seen looking into the matching network is *Z0*.

Then **reflections will be eliminated** on the transmission line to the left of the matching network, although there will usually be multiple reflections between the matching network and the load. This procedure is sometimes referred to as ***tuning***.

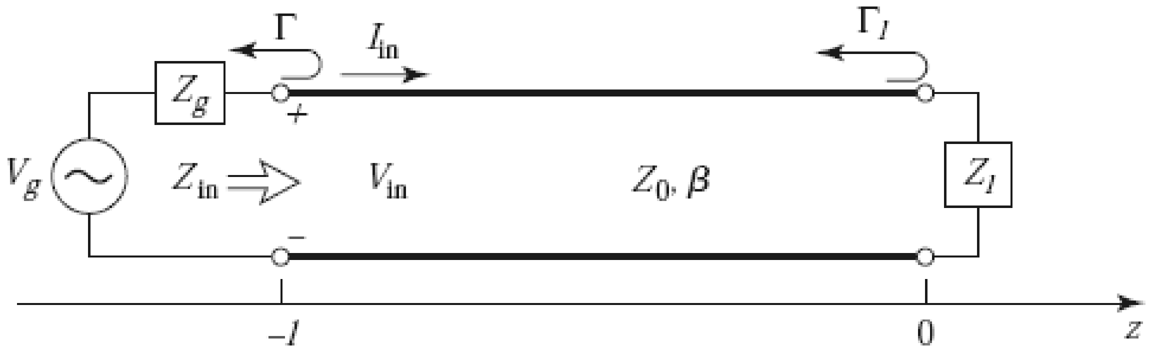
Impedance matching or tuning is important

- **Maximum power is delivered** when the load is matched to the line (assuming **the generator is matched**), and power loss in the feed line is minimized.

- Impedance matching sensitive receiver components (antenna, low-noise amplifier, etc.) may **improve the signal-to-noise ratio (信噪比)** of the system.

- Impedance matching in a power distribution network (such as an antenna array feed network) may **reduce amplitude and phase errors**.

Matching review



The power delivered to the load is

$$P = \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2+(X_{in}+X_g)^2}.$$

Load Matched to Line $Z_l = Z_0$, so $\Gamma_\ell = 0$, $P = \frac{1}{2}|V_g|^2 \frac{Z_0}{(Z_0+R_g)^2+X_g^2}.$

Generator Matched to Loaded Line $Z_{in} = Z_g$, $P = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g^2+X_g^2)}.$

Conjugate Matching $Z_{in} = Z_g^*.$ $P = \frac{1}{2}|V_g|^2 \frac{1}{4R_g},$

the maximum power delivered to the load.

As long as the load impedance, Z_L, has a positive real part, a matching network can always be found.

$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_o^+ \left(e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right)}{V_o^+ \left(e^{j\beta\ell} - \Gamma e^{-j\beta\ell}\right)} Z_0 = \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}} Z_0,$$

if $Z_L = 0.$ $\quad Z_{\text{in}} = j Z_0 \tan \beta\ell,$

$Z_L \to \infty \quad Z_{\text{in}} = -j Z_0 \cot \beta\ell,$

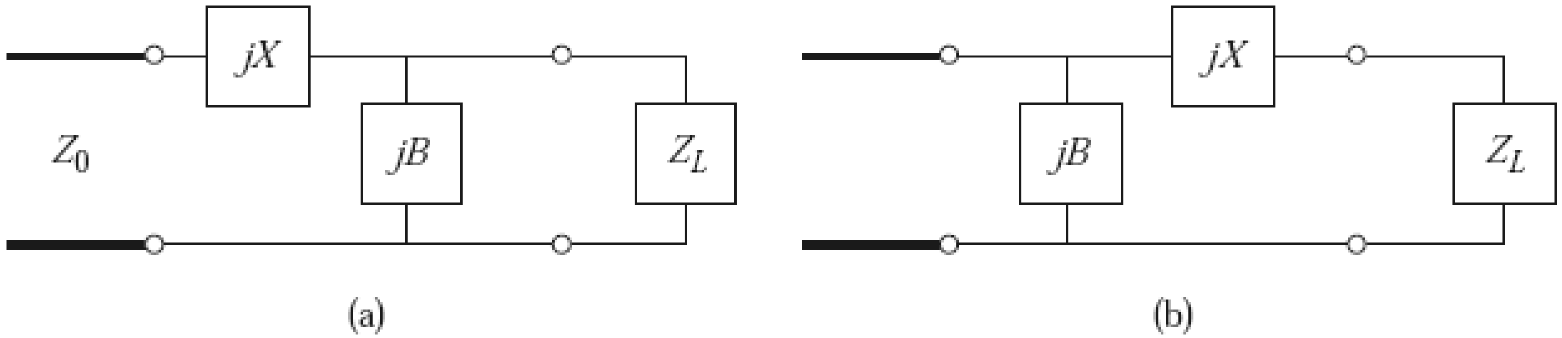
$Z_L = jX_L \quad Z_{\text{in}} = jZ_0 \tan\left(\beta l + \theta\right)$

Is a matching network unique for a particular system?
How to select a particular matching network?

- **Complexity**—As with most engineering solutions, the simplest design that satisfies the required specifications is generally preferable.
- **Bandwidth**—Any type of matching network can ideally give a perfect match (zero reflection) at a single frequency. In many applications, however, it is desirable to match a load over a band of frequencies.
- ***Implementation***—*Depending on the type of transmission line or waveguide being used,* one type of matching network may be preferable to another. For example, tuning stubs are much easier to implement in waveguide than are multisection quarter-wave transformers.
- **Adjustability**—In some applications the matching network may require adjustment to match a variable load impedance.

5.1 MATCHING WITH LUMPED ELEMENTS (L NETWORKS)

The simplest type of matching network is the *L-section,* which uses two reactive elements to match an arbitrary load impedance to a transmission line.



L-section matching networks. (a) Network for z_L inside the 1 + j x circle. (b) Network for z_L outside the 1 + j x circle.

The reactive elements may be either inductors or capacitors, depending on the load impedance.

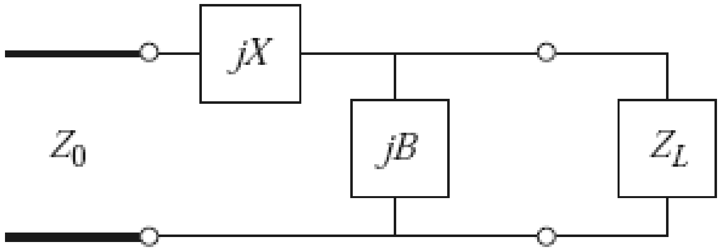
If the frequency is low enough and/or the circuit size is small enough, actual lumped-element capacitors and inductors can be used.

There are a large range of frequencies (usually >1GHz) and circuit sizes where lumped elements may not be realizable. This is **a limitation of the *L-section* matching technique.**

Two types of solutions:

- Analytic Solutions
- Smith Chart Solutions

Analytic Solutions



(a)

$$Z_L = R_L + jX_L.$$

The circuit (a) should be used when $z_L = Z_L/Z_0$ is inside the 1 + j x circle on the Smith chart, which implies that $R_L > Z_0$ *for this case.*

The impedance seen looking into the matching network, followed by the load impedance, must be equal to *Z0*

$$Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}.$$

then

$$B(XR_L - X_L Z_0) = R_L - Z_0,$$

$$X(1 - BX_L) = BZ_0R_L - X_L.$$

The solution is

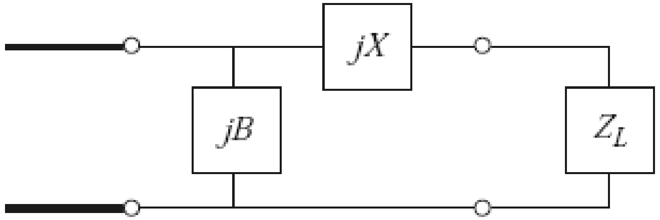
$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}.$$

Note that since *RL > Z0*, the argument of the second square root is always positive. Then the series reactance can be found as

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}.$$

Two solutions are possible for *B and X*.

One solution, however, may result in significantly smaller values for the reactive components, or may be the preferred solution if the **bandwidth of the match is better**, or if **the SWR** on the line between the matching network and the load is **smaller**.



(b)

The circuit (b) is used when zL is outside the 1 + j x circle on the Smith chart, which implies that RL < Z0.

The admittance seen looking into the matching network, followed by the load impedance, must be equal to 1/*Z0*

$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}.$$

Rearranging and separating into real and imaginary parts gives two equations for the two unknowns, *X and B:*

$$BZ_0(X + X_L) = Z_0 - R_L,$$

$$(X + X_L) = BZ_0R_L.$$

Solving for *X and B gives*

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L,$$

$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}.$$

Smith Chart Solutions

EXAMPLE 5.1 L-SECTION IMPEDANCE MATCHING

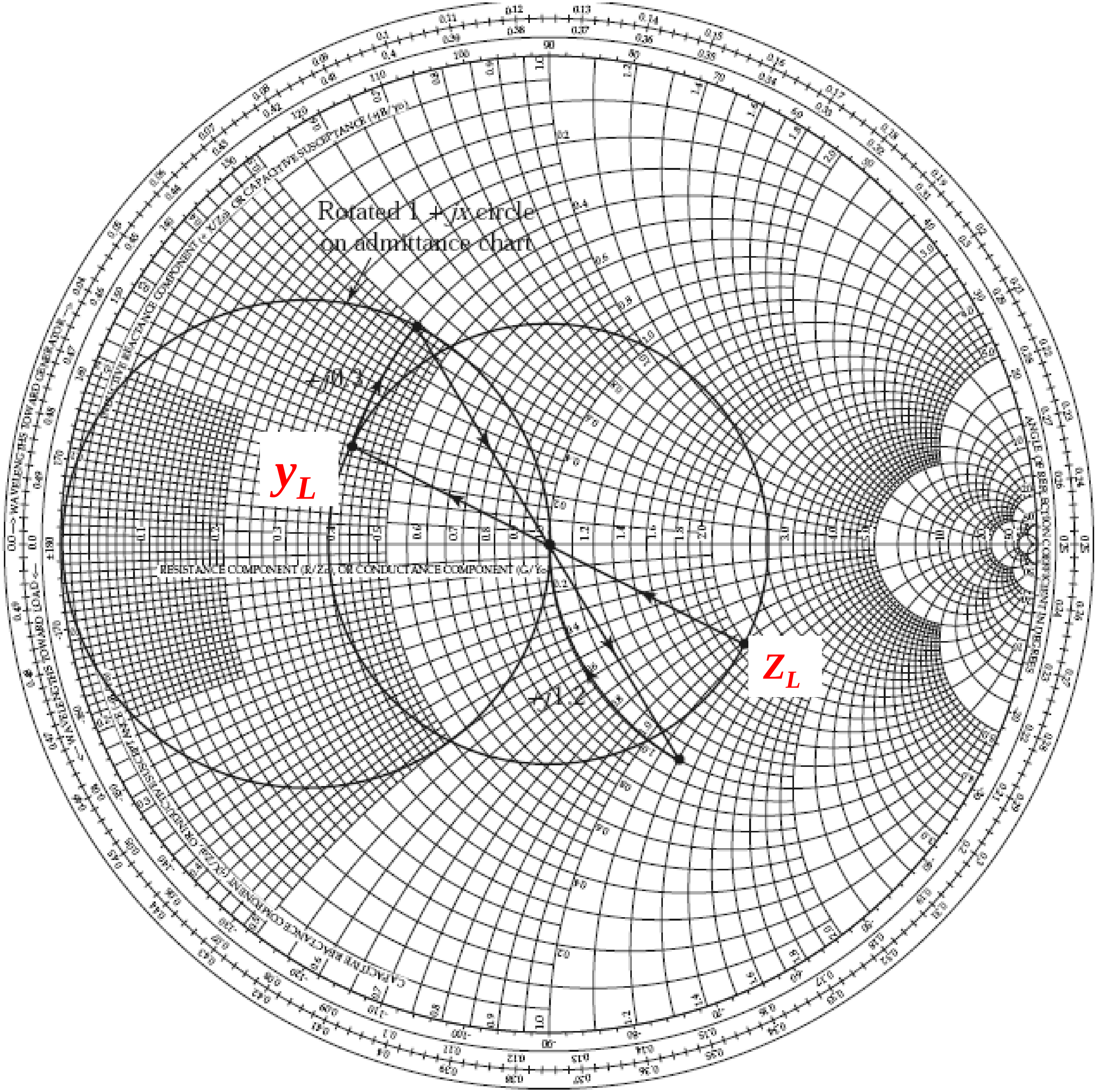
Design an L-section matching network to match a series RC load with an impedance $Z_L = 200 - j100$ to a 100 line at a frequency of 500 MHz.

Solution

1) The normalized load impedance is $Z_L = 2 - j1$, which is plotted on *the Smith* chart. This point is inside the $1 + jx$ circle, so we use the matching circuit of Figure a.

2) Because the first element from the load is a **shunt susceptance** (电纳), it makes sense to **convert to admittance** by drawing the SWR circle through the load, and a straight line from the load through the center of the chart,

$$y_L = 0.4 + j0.2$$


Rotated 1 + jx circle
on admittance chart
+j0.3
−j1.2
y_L
z_L
RESISTANCE COMPONENT (R/Z₀), OR CONDUCTANCE COMPONENT (G/Y₀)

3) we want to be on the 1 + *j x* circle so that we can add a series reactance to cancel *jx* and match the load. This means that the shunt susceptance must move us from y_L to the 1 + j x circle on the admittance Smith chart.

construct the rotated **1 + j x** circle on the admittance Smith chart.

4) Adding a susceptance will move from y_L to the 1 + j x circle on the admittance Smith chart. Clock-wise move along a constant-conductance circle to **y = 0.4 + j0.5** (this choice is the shortest distance from y_L to the shifted 1 + j x circle).

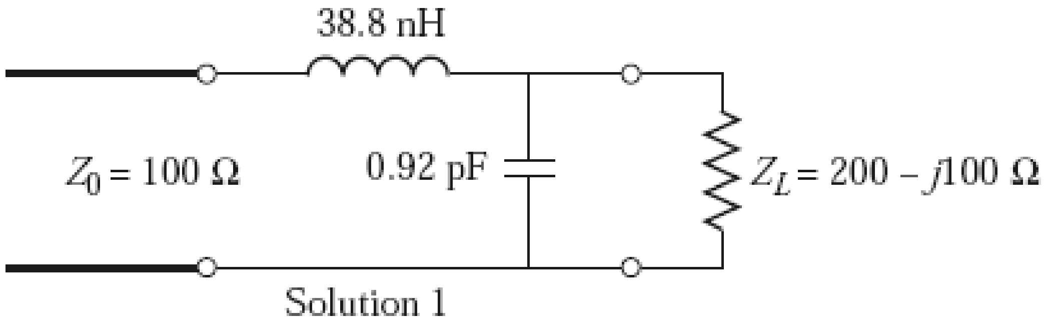
$$\mathbf{jb = y - y_L = j\,0.3}$$

5) Converting back to impedance leaves us at ***z* = 1 − j1.2**.

6) Move along a constant-resistance circle to the center of the chart.

A series reactance of **x = j1.2** will bring us to the center of the chart.

For comparison, the analytical formulas give the solution as $b = 0.29$, $x = 1.22$.



For a matching frequency of 500 MHz, the capacitor has a value of

$$C = \frac{b}{2\pi f Z_0} = 0.92 \text{ pF},$$

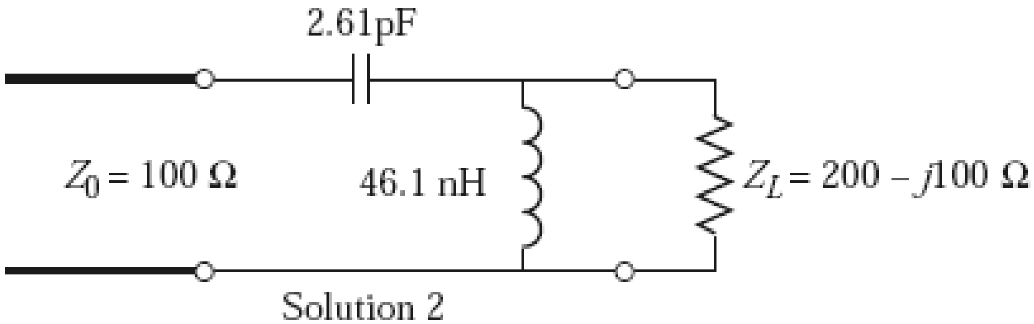
and the inductor has a value of

$$L = \frac{x Z_0}{2\pi f} = 38.8 \text{ nH}.$$

the second solution to this matching problem.

1) we will move to a point on the lower half of the shifted 1 + *j x circle*, to $y = 0.4 - j0.5$. so $\mathbf{b = y - y_L = -0.7}$

2) Then converting to impedance z=1+1.2 and adding a series reactance of ***x = −1.2*** leads to a match as well.

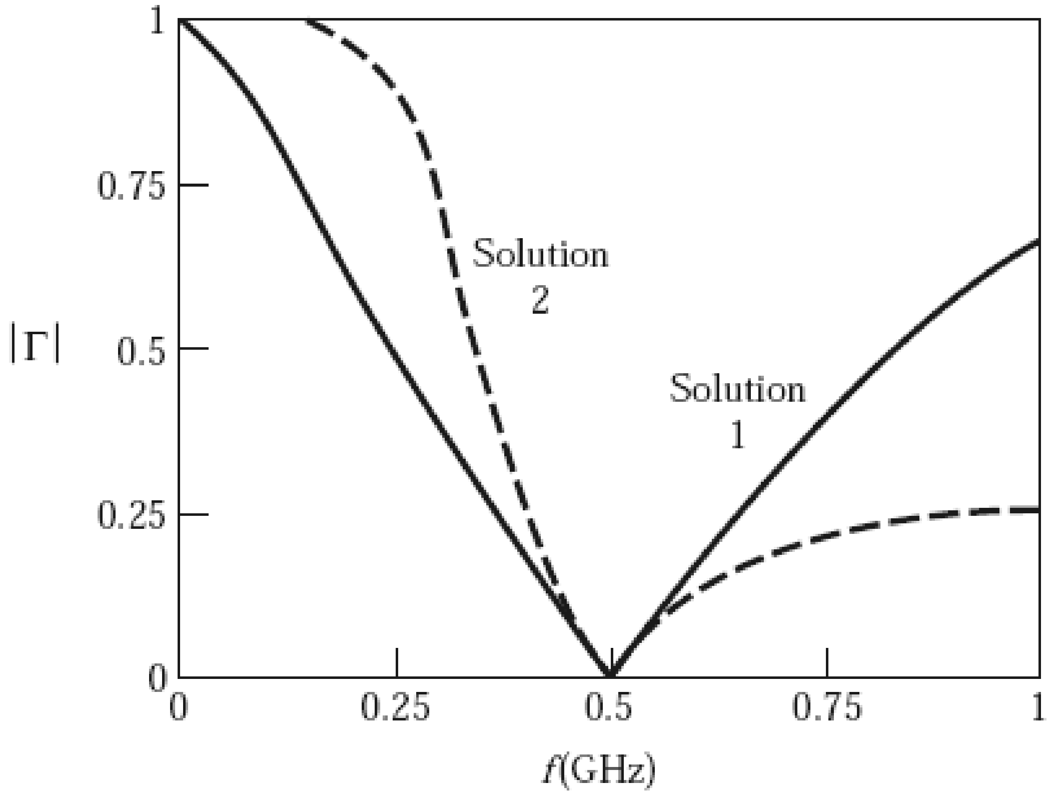


(b)

Formulas (5.3a) and (5.3b) give this solution as $b = -0.69$, $x = -1.22$.

$$C = \frac{-1}{2\pi f x Z_0} = 2.61 \text{ pF},$$

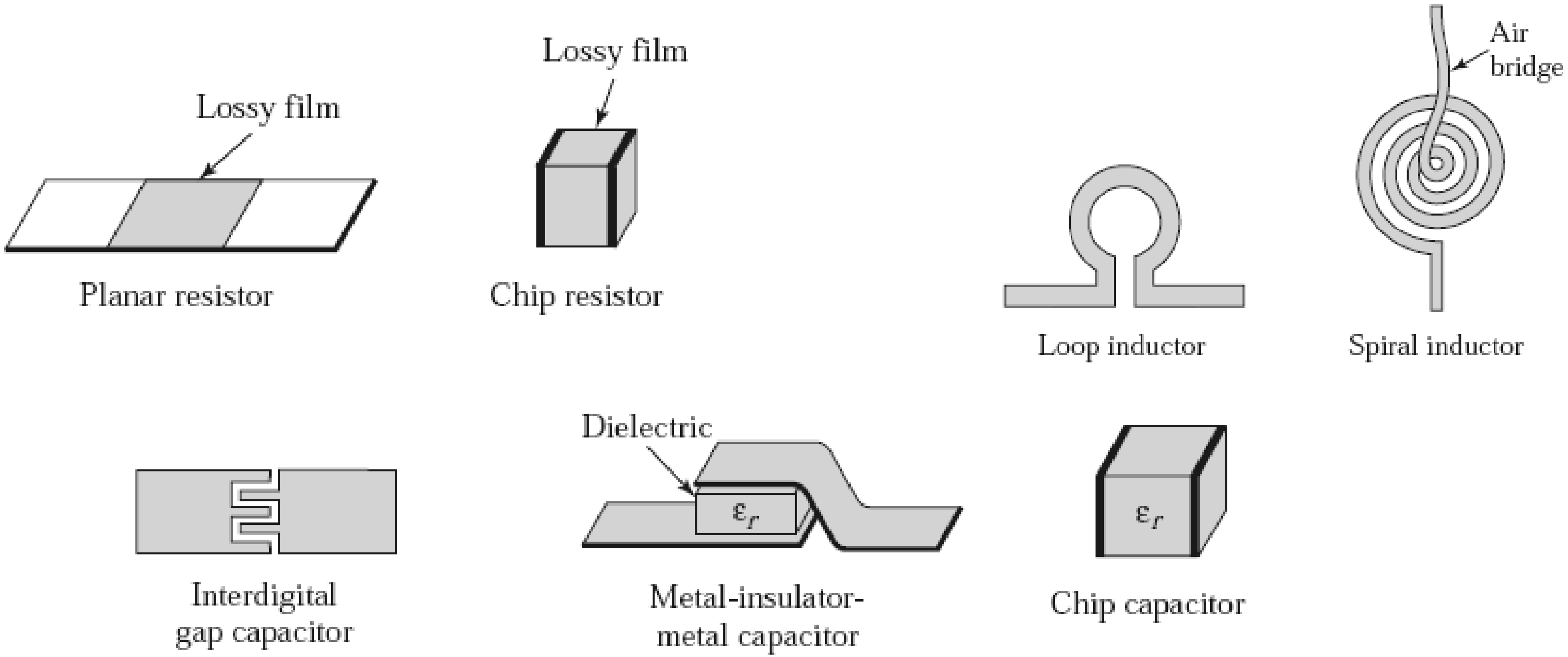
$$L = \frac{-Z_0}{2\pi f b} = 46.1 \text{ nH}.$$



the reflection coefficient magnitude versus frequency for these two matching networks, assuming that the load impedance of ZL = 200 − j100 at 500 MHz consists of a 200 resistor and a 3.18 pF capacitor in series.

There is not a substantial difference in bandwidth for these two solutions.

Lumped Elements for Microwave Integrated Circuits



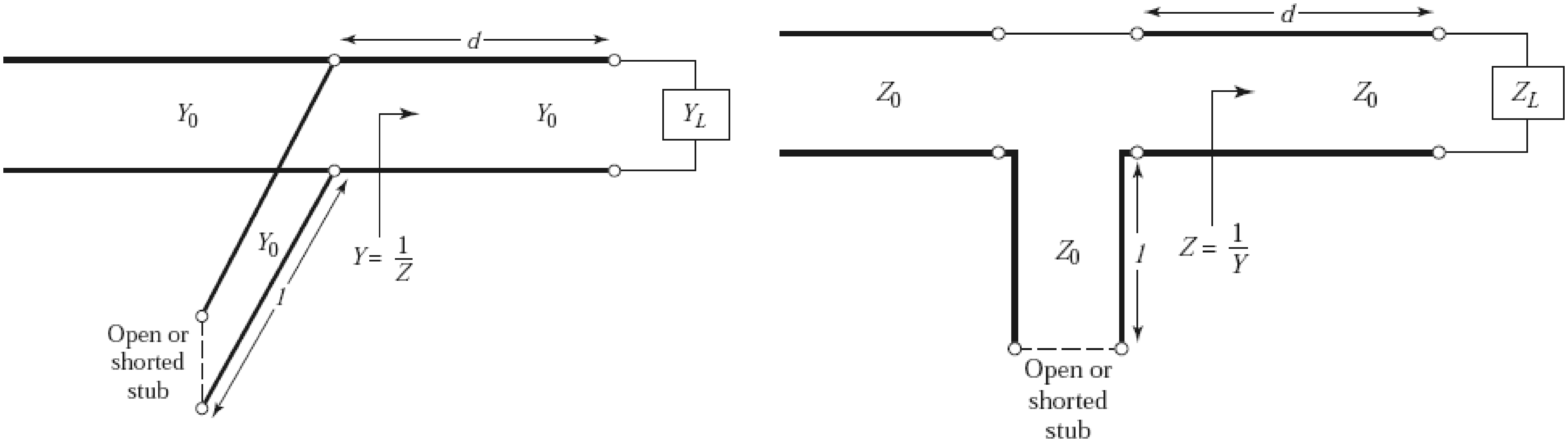
such components can be used in hybrid and monolithic microwave integrated circuits at frequencies up to 60 GHz, or higher, if the condition that $< \lambda/10$ *is* satisfied.

5.2 SINGLE-STUB TUNING (单短截线调谐)

Another popular matching technique uses a **single open-circuited or short-circuited length of transmission line (a *stub*)** connected either **in parallel （并联）** **or in series （串联）** with the **transmission feed line （传输馈线）** at a certain distance from the load,

Such a single-stub tuning circuit is often very convenient because the stub can be fabricated as part of the transmission line media of the circuit, **and lumped elements are avoided**.

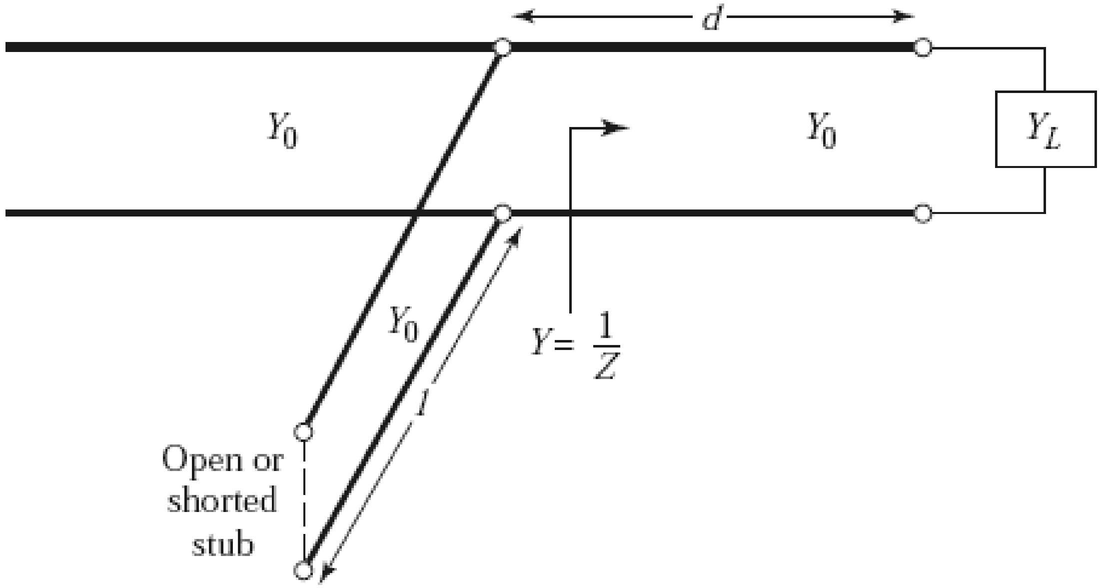
Shunt stubs （并联短截线） are preferred for **microstrip line or stripline**, while **series stubs** （串联短截线） are preferred for **slotline or coplanar waveguide**.



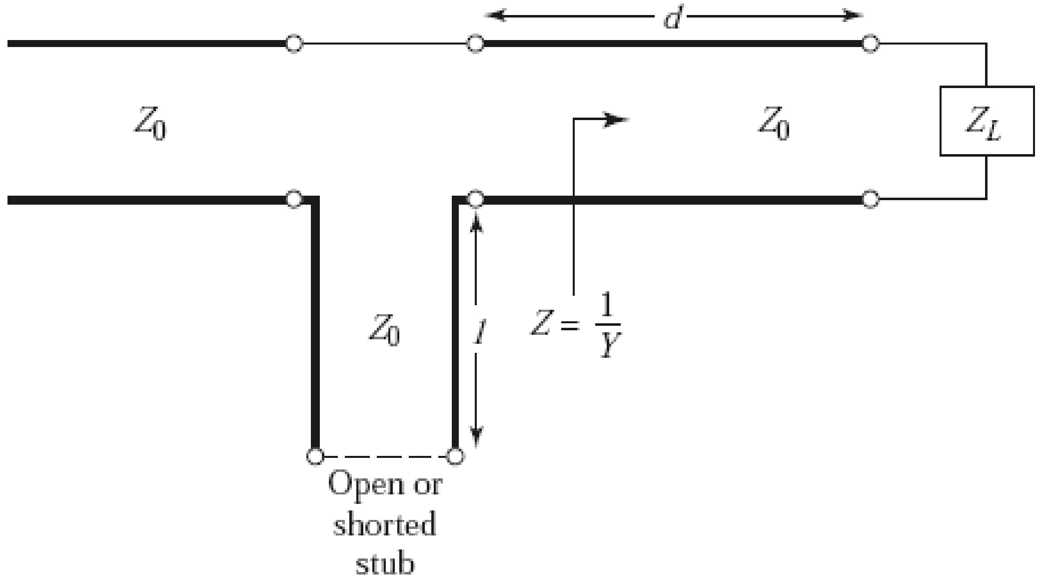
Single-stub tuning circuits. (a) Shunt stub. (b) Series stub.

In single-stub tuning **the two adjustable parameters** are：
the distance, *d*, from the load to the stub position,
the value of susceptance or reactance provided by the stub.

For the shunt-stub case, the basic idea is to select *d* so that the admittance, *Y* , seen looking into the line at distance d from the load is of the form $\mathbf{Y=Y_0 + j\,B}$. Then the stub susceptance is chosen as $\mathbf{-j\,B}$, resulting in a matched condition.



For the series-stub case, the distance d is selected so that the impedance, Z, seen looking into the line at a distance d from the load is of the form Z=**Z0 + j X**. Then the stub reactance is chosen as **−j X**, resulting in a matched condition.



The proper length of an open or shorted transmission line section can provide any desired value of reactance or susceptance.

For a given susceptance or reactance, the difference in lengths of an open- or short-circuited stub is $\lambda/4$.

Shunt Stubs

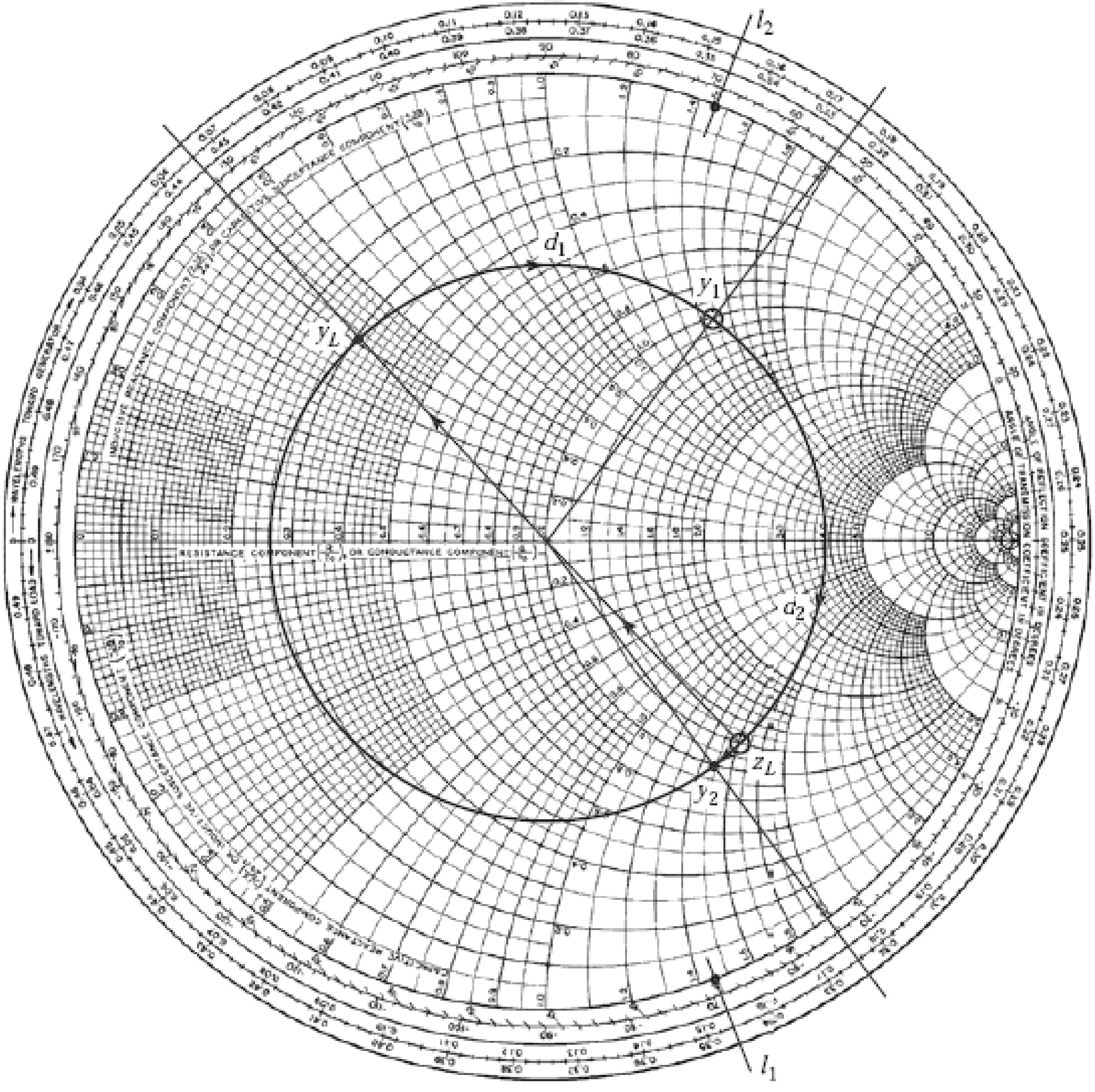
EXAMPLE 5.2 SINGLE-STUB SHUNT TUNING

For a load impedance $Z_L = 60 - j80$, design two single-stub (short circuit) shunt tuning networks to match this load to a 50 *line*. Assuming that the load is matched at 2 GHz and that the load consists of a resistor and capacitor in series, plot the reflection coefficient magnitude from 1 to 3 GHz for each solution.

Solution

(1) The first step is to plot the normalized load impedance $\mathbf{z_L = 1.2 - j1.6}$, *construct* the appropriate SWR circle, and convert to the **load admittance,** y_L For the remaining steps we consider the Smith chart as an admittance chart.

(2) Notice that the SWR circle intersects the **1 + *jb* circle** at two points, denoted as ***y1*** *and* ***y2*** in Figure 5.5a.



(a)

Thus the distance *d from* the load to the stub is given by either of these two intersections. Reading the WTG scale, we obtain

$$d_1 = 0.176 - 0.065 = 0.110\lambda,$$
$$d_2 = 0.325 - 0.065 = 0.260\lambda.$$

Actually, there is an infinite number of distances *d around the SWR circle* that intersect the 1 + *jb circle*. Usually it is desired to **keep the matching stub as close as possible to the load** to **improve the bandwidth** of the match and to **reduce loss**es caused by a possibly large standing wave ratio on the line between the stub and the load.

At the two intersection points, the normalized admittances are

$$y_1 = 1.00 + j1.47,$$
$$y_2 = 1.00 - j1.47.$$

(3) Thus, the first tuning solution requires a stub with a susceptance of **−j1.47**.

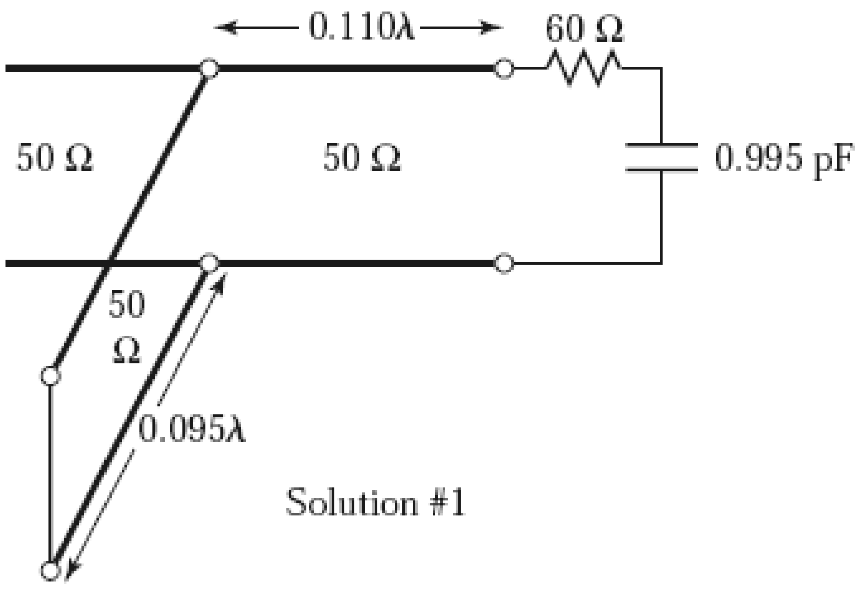
The length of a short-circuited stub that gives this susceptance can be found on the Smith chart by starting at *y* =∞ *(the short circuit) and* moving along the outer edge of the chart (*g = 0)* toward the generator to the −j1.47 point.

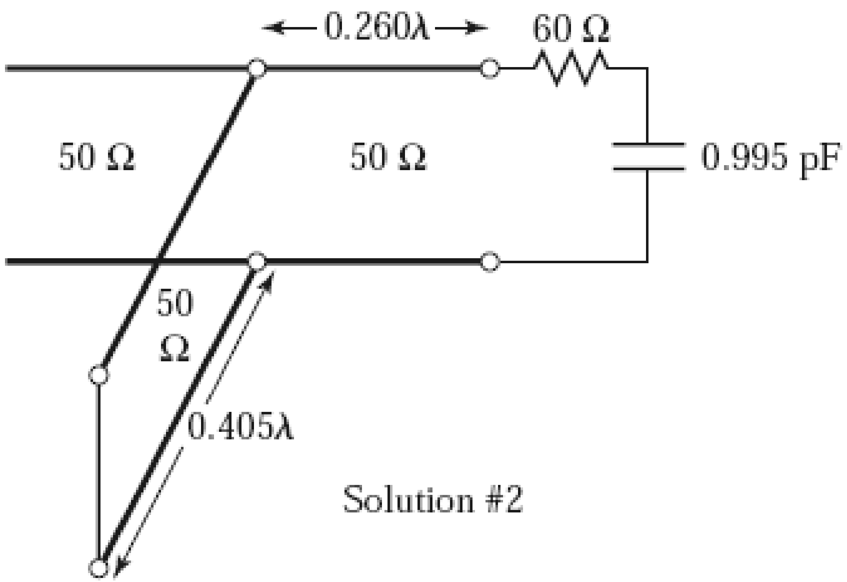
The stub length is then

$$\ell_1 = 0.095\lambda.$$

Similarly, the required short-circuit stub length for the second solution is

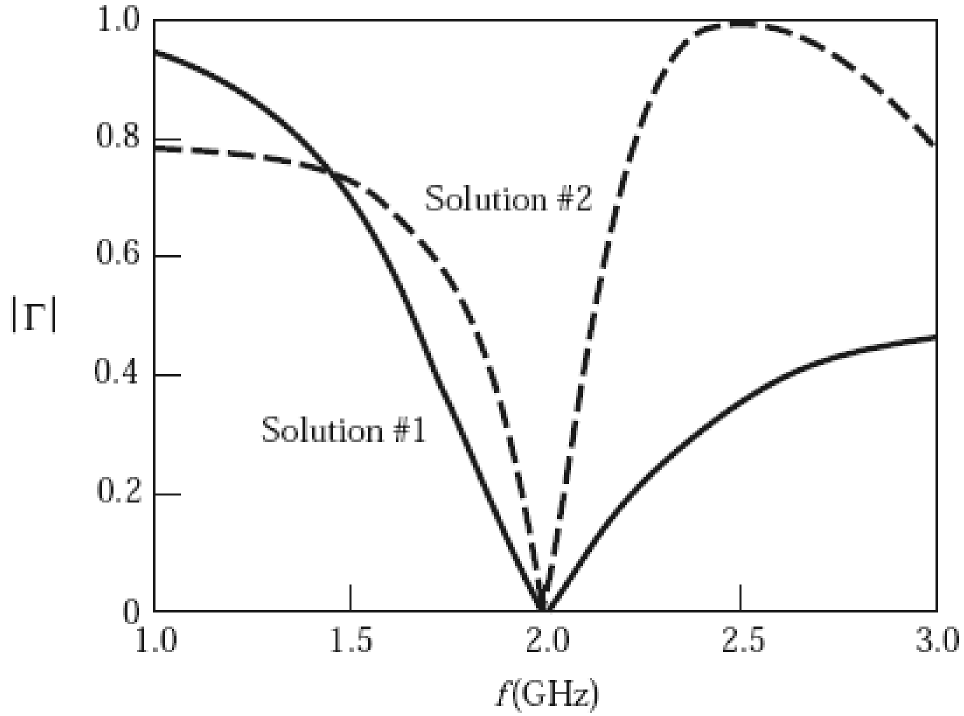
$$\ell_2 = 0.405\lambda.$$





To analyze the frequency dependence of these two designs, we need to know the load impedance as a function of frequency.

The series-RC load impedance is $Z_L = 60 - j80$ *at 2 GHz, so* $R = 60$ *and* $C = 0.995$ *pF.*



(c)

Reflection coefficient magnitudes versus frequency for the tuning circuits

Observe that solution 1 has a significantly better bandwidth than solution 2; this is because both *d and stub length are shorter* for solution 1, which reduces the frequency variation of the match.

Analytical solution

To derive formulas for *d* and, let the load impedance be written as $Z_L = 1/Y_L = R_L + j\,X_L$.

$$Z = Z_0 \frac{(R_L + jX_L) + jZ_0 t}{Z_0 + j(R_L + jX_L)t},$$

where $t = \tan \beta d$. The admittance at this point is

$$Y = G + jB = \frac{1}{Z},$$

where

$$G = \frac{R_L(1+t^2)}{R_L^2 + (X_L + Z_0 t)^2},$$

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0\left[R_L^2 + (X_L + Z_0 t)^2\right]}.$$

Now *d (which implies t*) is chosen so that G = Y0 = 1/Z0.

$$Z_0(R_L - Z_0)t^2 - 2X_L Z_0 t + \left(R_L Z_0 - R_L^2 - X_L^2\right) = 0.$$

Solving for *t gives*

$$t = \frac{X_L \pm \sqrt{R_L\left[(Z_0 - R_L)^2 + X_L^2\right]/Z_0}}{R_L - Z_0} \quad \text{for } R_L \neq Z_0.$$

If $R_L = Z_0$, then $t = -X_L/2Z_0$.

The two principal solutions for *d are*

$$\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t & \text{for } t \geq 0 \\ \dfrac{1}{2\pi}(\pi + \tan^{-1} t) & \text{for } t < 0. \end{cases}$$

To find the required stub lengths, first use *t in (5.8b)* to find the stub susceptance, $B_s = -B$. Then, for an open-circuited stub,

$$\frac{\ell_o}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{B_s}{Y_0}\right) = \frac{-1}{2\pi}\tan^{-1}\left(\frac{B}{Y_0}\right),$$

$$Z_{in} = -jZ_0 \cot \beta\ell,$$

for $Z_L \to \infty$

and for a short-circuited stub,

$$\frac{\ell_s}{\lambda} = \frac{-1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B_s}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B}\right).$$

$$Z_{in} = jZ_0 \tan \beta\ell,$$

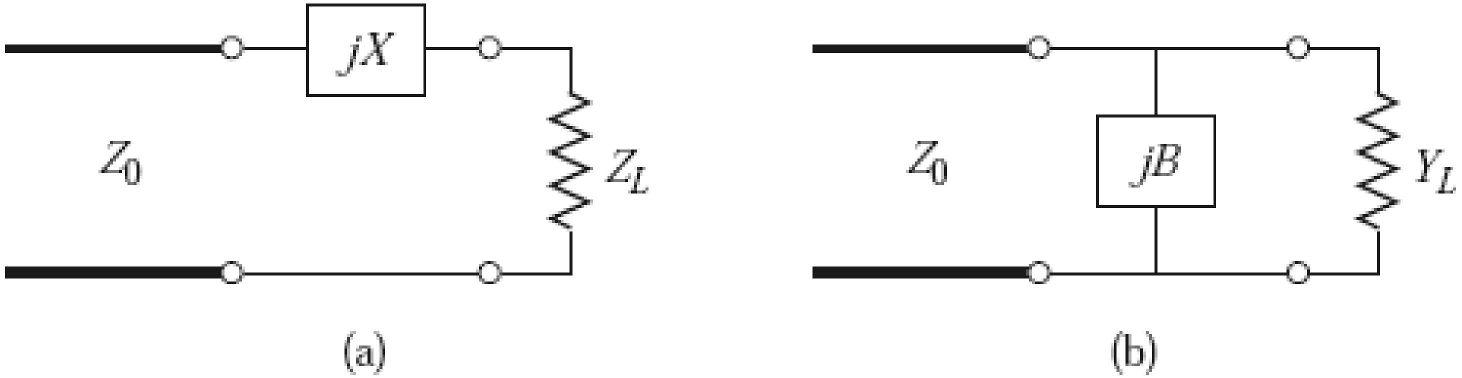
for $Z_L = 0.$

If the length is negative, $\lambda/2$ can be added to give a positive result.

Homework

5.1 Design two lossless L-section matching circuits to match each of the following loads to a 100 Ω generator at 3 GHz. (a) $Z_L = 150 - j200\ \Omega$ and (b) $Z_L = 20 - j90\ \Omega$.

5.2 We have seen that the matching of an arbitrary load impedance requires a network with at least two degrees of freedom. Determine the types of load impedances/admittances that can be matched with the two single-element networks shown below.



5.3 A load impedance $Z_L = 100 + j80\ \Omega$ is to be matched to a 75 Ω line using a single shunt-stub tuner. Find two designs using open-circuited stubs.

5.4 Repeat Problem 5.3 using short-circuited stubs.