## Lec15 Impedance Matching and Tuning（II）阻抗匹配和调谐

## Series Stubs（串行短截线）

## EXAMPLE 5．3 SINGLE－STUB SERIES TUNING

Match a load impedance of $Z_{L}=100+\mathrm{j} 80$ to a 50 ohm line using a single series open－circuit stub．Assuming that the load is matched at 2 GHz and that the load consists of a resistor and inductor in series，plot the reflection coefficient magnitude from 1 to 3 GHz ．

## Smith chart Solution

1）First plot the normalized load impedance， $\mathrm{zL}=2+j 1.6$ ，and draw the SWR circle．
2）The SWR circle intersects the $1+j x$ circle at two points，denoted as $z 1$ and $z 2$ ．

3) The shortest distance, $d l$, from the load to the stub is, from the WTG scale,

$$
d_{1}=0.328-0.208=0.120 \lambda,
$$

and the second distance is

$$
d_{2}=(0.5-0.208)+0.172=0.463 \lambda .
$$

The normalized impedances at the two intersection points are

$$
\begin{aligned}
& z_{1}=1-j 1.33 \\
& z_{2}=1+j 1.33 .
\end{aligned}
$$

4) the first solution requires a stub with a reactance of $j 1.33$.

The length of an open-circuited stub that gives this reactance can be found on the Smith chart by starting at $z=\infty$ (open circuit), and moving along the outer edge of the chart $(r=0)$ toward the generator to the $j 1.33$ point. This gives a stub length of

$$
\ell_{1}=0.397 \lambda .
$$

the required open-circuited stub length for the second solution is

$$
\ell_{2}=0.103 \lambda
$$

If the load is a series resistor and inductor with $\mathrm{ZL}=100+\mathrm{j} 80$ at 2 GHz , then $\mathrm{R}=100$ and $\mathrm{L}=6.37 \mathrm{nH}$. The two matching circuits are shown in the following figure.

## Analytical solution

let the load admittance be written as $\quad Y_{L}=1 / Z_{L}=G_{L}+j B_{L}$.
Then the admittance Y down a length d of line from the load is

$$
Y=Y_{0} \frac{\left(G_{L}+j B_{L}\right)+j t Y_{0}}{Y_{0}+j t\left(G_{L}+j B_{L}\right)}
$$

where $t=\tan \beta d$ and $Y_{0}=1 / Z_{0}$.



The impedance at this point is

$$
Z=R+j X=\frac{1}{Y},
$$

where

$$
\begin{aligned}
R & =\frac{G_{L}\left(1+t^{2}\right)}{G_{L}^{2}+\left(B_{L}+Y_{0} t\right)^{2}}, \\
X & =\frac{G_{L}^{2} t-\left(Y_{0}-t B_{L}\right)\left(B_{L}+t Y_{0}\right)}{Y_{0}\left[G_{L}^{2}+\left(B_{L}+Y_{0} t\right)^{2}\right]} .
\end{aligned}
$$

Now d (which implies t ) is chosen so that $\mathrm{R}=\mathrm{Z} 0=1 / \mathrm{Y} 0$.

$$
Y_{0}\left(G_{L}-Y_{0}\right) t^{2}-2 B_{L} Y_{0} t+\left(G_{L} Y_{0}-G_{L}^{2}-B_{L}^{2}\right)=0
$$

Solving for t gives

$$
\begin{aligned}
& t=\frac{B_{L} \pm \sqrt{G_{L}\left[\left(Y_{0}-G_{L}\right)^{2}+B_{L}^{2}\right] / Y_{0}}}{G_{L}-Y_{0}} \text { for } G_{L} \neq Y_{0} . \\
& \text { If } G_{L}=Y_{0} \text {, then } t=-B_{L} / 2 Y_{0} .
\end{aligned}
$$

$$
d / \lambda= \begin{cases}\frac{1}{2 \pi} \tan ^{-1} t & \text { for } t \geq 0 \\ \frac{1}{2 \pi}\left(\pi+\tan ^{-1} t\right) & \text { for } t<0\end{cases}
$$

The required stub lengths are determined by first using $t$ to find the reactance X . This reactance is the negative of the necessary stub reactance, Xs .

Thus, for a short circuited stub,

$$
\frac{\ell_{s}}{\lambda}=\frac{1}{2 \pi} \tan ^{-1}\left(\frac{X_{s}}{Z_{0}}\right)=\frac{-1}{2 \pi} \tan ^{-1}\left(\frac{X}{Z_{0}}\right),
$$

and for an open-circuited stub,

$$
\frac{\ell_{0}}{\lambda}=\frac{-1}{2 \pi} \tan ^{-1}\left(\frac{Z_{0}}{X_{s}}\right)=\frac{1}{2 \pi} \tan ^{-1}\left(\frac{Z_{0}}{X}\right) .
$$

If the length given is negative, $\lambda / 2$ can be added to give a positive result.

## 5．3 DOUBLE－STUB TUNING（双短截线调谐）

The single－stub tuner is able to match any load impedance（having a positive real part）to a transmission line，but suffers from the disadvantage of requiring a variable length of line between the load and the stub．

The double－stub tuner uses two tuning stubs in fixed positions．


## Smith Chart Solution



## 1) Find $y_{L}$

2) Rotate $1+j b$ circle, the amount of rotation is d wavelengths toward the load.
3) move the load admittance to $y 1$ and yl '.
4) Move y1 to $y 2$; move $y 1$ ' to $y 2$ '.
5) Move y2 and y2' to the center of smith chart.

The forbidden area is inside the shaded region of the $\mathrm{g} 0+\mathrm{jb}$ circle, no value of stub susceptance b1 could ever bring the load point to intersect the rotated $1+\mathrm{jb}$ circle.

A simple way of reducing the forbidden range is to reduce the distance $d$ between the stubs. This has the effect of swinging the rotated $1+j b$ circle back toward the $y$ $=\infty$ point, but d must be kept large enough for the practical purpose of fabricating the two separate stubs. In addition, stub spacings near 0 or $\lambda / 2$ lead to matching networks that are very frequency sensitive. In practice, stub spacings are usually chosen as $\lambda / \mathbf{8}$ or $3 \lambda / \mathbf{8}$.

## EXAMPLE 5.4 DOUBLE-STUB TUNING

Design a double-stub shunt tuner to match a load impedance $Z L=60-j 80$ to a 50 line. The stubs are to be open-circuited stubs and are spaced $\lambda / 8$ apart. Assuming that this load consists of a series resistor and capacitor and that the match frequency is 2 GHz , plot the reflection coefficient magnitude versus frequency from 1 to 3 GHz .


1) $y_{L}=0.3+j 0.4$ :
2) move yL to yl and yl'and then

$$
b_{1}=1.314 \quad \text { or } \quad b_{1}^{\prime}=-0.114
$$

3) transform through the $\lambda / 8$ section of line by rotating along a constant radius (SWR) circle $\lambda / 8$ toward the generator.

$$
y_{2}=1-j 3.38 \quad \text { or } \quad y_{2}^{\prime}=1+j 1.38
$$

4)Then the susceptance of the second stub should be

$$
b_{2}=3.38 \quad \text { or } \quad b_{2}^{\prime}=-1.38
$$

The lengths of the open-circuited stubs are then found as

$$
\ell_{1}=0.146 \lambda, \ell_{2}=0.204 \lambda \quad \text { or } \quad \ell_{1}^{\prime}=0.482 \lambda, \ell_{2}^{\prime}=0.350 \lambda
$$

This completes both solutions for the double-stub tuner design.
At $\mathrm{f}=2 \mathrm{GHz}$ the resistor-capacitor load of $\mathrm{ZL}=60-\mathrm{j} 80$ implies that $\mathrm{R}=60$ and $\mathrm{C}=0.995 \mathrm{pF}$.

(b)

(c)

Note that the first solution has a much narrower bandwidth than the second (primed) solution due to the fact that both stubs for the first solution are somewhat longer (and closer to $\lambda / 2$ ) than the stubs of the second solution.

## Analytic Solution

The admittance just to the left of the first stub is

$$
Y_{1}=G_{L}+j\left(B_{L}+B_{1}\right),
$$

where $\mathrm{YL}=\mathrm{GL}+\mathrm{j}$ BL is the load admittance, and B1 is the susceptance of the first stub.
After transforming through a length $d$ of transmission line, we find that the admittance just to the right of the second stub is

$$
Y_{2}=Y_{0} \frac{G_{L}+j\left(B_{L}+B_{1}+Y_{0} t\right)}{Y_{0}+j t\left(G_{L}+j B_{L}+j B_{1}\right)},
$$

where $t=\tan \beta d$ and $Y_{0}=1 / Z_{0}$.
At this point the real part of Y2 must equal Y0,

$$
G_{L}^{2}-G_{L} Y_{0} \frac{1+t^{2}}{t^{2}}+\frac{\left(Y_{0}-B_{L} t-B_{1} t\right)^{2}}{t^{2}}=0 .
$$

Solving for $G L$ gives $\quad G_{L}=Y_{0} \frac{1+t^{2}}{2 t^{2}}\left[1 \pm \sqrt{1-\frac{4 t^{2}\left(Y_{0}-B_{L} t-B_{1} t\right)^{2}}{Y_{0}^{2}\left(1+t^{2}\right)^{2}}}\right]$.
Because GL is real, the quantity within the square root must be nonnegative, and so

$$
0 \leq \frac{4 t^{2}\left(Y_{0}-B_{L} t-B_{1} t\right)^{2}}{Y_{0}^{2}\left(1+t^{2}\right)^{2}} \leq 1 .
$$

This implies that

$$
0 \leq G_{L} \leq Y_{0} \frac{1+t^{2}}{t^{2}}=\frac{Y_{0}}{\sin ^{2} \beta d}
$$

which gives the range on GL that can be matched for a given stub spacing d .
After $d$ has been set, the first stub susceptance can be determined as

$$
B_{1}=-B_{L}+\frac{Y_{0} \pm \sqrt{\left(1+t^{2}\right) G_{L} Y_{0}-G_{L}^{2} t^{2}}}{t} .
$$

Then the second stub susceptance can be found from the negative of the imaginary part of Y2

$$
B_{2}=\frac{ \pm Y_{0} \sqrt{Y_{0} G_{L}\left(1+t^{2}\right)-G_{L}^{2} t^{2}}+G_{L} Y_{0}}{G_{L} t}
$$

The open-circuited stub length is found as

$$
\frac{\ell_{0}}{\lambda}=\frac{1}{2 \pi} \tan ^{-1}\left(\frac{B}{Y_{0}}\right)
$$

and the short-circuited stub length is found as

$$
\frac{\ell_{s}}{\lambda}=\frac{-1}{2 \pi} \tan ^{-1}\left(\frac{Y_{0}}{B}\right)
$$

where $\mathrm{B}=\mathrm{B} 1$ or B 2 .

## Homework

5.9 Design a double-stub tuner using open-circuited stubs with a $\lambda / 8$ spacing to match a load admittance $Y_{L}=(0.4+j 1.2) Y_{0}$.

