Lec17 Microwave Resonators (II)
微波谐振器

6.3 RECTANGULAR WAVEGUIDE CAVITY RESONATORS

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Microwave resonators can also be constructed **from closed sections of waveguide**.

➢Because radiation loss from an open-ended waveguide can be significant, waveguide resonators are usually **short circuited at both ends**, thus forming a closed box, or **cavity**.

➢**Electric and magnetic energy** is **stored** within the cavity enclosure, and power is **dissipated** in the metallic walls of the cavity as well as in the dielectric material that may fill the cavity.

Main contents:

1) the resonant frequencies for a general TE or TM resonant mode

2) an expression for the unloaded *Q* of the TE10 mode.

Summary of Results for Rectangular Waveguide

Quantity	TE_{mn} Mode	TM_{mn} Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	$\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$	$\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$
β	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$
λ_c	$\frac{2\pi}{k_c}$	$\frac{2\pi}{k_c}$
λ_g	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$
v_p	$\frac{\omega}{\beta}$	$\frac{\omega}{\beta}$
α_d	$\frac{k^2 \tan\delta}{2\beta}$	$\frac{k^2 \tan\delta}{2\beta}$

Quantity	TE_{mn} Mode	TM_{mn} Mode
H_z	$A\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	0
E_x	$\frac{j\omega\mu n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\beta m\pi}{k_c^2 a}B\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
E_y	$\frac{-j\omega\mu m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\beta n\pi}{k_c^2 b}B\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
H_x	$\frac{j\beta m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{j\omega\epsilon n\pi}{k_c^2 b}B\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
H_y	$\frac{j\beta n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\omega\epsilon m\pi}{k_c^2 a}B\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
Z	$Z_{\text{TE}} = \frac{k\eta}{\beta}$	$Z_{\text{TM}} = \frac{\beta\eta}{k}$

Resonant Frequencies

The resonant frequencies of this cavity are found under the assumption that the cavity is lossless. Q is determined using the perturbation method.

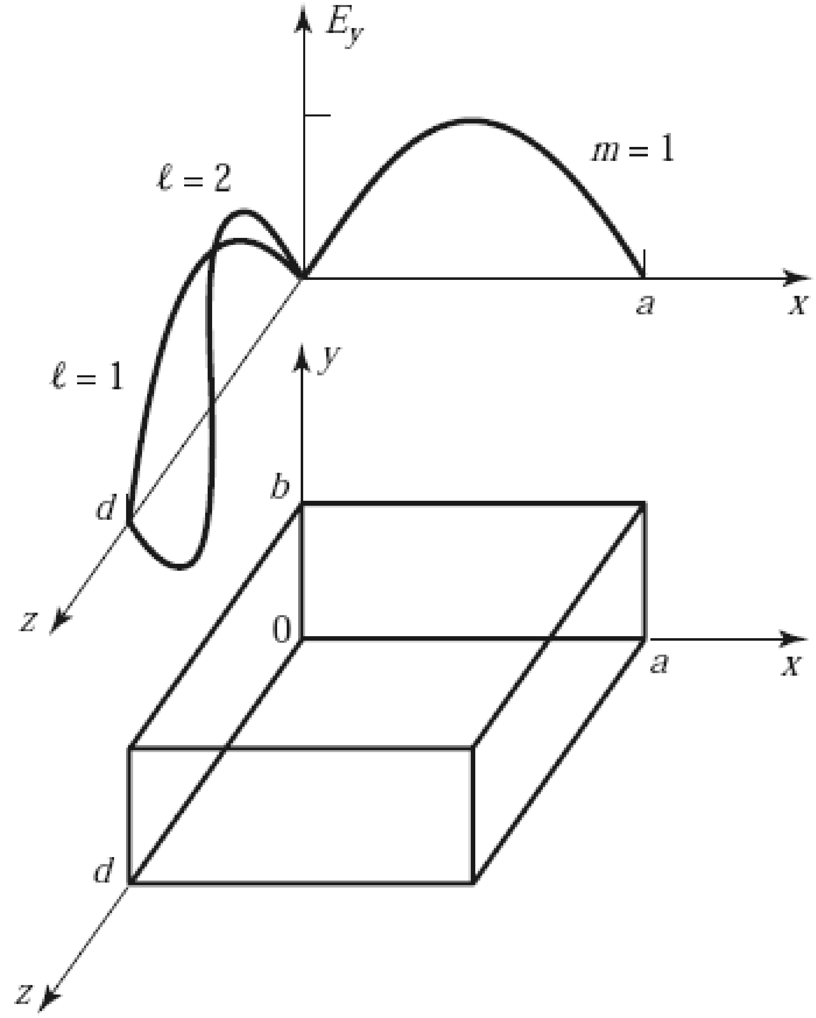
The transverse electric fields (Ex , Ey) of the TEmn or TMmn rectangular waveguide mode can be written as

$$\bar{E}_t(x, y, z) = \bar{e}(x, y) \left(A^+ e^{-j\beta_{mn}z} + A^- e^{j\beta_{mn}z}\right),$$

The propagation constant of the *m, nth TE* or TM mode is

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2},$$

where $k = \omega\sqrt{\mu\epsilon}$, and μ and ϵ are the permeability and permittivity of the material filling the cavity.



A rectangular cavity resonator, and the electric field variations for the TE101 and TE102 resonant modes.

Applying the condition that $\bar{E}_t = 0$ at $z = 0$

$$A^+ = -A^-$$

Then the condition that $\bar{E}_t = 0$ at $z = d$ leads to the equation

$$\bar{E}_t(x, y, d) = -\bar{e}(x, y) A^+ 2j \sin \beta_{mn} d = 0.$$

The only nontrivial (*A+ = 0)* solution occurs for

$$\beta_{mn} d = \ell\pi, \quad \ell = 1, 2, 3, \ldots,$$

which implies that the cavity must be an integer multiple of **a half-guide wavelength long at the resonant frequency**.

A resonance wave number for the rectangular cavity can be defined as

$$k_{mn\ell} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2}.$$

TE_{mnl} or TM_{mnl} are resonant modes of the cavity, where *m*, *n*, *l* indicate the number of variations in the standing wave pattern in the x, y, z directions, respectively.

$$f_{mn\ell} = \frac{ck_{mn\ell}}{2\pi\sqrt{\mu_r\epsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2}.$$

If $b < a < d$, **the dominant resonant mode** (lowest resonant frequency) will be **the TE101 mode**, corresponding to the **TE10 dominant waveguide mode in a shorted guide of length λg/2**, and is **similar to the short-circuited λ/2 transmission line resonator**.

Unloaded Q of the TE10 *Mode*

The total fields for the TE10 *resonant* mode can be written as

$$E_y = A^+ \sin\frac{\pi x}{a}\left(e^{-j\beta z} - e^{j\beta z}\right),$$

$$H_x = \frac{-A^+}{Z_{\mathrm{TE}}}\sin\frac{\pi x}{a}\left(e^{-j\beta z} + e^{j\beta z}\right),$$

$$H_z = \frac{j\pi A^+}{k\eta a}\cos\frac{\pi x}{a}\left(e^{-j\beta z} - e^{j\beta z}\right).$$

Letting $E_0 = -2jA^+$

$$E_y = E_0 \sin\frac{\pi x}{a}\sin\frac{\ell\pi z}{d},$$

$$H_x = \frac{-jE_0}{Z_{\mathrm{TE}}}\sin\frac{\pi x}{a}\cos\frac{\ell\pi z}{d},$$

$$H_z = \frac{j\pi E_0}{k\eta a}\cos\frac{\pi x}{a}\sin\frac{\ell\pi z}{d},$$

The fields form standing waves inside the cavity.

The stored electric energy is,

$$W_e = \frac{\epsilon}{4} \int_V E_y E_y^* dv = \frac{\epsilon abd}{16} E_0^2,$$

and the stored magnetic energy is,

$$W_m = \frac{\mu}{4} \int_V (H_x H_x^* + H_z H_z^*) dv$$

$$= \frac{\mu abd}{16} E_0^2 \left(\frac{1}{Z_{\text{TE}}^2} + \frac{\pi^2}{k^2 \eta^2 a^2} \right).$$

Because $Z_{\text{TE}} = k\eta/\beta$, with $\beta = \beta_{10} = \sqrt{k^2 - (\pi/a)^2}$,

$$\left(\frac{1}{Z_{\text{TE}}^2} + \frac{\pi^2}{k^2 \eta^2 a^2} \right) = \frac{\beta^2 + (\pi/a)^2}{k^2 \eta^2} = \frac{1}{\eta^2} = \frac{\epsilon}{\mu},$$

We = Wm at resonance.

For small losses we can find the power dissipated in the cavity walls using the perturbation method.

The power lost in the conducting walls is

$$P_c = \frac{R_s}{2} \int_{\text{walls}} |H_t|^2 ds,$$

where $R_s = \sqrt{\omega\mu_0/2\sigma}$ is the surface resistivity of the metallic walls,

and *Ht is the* tangential magnetic field at the surface of the walls.

$$\begin{aligned} P_c = \frac{R_s}{2} \bigg\{ & 2 \int_{y=0}^{b} \int_{x=0}^{a} |H_x(z=0)|^2 dxdy + 2 \int_{z=0}^{d} \int_{y=0}^{b} |H_z(x=0)|^2 dydz \\ & + 2 \int_{z=0}^{d} \int_{x=0}^{a} \left[|H_x(y=0)|^2 + |H_z(y=0)|^2 \right] dxdz \bigg\} \\ = & \frac{R_s E_0^2 \lambda^2}{8\eta^2} \left(\frac{\ell^2 ab}{d^2} + \frac{bd}{a^2} + \frac{\ell^2 a}{2d} + \frac{d}{2a} \right), \end{aligned} \tag{6.45}$$

The unloaded Q of the cavity with lossy conducting walls but lossless dielectric can be found as

$$Q_c = \frac{2\omega_0 W_e}{P_c}$$

$$= \frac{k^3 abd\eta}{4\pi^2 R_s} \frac{1}{[(\ell^2 ab/d^2) + (bd/a^2) + (\ell^2 a/2d) + (d/2a)]}$$

$$= \frac{(kad)^3 b\eta}{2\pi^2 R_s} \frac{1}{(2\ell^2 a^3 b + 2bd^3 + \ell^2 a^3 d + ad^3)}.$$

Next we compute the power lost in the dielectric material that may fill the cavity.

A lossy dielectric has $\epsilon = \epsilon' - j\epsilon'' = \epsilon_r \epsilon_0 (1 - j \tan\delta)$,

The power dissipated in the dielectric is,

$$P_d = \frac{1}{2} \int_V \bar{J} \cdot \bar{E}^* dv = \frac{\omega\epsilon''}{2} \int_V |\bar{E}|^2 dv = \frac{abd\omega\epsilon''|E_0|^2}{8},$$

The unloaded Q of the cavity with a lossy dielectric filling, but with perfectly conducting walls, is

$$Q_d = \frac{2\omega W_e}{P_d} = \frac{\epsilon'}{\epsilon''} = \frac{1}{\tan\delta}.$$

When both wall losses and dielectric losses are present, the total power loss is $P_c + P_d$, so the total unloaded Q as

$$Q_0 = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}.$$

EXAMPLE 6.3 DESIGN OF A RECTANGULAR CAVITY RESONATOR

A rectangular waveguide cavity is made from a piece of copper WR-187 H-band waveguide, with $a = 4.755$ cm and $b = 2.215$ cm. The cavity is filled with polyethylene ($\epsilon_r = 2.25$, $\tan\delta = 0.0004$). If resonance is to occur at $f = 5$ GHz, find the required length, d, and the resulting unloaded Q for the $\ell = 1$ and $\ell = 2$ resonant modes.

Solution

The wave number *k is*

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = 157.08 \text{ m}^{-1}.$$

the required length for resonance *(m = 1, n = 0)*

$$d = \frac{\ell\pi}{\sqrt{k^2 - (\pi/a)^2}},$$

$$\text{for } \ell = 1, \quad d = \frac{\pi}{\sqrt{(157.08)^2 - (\pi/0.04755)^2}} = 2.20 \text{ cm},$$

$$\text{for } \ell = 2, \quad d = 2(2.20) = 4.40 \text{ cm}.$$

From Example 6.1, the surface resistivity of copper at 5 GHz is $R_s = 1.84 \times 10^{-2}\ \Omega$. The intrinsic impedance is

$$\eta = \frac{377}{\sqrt{\epsilon_r}} = 251.3\ \Omega.$$

The Q due to conductor loss only is

$$\text{for } \ell = 1, \quad Q_c = 8{,}403,$$
$$\text{for } \ell = 2, \quad Q_c = 11{,}898.$$

the Q due to dielectric loss only is, for both l = 1 and l= 2,

$$Q_d = \frac{1}{\tan\delta} = \frac{1}{0.0004} = 2500.$$

Then total unloaded *Qs are*,

$$\text{for } \ell = 1, \quad Q_0 = \left(\frac{1}{8403} + \frac{1}{2500}\right)^{-1} = 1927,$$
$$\text{for } \ell = 2, \quad Q_0 = \left(\frac{1}{11{,}898} + \frac{1}{2500}\right)^{-1} = 2065.$$

6.4 CIRCULAR WAVEGUIDE CAVITY RESONATORS

A cylindrical cavity resonator can be constructed from a section of circular waveguide shorted at both ends.

Because the dominant circular waveguide mode is the TE11 mode, the dominant cylindrical cavity mode is the TE111 mode.

Circular cavities are often used for microwave frequency meters.

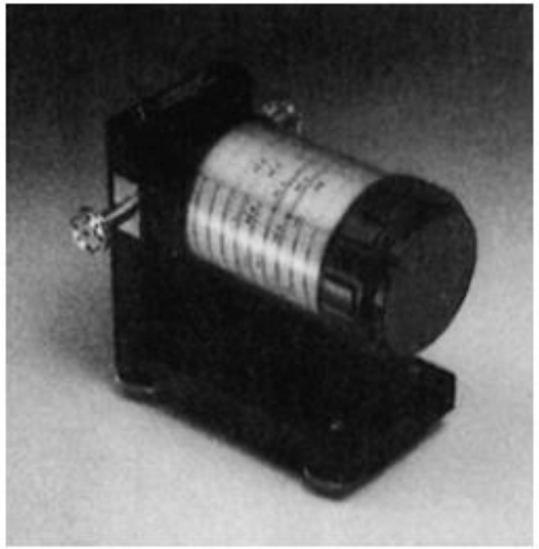
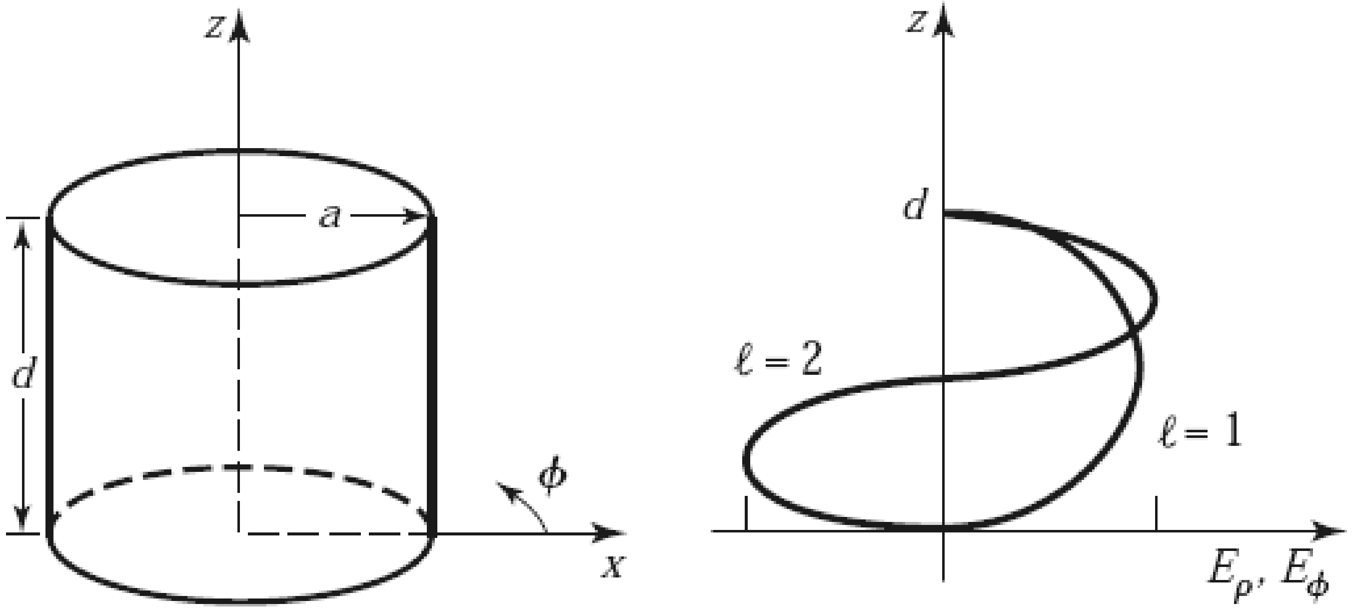


TABLE 3.5 Summary of Results for Circular Waveguide

Quantity	TE_{nm} Mode	TM_{nm} Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	$\dfrac{p'_{nm}}{a}$	$\dfrac{p_{nm}}{a}$
β	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$
λ_c	$\dfrac{2\pi}{k_c}$	$\dfrac{2\pi}{k_c}$
λ_g	$\dfrac{2\pi}{\beta}$	$\dfrac{2\pi}{\beta}$
v_p	$\dfrac{\omega}{\beta}$	$\dfrac{\omega}{\beta}$
α_d	$\dfrac{k^2 \tan\delta}{2\beta}$	$\dfrac{k^2 \tan\delta}{2\beta}$

Quantity	TE_{nm} Mode	TM_{nm} Mode
E_z	0	$(A\sin n\phi + B\cos n\phi)J_n(k_c\rho)e^{-j\beta z}$
H_z	$(A\sin n\phi + B\cos n\phi)J_n(k_c\rho)e^{-j\beta z}$	0
E_ρ	$\frac{-j\omega\mu n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$	$\frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$
E_ϕ	$\frac{j\omega\mu}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$	$\frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$
H_ρ	$\frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$	$\frac{j\omega\epsilon n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$
H_ϕ	$\frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$	$\frac{-j\omega\epsilon}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$
Z	$Z_{\mathrm{TE}} = \frac{k\eta}{\beta}$	$Z_{\mathrm{TM}} = \frac{\beta\eta}{k}$

Resonant Frequencies



A cylindrical resonant cavity, and the electric field distribution for resonant modes with $\ell = 1$ or $\ell = 2$.

the transverse electric fields ($E\rho$, $E\varphi$) *of the TEnm or TMnm circular* waveguide mode can be written as

$$\bar{E}_t(\rho, \phi, z) = \bar{e}(\rho, \phi)\left(A^+ e^{-j\beta_{nm} z} + A^- e^{j\beta_{nm} z}\right),$$

where $\bar{e}(\rho, \phi)$ represents the transverse variation of the mode, and A^+ and A^- are arbitrary amplitudes of the forward and backward traveling waves. The propagation constant of the

TEnm mode is,

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2},$$

while the propagation constant of the TM*nm mode is,*

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{p_{nm}}{a}\right)^2},$$

where $k = \omega\sqrt{\mu\epsilon}$.

In order to have $\bar{E}_t = 0$ at $z = 0, d$, we must choose $A^+ = -A^-$, and $A^+ \sin\beta_{nm}$ $d = 0$,

$$\beta_{nm} d = \ell\pi, \quad \text{for } \ell = 0, 1, 2, 3, \ldots,$$

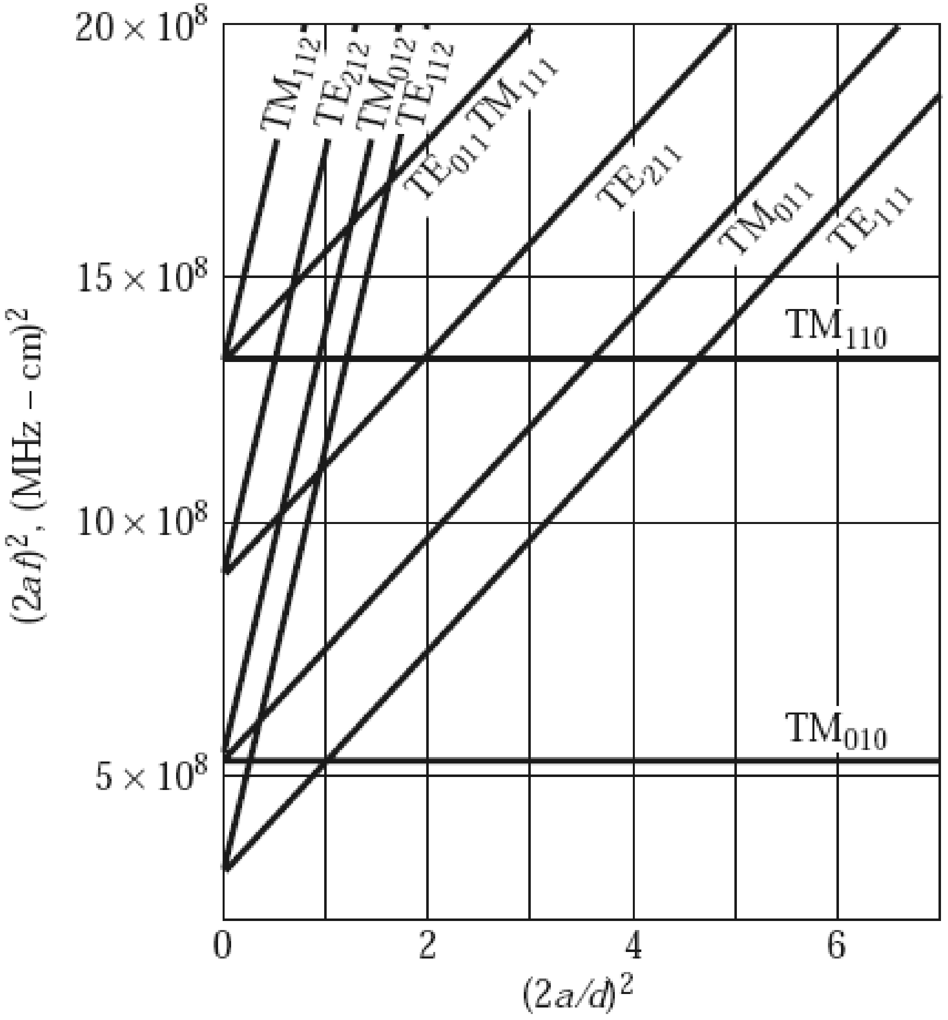
which implies that the waveguide must be an integer number of half-guide wavelengths long.

The resonant frequency of the TE*nm mode is*

$$f_{nm\ell} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2},$$

the resonant frequency of the TM*nm mode is*

$$f_{nm\ell} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2}.$$



Thus the dominant TE mode is the TE111 mode, while the dominant TM mode is the TM010 mode.

Resonant mode chart for a cylindrical cavity.

Unloaded Q of the TEnm *Mode*

From the fact that $A+ = -A-$, *the fields of the TEnm mode can be* written as

$$H_z = H_0 J_n\left(\frac{p'_{nm}\rho}{a}\right)\cos n\phi \sin\frac{\ell\pi z}{d},$$

$$H_\rho = \frac{\beta a H_0}{p'_{nm}} J'_n\left(\frac{p'_{nm}\rho}{a}\right)\cos n\phi \cos\frac{\ell\pi z}{d},$$

$$H_\phi = \frac{-\beta a^2 n H_0}{(p'_{nm})^2\rho} J_n\left(\frac{p'_{nm}\rho}{a}\right)\sin n\phi \cos\frac{\ell\pi z}{d},$$

$$E_\rho = \frac{jk\eta a^2 n H_0}{(p'_{nm})^2\rho} J_n\left(\frac{p'_{nm}\rho}{a}\right)\sin n\phi \sin\frac{\ell\pi z}{d},$$

$$E_\phi = \frac{jk\eta a H_0}{p'_{nm}} J'_n\left(\frac{p'_{nm}\rho}{a}\right)\cos n\phi \sin\frac{\ell\pi z}{d},$$

$$E_z = 0,$$

where $\eta = \sqrt{\mu/\epsilon}$ and $H_0 = -2jA^+$.

Because the time-average stored electric and magnetic energies are equal, the total stored energy is

$$W = 2W_e = \frac{\epsilon}{2}\int_{z=0}^{d}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\left(|E_\rho|^2 + |E_\phi|^2\right)\rho d\rho d\phi dz$$
$$= \frac{\epsilon k^2\eta^2 a^2 \pi d H_0^2}{4(p'_{nm})^2}\int_{\rho=0}^{a}\left[J_n'^2\left(\frac{p'_{nm}\rho}{a}\right) + \left(\frac{na}{p'_{nm}\rho}\right)^2 J_n^2\left(\frac{p'_{nm}\rho}{a}\right)\right]\rho d\rho$$
$$= \frac{\epsilon k^2\eta^2 a^4 H_0^2 \pi d}{8(p'_{nm})^2}\left[1 - \left(\frac{n}{p'_{nm}}\right)^2\right]J_n^2(p'_{nm}),$$

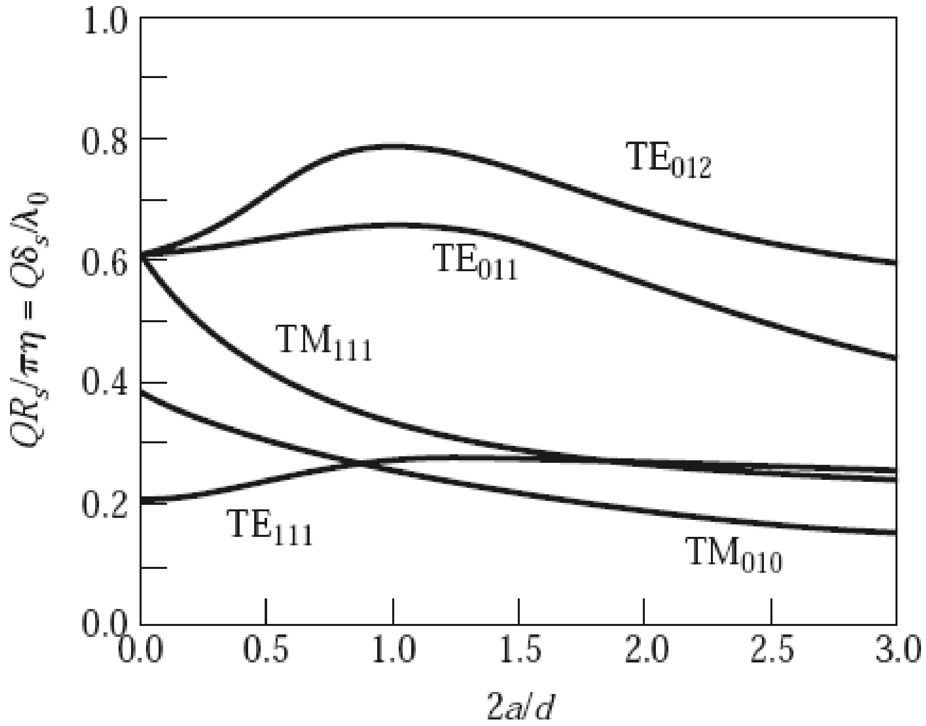
The power loss in the conducting walls is

$$P_c = \frac{R_s}{2}\int_S |\bar{H}_{\tan}|^2 ds$$
$$= \frac{R_s}{2}\left\{\int_{z=0}^{d}\int_{\phi=0}^{2\pi}\left[|H_\phi(\rho=a)|^2 + |H_z(\rho=a)|^2\right] a d\phi dz\right.$$
$$\left. + 2\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\left[|H_\rho(z=0)|^2 + |H_\phi(z=0)|^2\right]\rho d\rho d\phi\right\}$$
$$= \frac{R_s}{2}\pi H_0^2 J_n^2(p'_{nm})\left\{\frac{da}{2}\left[1 + \left(\frac{\beta a n}{(p'_{nm})^2}\right)^2\right] + \left(\frac{\beta a^2}{p'_{nm}}\right)^2\left(1 - \frac{n^2}{(p'_{nm})^2}\right)\right\}.$$

the unloaded Q of the cavity with imperfectly conducting walls but lossless dielectric is

$$Q_c = \frac{\omega_0 W}{P_c} = \frac{(ka)^3 \eta a d}{4(p'_{nm})^2 R_s} \frac{1 - \left(\frac{n}{p'_{nm}}\right)^2}{\left\{\frac{ad}{2}\left[1 + \left(\frac{\beta a n}{(p'_{nm})^2}\right)^2\right] + \left(\frac{\beta a^2}{p'_{nm}}\right)^2 \left(1 - \frac{n^2}{(p'_{nm})^2}\right)\right\}}.$$

the frequency dependence of Qc is given by k/Rs , which varies as 1/√ f ; this gives the variation in Qc for a given resonant mode and cavity shape.



the TE011 mode has an unloaded *Q* significantly higher than that of the lower order TE111, TM010, or TM111 mode.

To compute the unloaded Q due to dielectric loss, we must compute the power dissipated in the dielectric. Thus,

$$P_d = \frac{1}{2}\int_V \bar{J}\cdot\bar{E}^* dv = \frac{\omega\epsilon''}{2}\int_V \left[|E_\rho|^2 + |E_\phi|^2\right] dv$$

$$= \frac{\omega\epsilon'' k^2\eta^2 a^2 H_0^2 \pi d}{4(p'_{nm})^2}\int_{\rho=0}^{a}\left[\left(\frac{na}{p'_{nm}\rho}\right)^2 J_n^2\left(\frac{p'_{nm}\rho}{a}\right) + J_n'^2\left(\frac{p'_{nm}\rho}{a}\right)\right]\rho d\rho$$

$$= \frac{\omega\epsilon'' k^2\eta^2 a^4 H_0^2}{8(p'_{nm})^2}\left[1 - \left(\frac{n}{p'_{nm}}\right)^2\right] J_n^2(p'_{nm}).$$

the unloaded *Q* due to dielectric loss is

$$Q_d = \frac{\omega W}{P_d} = \frac{\epsilon}{\epsilon''} = \frac{1}{\tan\delta},$$

EXAMPLE 6.4 DESIGN OF A CIRCULAR CAVITY RESONATOR

A circular cavity resonator with $d = 2a$ is to be designed to resonate at 5.0 GHz in the TE_{011} mode. If the cavity is made from copper and is Teflon filled ($\epsilon_r = 2.08$, $\tan\delta = 0.0004$), find its dimensions and unloaded Q.

Solution

$$k = \frac{2\pi f_{011}\sqrt{\epsilon_r}}{c} = \frac{2\pi(5 \times 10^9)\sqrt{2.08}}{3 \times 10^8} = 151.0 \text{ m}^{-1}$$

the resonant frequency of the TE011 mode is

$$f_{011} = \frac{c}{2\pi\sqrt{\epsilon_r}}\sqrt{\left(\frac{p'_{01}}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2},$$

with $p'_{01} = 3.832$. Then, since $d = 2a$

$$\frac{2\pi f_{011}\sqrt{\epsilon_r}}{c} = k = \sqrt{\left(\frac{p'_{01}}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2}.$$

Solving for a gives

$$a = \frac{\sqrt{(p'_{01})^2 + (\pi/2)^2}}{k} = \frac{\sqrt{(3.832)^2 + (\pi/2)^2}}{151.0} = 2.74 \text{ cm},$$

so we have $d = 5.48$ cm.

The surface resistivity of copper at 5 GHz is $R_s = 0.0184\ \Omega$. Then from (6.57), with $n = 0$, $m = \ell = 1$, and $d = 2a$, the unloaded Q due to conductor losses is

$$Q_c = \frac{(ka)^3 \eta a d}{4(p'_{01})^2 R_s} \frac{1}{[ad/2 + (\beta a^2/p'_{01})^2]} = \frac{ka\eta}{2R_s} = 29{,}390,$$

$$Q_d = \frac{1}{\tan\delta} = \frac{1}{0.0004} = 2500,$$

$$Q_0 = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1} = 2300.$$

Homework

6.9 A rectangular cavity resonator is constructed from a 2.0 cm length of aluminum X-band waveguide. The cavity is air filled. Find the resonant frequency and unloaded Q of the TE_{101} and TE_{102} resonant modes.

6.15 An air-filled rectangular cavity resonator has its first three resonant modes at the frequencies 5.2, 6.5, and 7.2 GHz. Find the dimensions of the cavity.