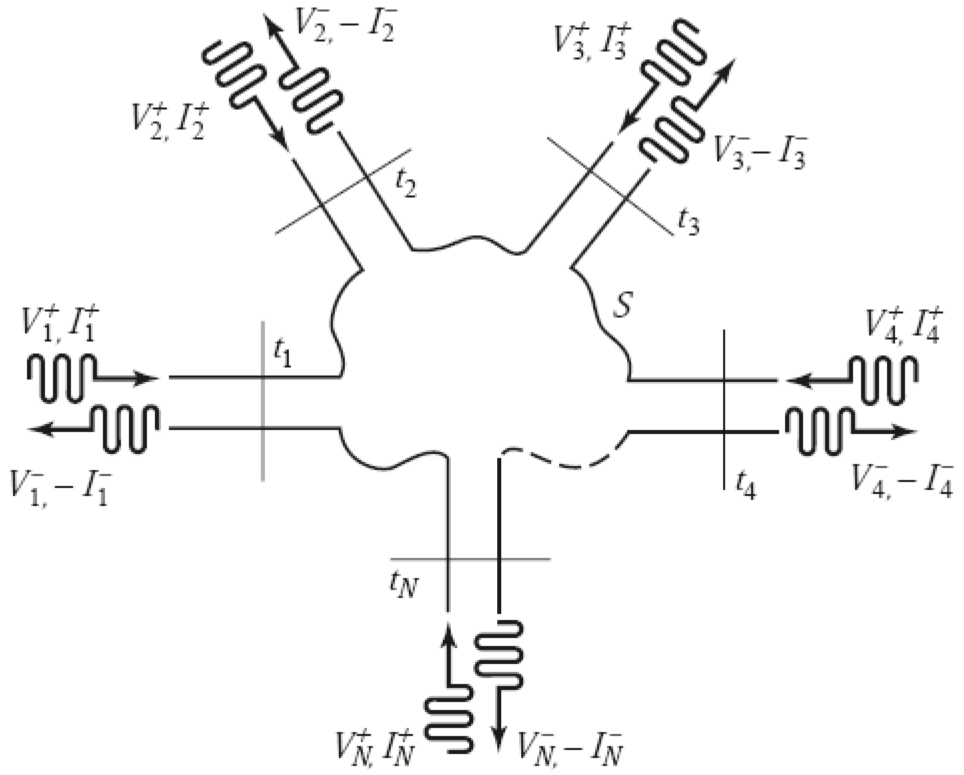
Lec10 Microwave Network Analysis （II）

4.2 IMPEDANCE AND ADMITTANCE MATRICES

Equivalent voltages and currents can be defined for TEM and non-TEM waves in a microwave network. Then, we can use the impedance and/or admittance matrices of circuit theory to relate these terminal or port quantities to each other,



an arbitrary N-port microwave network

At a specific point on the *nth port, a* terminal plane, *tn,* is defined along with equivalent voltages and currents for the incident (V^+_n, I^+_n) and reflected (V^-_n, I^-_n) waves.

$$V_n = V_n^+ + V_n^-,$$

$$I_n = I_n^+ - I_n^-,$$

The impedance matrix [Z] of the microwave network then relates these voltages and currents:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

or in matrix form as $[V] = [Z][I].$

Similarly, we can define an admittance matrix [Y] as

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & & & \vdots \\ \vdots & & & \vdots \\ Y_{N1} & \cdots & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

or in matrix form as $[I] = [Y][V].$

[Z] and [Y] matrices are the inverses of each other: $[Y] = [Z]^{-1}.$

Z_{ij} can be found by driving port j with the current I_j , open circuiting all other ports (so $I_k = 0$ for $k \neq j$), and measuring the open-circuit voltage at port i .

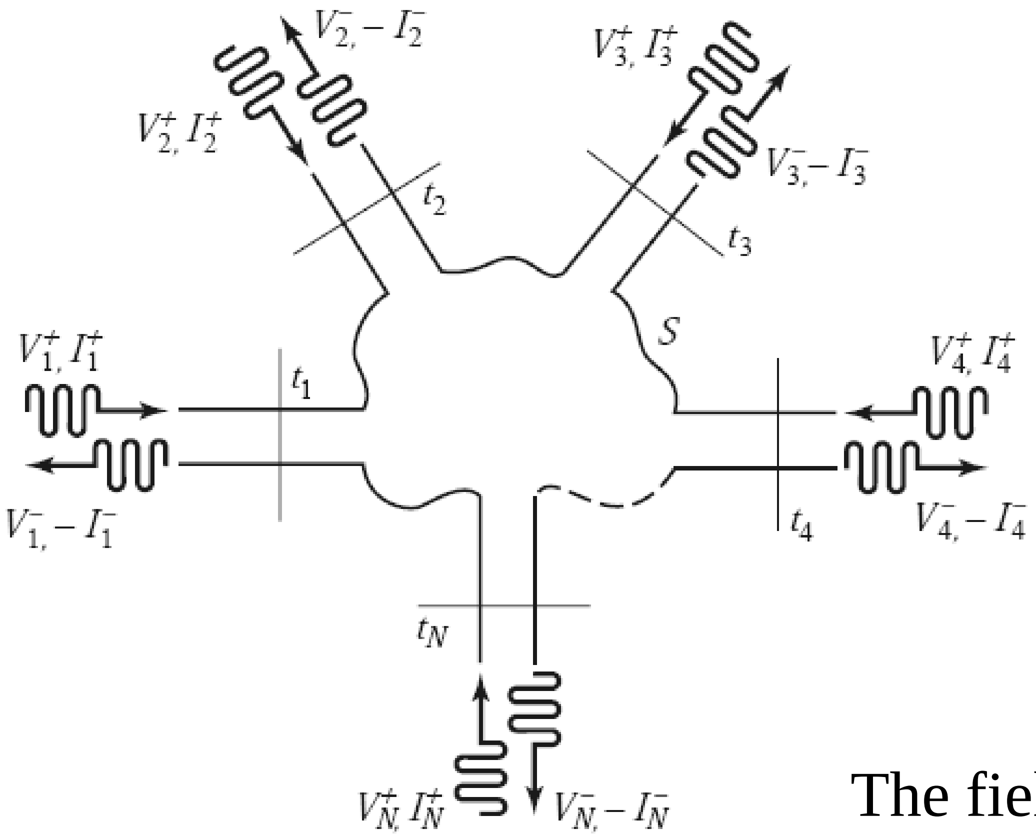
$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k = 0 \text{ for } k \neq j}.$$

Similaly, Y_{ij} can be determined by driving port j with the voltage V_j , short circuiting all other ports (so $V_k = 0$ for $k \neq j$), and measuring the short-circuit current at port i .

In general, each Z_{ij} or Y_{ij} element may be complex. For an arbitrary N-port network, the impedance and admittance matrices are $N \times N$ in size, so there are $2N^2$ independent quantities or degrees of freedom.

- If the network is reciprocal (not containing any active devices or nonreciprocal media, such as ferrites or plasmas), the impedance and admittance matrices are symmetric, so that $Z_{ij} = Z_{ji}$, and $Y_{ij} = Y_{ji}$.
- If the network is lossless, we can show that all the Z_{ij} or Y_{ji} elements are purely imaginary.

Reciprocal Networks （互易网络）



Consider the arbitrary network to be reciprocal with **short circuits** placed at all terminal planes except those of ports 1 and 2.
Let E_a, H_a and E_b, H_b be the fields in the network due **to two independent sources a and b**.

From the reciprocity theorem

$$\oint_S \bar{E}_a \times \bar{H}_b \cdot d\bar{s} = \oint_S \bar{E}_b \times \bar{H}_a \cdot d\bar{s},$$

The fields due to sources *a* and *b* at the terminal planes *t1 and t2* are

$$\bar{E}_{1a} = V_{1a}\bar{e}_1,\ \bar{H}_{1a} = I_{1a}\bar{h}_1,$$
$$\bar{E}_{1b} = V_{1b}\bar{e}_1,\ \bar{H}_{1b} = I_{1b}\bar{h}_1,$$
$$\bar{E}_{2a} = V_{2a}\bar{e}_2,\ \bar{H}_{2a} = I_{2a}\bar{h}_2,$$
$$\bar{E}_{2b} = V_{2b}\bar{e}_2,\ \bar{H}_{2b} = I_{2b}\bar{h}_2,$$

Substituting the fields into the reciprocity equation gives

$$(V_{1a}I_{1b} - V_{1b}I_{1a}) \int_{S_1} \bar{e}_1 \times \bar{h}_1 \cdot d\bar{s} + (V_{2a}I_{2b} - V_{2b}I_{2a}) \int_{S_2} \bar{e}_2 \times \bar{h}_2 \cdot d\bar{s} = 0,$$

where S1 and S2 are the cross-sectional areas at the terminal planes of ports 1 and 2.

Since $$\int_{S_1} \bar{e}_1 \times \bar{h}_1 \cdot d\bar{s} = \int_{S_2} \bar{e}_2 \times \bar{h}_2 \cdot d\bar{s} = 1.$$

Then $$V_{1a}I_{1b} - V_{1b}I_{1a} + V_{2a}I_{2b} - V_{2b}I_{2a} = 0.$$

Now use the 2 × 2 admittance matrix of the (effectively) two-port network to eliminate the I_{1a}, I_{1b}, I_{2a}, I_{2b}, :

$$I_1 = Y_{11}V_1 + Y_{12}V_2,$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2.$$

Then $$(V_{1a}V_{2b} - V_{1b}V_{2a})(Y_{12} - Y_{21}) = 0.$$

Because the sources *a and b* are independent, the voltages V_{1a}, V_{1b}, V_{2a}, and V_{2b} can take on arbitrary values. We must have $\boldsymbol{Y}_{12} = \boldsymbol{Y}_{\mathbf{21}}$

Since the choice of which ports are labeled as 1 and 2 is arbitrary, we have the general result that

$$Y_{ij} = Y_{ji}.$$

Then if $[Y]$ is a symmetric matrix, its inverse, $[Z]$, is also symmetric.

Lossless Networks

If the network is lossless, then the net real power delivered to the network must be zero. Thus, Re{P_{avg}} = *0, where*

$$P_{\text{avg}} = \frac{1}{2}[V]^t[I]^* = \frac{1}{2}([Z][I])^t[I]^* = \frac{1}{2}[I]^t[Z][I]^* \qquad t$$

$$= \frac{1}{2}(I_1 Z_{11} I_1^* + I_1 Z_{12} I_2^* + I_2 Z_{21} I_1^* + \cdots)$$

$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} I_m Z_{mn} I_n^*.$$

更正公式（4.37）

Since we could set all port currents equal to zero except for the *nth current*. *So*,

$$\mathrm{Re}\{I_n Z_{nn} I_n^*\} = |I_n|^2 \,\mathrm{Re}\{Z_{nn}\} = 0,$$

or

$$\mathrm{Re}\{Z_{nn}\} = 0.$$

let all port currents be zero except for *Im and In*. Then

$$\mathrm{Re}\left\{(I_n I_m^* + I_m I_n^*) Z_{mn}\right\} = 0,$$

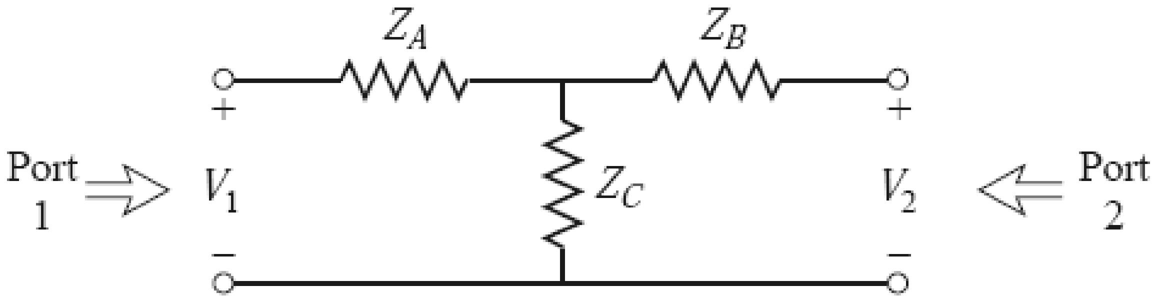
since $Z_{mn} = Z_{nm}$. However, $(I_n I_m^* + I_m I_n^*)$ is a purely real quantity that is, in general, nonzero. Thus we must have that

$$\mathrm{Re}\{Z_{mn}\} = 0.$$

The elements of the impedance and admittance matrices must be pure imaginary in a lossless network.

EXAMPLE 4.3 EVALUATION OF IMPEDANCE PARAMETERS

Find the Z parameters of the two-port T-network



Solution:

Z11 can be found as the input impedance of port 1 when port 2 is open-circuited:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_A + Z_C.$$

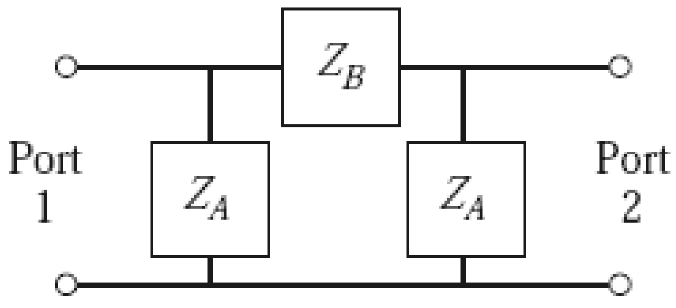
The transfer impedance *Z12* can be found measuring the open-circuit voltage at port 1 when a current *I2* is applied at port 2.

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_C. \qquad Z_{21} = Z_{12},$$

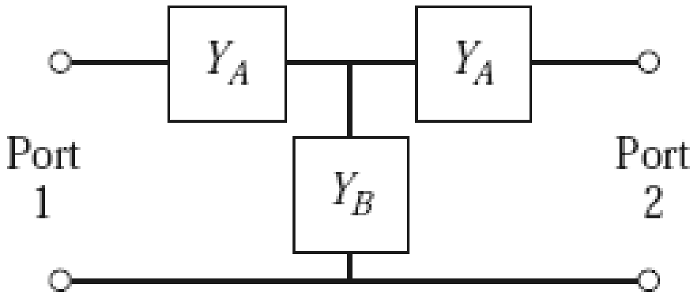
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_B + Z_C.$$

Homework

4.7 Derive the $[Z]$ and $[Y]$ matrices for the two-port networks shown in the figure below.



(a)



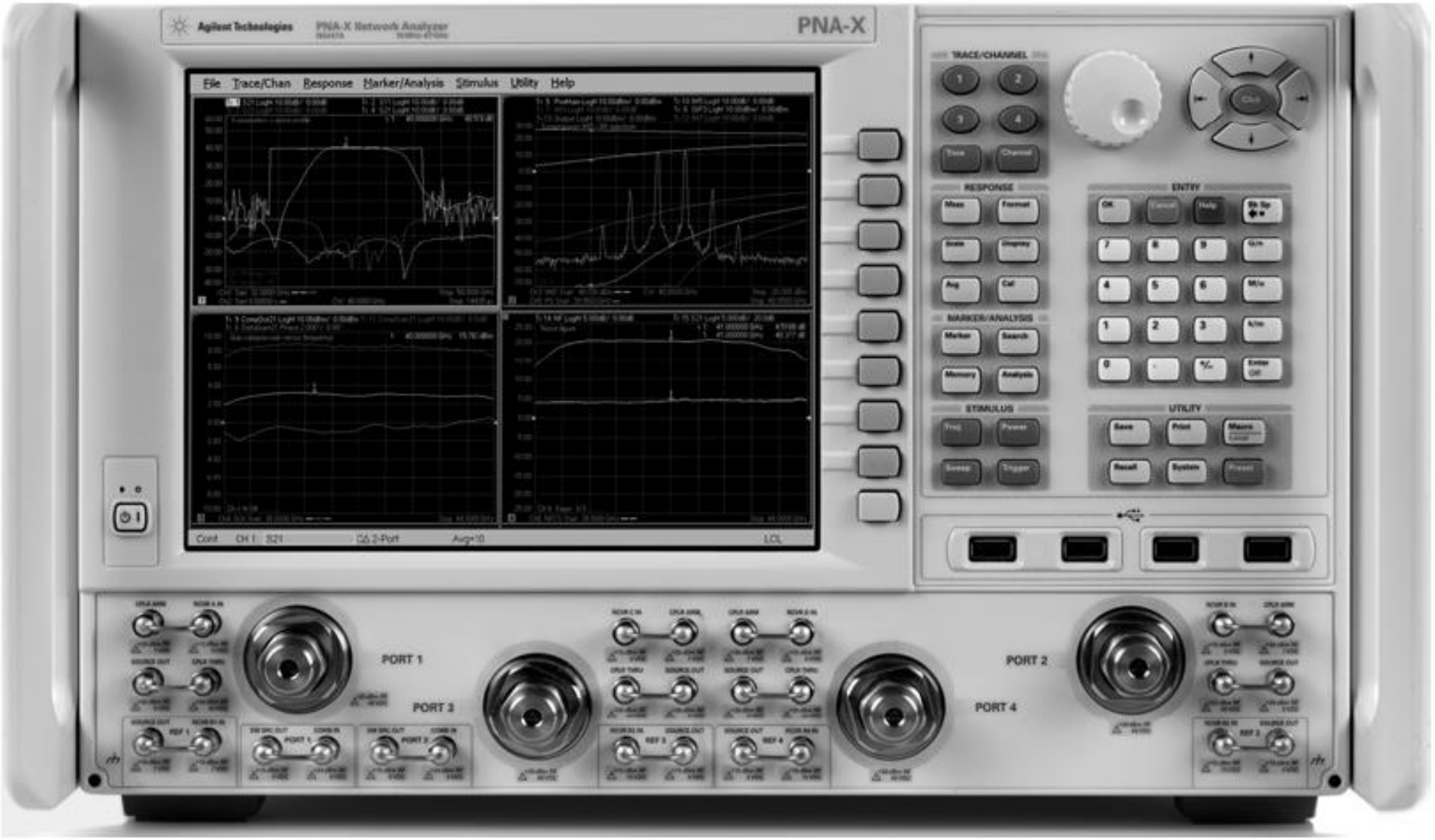
(b)

4.3 THE SCATTERING MATRIX

A practical problem exists when trying to measure voltages and currents at microwave frequencies because direct measurements usually involve the magnitude and phase of a wave traveling in a given direction or of a standing wave.

A representation more in accord with direct measurements, and with the ideas of incident, reflected, and transmitted waves, is given by the scattering matrix.

Once the scattering parameters of the network are known, conversion to other matrix parameters can be performed, if needed.



Photograph of the Agilent N5247A Programmable Network Analyzer. This instrument is used to measure the scattering parameters of RF and microwave networks from 10 MHz to 67 GHz.

The scattering matrix relates the voltage waves incident on the ports to those reflected from the ports.

Consider the N-port network shown in Figure 4.5, where V^+ n is the amplitude of the voltage wave incident on port n and V^- n is the amplitude of the voltage wave reflected from port n.

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix},$$

or

$$[V^-] = [S][V^+].$$

A specific element of the scattering matrix can be determined as

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+=0 \text{ for } k \neq j} .$$

S_{ij} is found by driving port j with an incident wave of voltage V^+_j and measuring the reflected wave amplitude V^-_i coming out of port i .

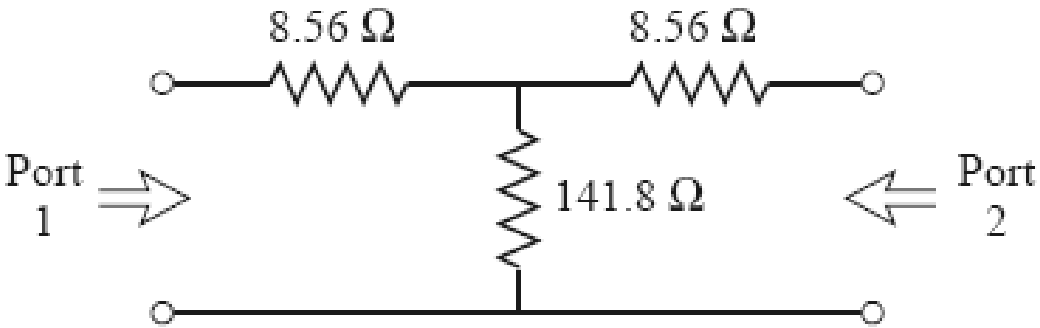
The incident waves on all ports except the j_{th} port are set to zero, which means that all ports should be terminated in matched loads to avoid reflections.

S_{ii} is the reflection coefficient seen looking into port i when all other ports are terminated in matched loads.
S_{ij} is the transmission coefficient from port j to port i when all other ports are terminated in matched loads.

EXAMPLE 4.4 EVALUATION OF SCATTERING PARAMETERS

Find the scattering parameters of the 3 dB attenuator circuit

Solution



S11 can be found as the reflection coefficient seen at port 1 when port 2 is terminated in a matched load (Z0 = 50):

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \Gamma^{(1)}|_{V_2^+=0} = \left. \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0} \right|_{Z_0 \text{ on port 2}},$$

$$Z_{in}^{(1)} = 8.56 + [141.8(8.56 + 50)]/(141.8 + 8.56 + 50) = 50\ \Omega, \text{ so } S_{11} = 0.$$

Because of the symmetry of the circuit, $S_{22} = 0$.

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}.$$

From the fact that $S11 = S22 = 0$, we know that $V^-_1 = 0$ when port 2 is terminated in $Z_0 = 50$, *and that* $V^+_2 = 0$.

$$V_1^+ = V_1 \qquad V_2^- = V_2.$$

By applying a voltage V1 at port 1 and using voltage division twice

$$V_2^- = V_2 = V_1\left(\frac{41.44}{41.44+8.56}\right)\left(\frac{50}{50+8.56}\right) = 0.707\,V_1,$$

$$S_{12} = S_{21} = 0.707.$$

the input power is $|V_1^+|^2/2Z_0$,

the output power is $|V_2^-|^2/2Z_0 = |S_{21}V_1^+|^2/2Z_0 = |S_{21}|^2/2Z_0|V_1^+|^2 = |V_1^+|^2/4Z_0$,

The output power is one-half (−3 dB) of the input power.

The scattering matrix can be determined from the [Z] (or [Y]) matrix and vice versa.

Assume that the characteristic impedances, Z_{0n}, of all the ports are identical and $Z_{0n} = 1.$

the total voltage and current at the n_{th} port can be written as

$$V_n = V_n^+ + V_n^-,$$

$$I_n = I_n^+ - I_n^- = V_n^+ - V_n^-.$$

Using the definition of [Z] gives

$$[Z][I] = [Z][V^+] - [Z][V^-] = [V] = [V^+] + [V^-],$$

That is, $$([Z] + [U])[V^-] = ([Z] - [U])[V^+],$$

where [U] is the unit, or identity, matrix.

$$[S] = ([Z] + [U])^{-1} ([Z] - [U]) ,$$

for a one-port network

$$S_{11} = \frac{z_{11} - 1}{z_{11} + 1},$$

in agreement with the result for the reflection coefficient seen looking into a load with a normalized input impedance of z_{11}.

Solving for [Z] gives

$$[Z] = ([U] + [S]) ([U] - [S])^{-1} .$$

Homework

4.11 Find the scattering parameters for the series and shunt loads shown below. Show that $S_{12} = 1 - S_{11}$ for the series case, and that $S_{12} = 1 + S_{11}$ for the shunt case. Assume a characteristic impedance Z_0.