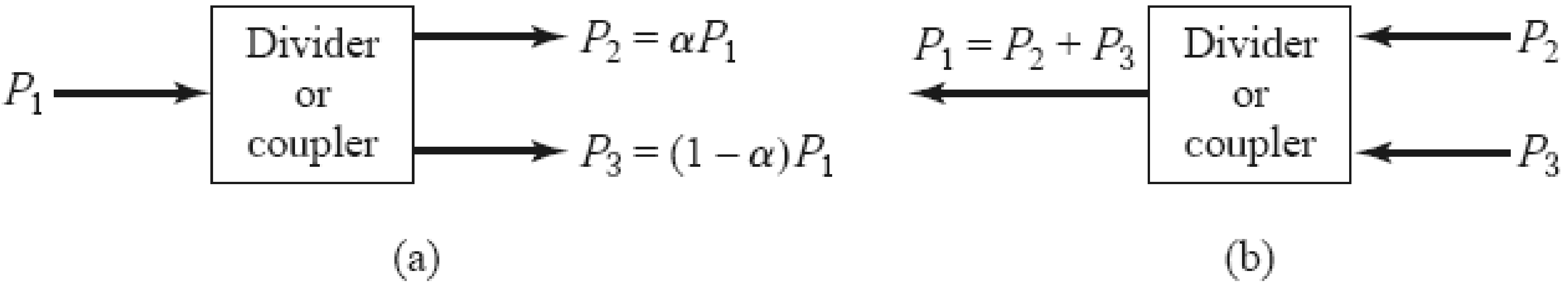
Lec18 Power Dividers and Directional Couplers (I)
功率分配器和定向耦合器

Power dividers and directional couplers are passive microwave components used for **power division or power combining**.



Power division and combining. (a) Power division. (b) Power combining.

The coupler or divider may have **three ports, four ports**, or more, and may be (ideally) lossless.
Three-port networks take the form **of T-junctions** and other power dividers, while four-port networks take the form of **directional couplers and hybrids**.

7.1 BASIC PROPERTIES OF DIVIDERS AND DOUPLERS

Three-Port Networks (T-Junctions)

The scattering matrix of an arbitrary three-port network has nine independent elements:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}.$$

If the device is passive and contains no anisotropic (各向异性) materials, then it must be **reciprocal** （互易）and its scattering matrix will be **symmetric** ($S_{ij} = S_{ji}$).

Usually, to avoid power loss, we would like to have a junction that is lossless and matched at all ports. However, that **it is impossible to construct such a three-port lossless reciprocal network that is matched at all ports**.

Proof

If all ports are matched, then $S_{ii} = 0$, and if the network is reciprocal, then

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}.$$

If the network is also lossless, then energy conservation requires that the scattering matrix satisfy the unitary (幺正) properties

$$[S]^t[S]^* = [U], \quad \Rightarrow$$

$$|S_{12}|^2 + |S_{13}|^2 = 1, \quad (7.3a)$$

$$|S_{12}|^2 + |S_{23}|^2 = 1, \quad (7.3b)$$

$$|S_{13}|^2 + |S_{23}|^2 = 1, \quad (7.3c)$$

$$S_{13}^* S_{23} = 0, \quad (7.3d)$$

$$S_{23}^* S_{12} = 0, \quad (7.3e)$$

$$S_{12}^* S_{13} = 0. \quad (7.3f)$$

(d)-(f) will always be inconsistent with one of equations (a)-(c).

A three-port network cannot be simultaneously lossless, reciprocal, and matched at all ports.

1. ***circulator***

If the three-port network is **nonreciprocal**, then *Si j is not equal to Sji , and the conditions of input* matching at all ports and energy conservation can be satisfied. Such a device is known as a ***circulator*** **（环形器）**, and generally relies on an anisotropic material, such as **ferrite(铁氧体)**, to achieve nonreciprocal behavior.

A circulator is a matched lossless **nonreciprocal** three-port network.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}.$$

$[S]$ must be unitary,

$$S_{31}^* S_{32} = 0,$$

$$S_{21}^* S_{23} = 0,$$

$$S_{12}^* S_{13} = 0,$$

$$|S_{12}|^2 + |S_{13}|^2 = 1,$$

$$|S_{21}|^2 + |S_{23}|^2 = 1,$$

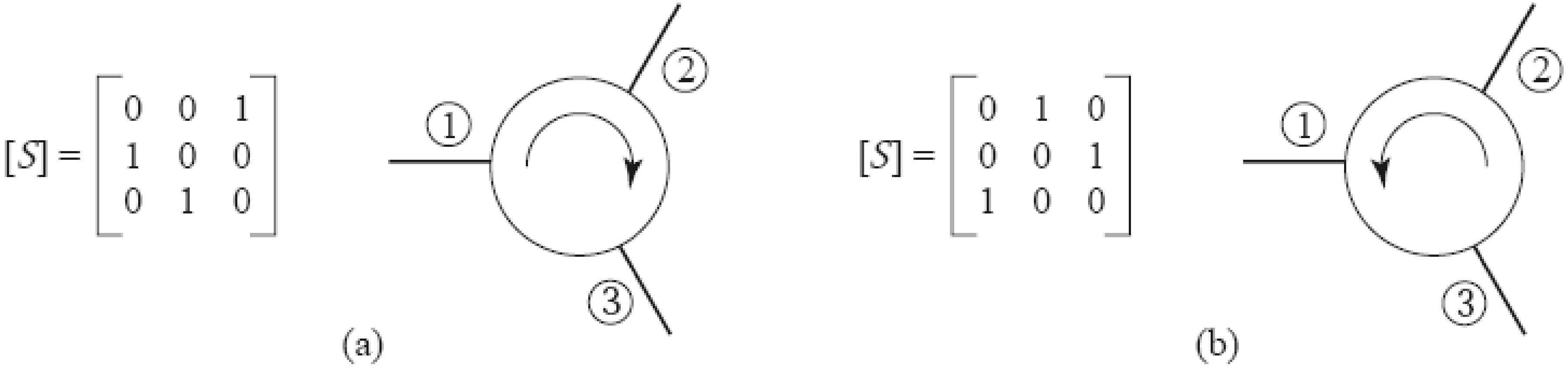
$$|S_{31}|^2 + |S_{32}|^2 = 1.$$

These equations can be satisfied in one of two ways.

Either $S_{12} = S_{23} = S_{31} = 0, \quad |S_{21}| = |S_{32}| = |S_{13}| = 1,$

or $S_{21} = S_{32} = S_{13} = 0, \quad |S_{12}| = |S_{23}| = |S_{31}| = 1.$

These results shows that $S_{ij} \neq S_{ji}$ for $i \neq j$



Two types of circulators and their scattering matrices. (a) Clockwise circulation. (b) Counterclockwise circulation.

2. **A lossless and reciprocal three-port network** can be physically realized if only two of its ports are matched.

If ports1and 2 are the matched ports, then

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}.$$

To be lossless, the following unitarity conditions must be satisfied:

$$S_{13}^* S_{23} = 0,$$

$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0,$$

$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0,$$

$$|S_{12}|^2 + |S_{13}|^2 = 1,$$

$$|S_{12}|^2 + |S_{23}|^2 = 1,$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1.$$

$$|S_{13}| = |S_{23}|$$

S13=S23=0

$$|S_{12}| = |S_{33}| = 1.$$

$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$

3. A lossy reciprocal matched three-port network

Resistive divider (电阻功分器)

A lossy three-port network can be made to have **isolation between its output ports** (e.g., *S23 = S32 = 0).*

Four-Port Networks (Directional Couplers)

The scattering matrix of a reciprocal four-port network matched at all ports

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}.$$

If the network is lossless, 10 equations result from the unitarity, or energy conservation.

Consider the multiplication of row 1 and row 2, and the multiplication of row 4 and row 3:

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0,$$

$$S_{14}^* S_{13} + S_{24}^* S_{23} = 0.$$

Multiply (7.10a) by S*24, and (7.10b) by S*13, and subtract to obtain

$$S_{14}^* (|S_{13}|^2 - |S_{24}|^2) = 0.$$

Similarly, the multiplication of row 1 and row 3, and the multiplication of row 4 and row 2, gives

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0,$$

$$S_{14}^* S_{12} + S_{34}^* S_{23} = 0.$$

Multiply (7.12a) by *S12, and (7.12b) by S34, and subtract to obtain*

$$S_{23} (|S_{12}|^2 - |S_{34}|^2) = 0.$$

$$S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0.$$
$$S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0.$$

$\Rightarrow$ $S_{14} = S_{23} = 0,$ A directional coupler.

Then the self-products (各行自乘) of the rows of the unitary scattering matrix yield the following equations:

$$|S_{12}|^2 + |S_{13}|^2 = 1,$$
$$|S_{12}|^2 + |S_{24}|^2 = 1,$$
$$|S_{13}|^2 + |S_{34}|^2 = 1,$$
$$|S_{24}|^2 + |S_{34}|^2 = 1,$$

$|S_{13}| = |S_{24}|$

$|S_{12}| = |S_{34}|$

we choose $S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$, and $S_{24} = \beta e^{j\phi}$, where α and β are real, and θ and ϕ are phase constants to be determined

The dot product of rows 2 and 3 gives

$$S_{12}^{*}S_{13} + S_{24}^{*}S_{34} = 0,$$

which yields a relation between the remaining phase constants as

$$\theta + \phi = \pi \pm 2n\pi.$$

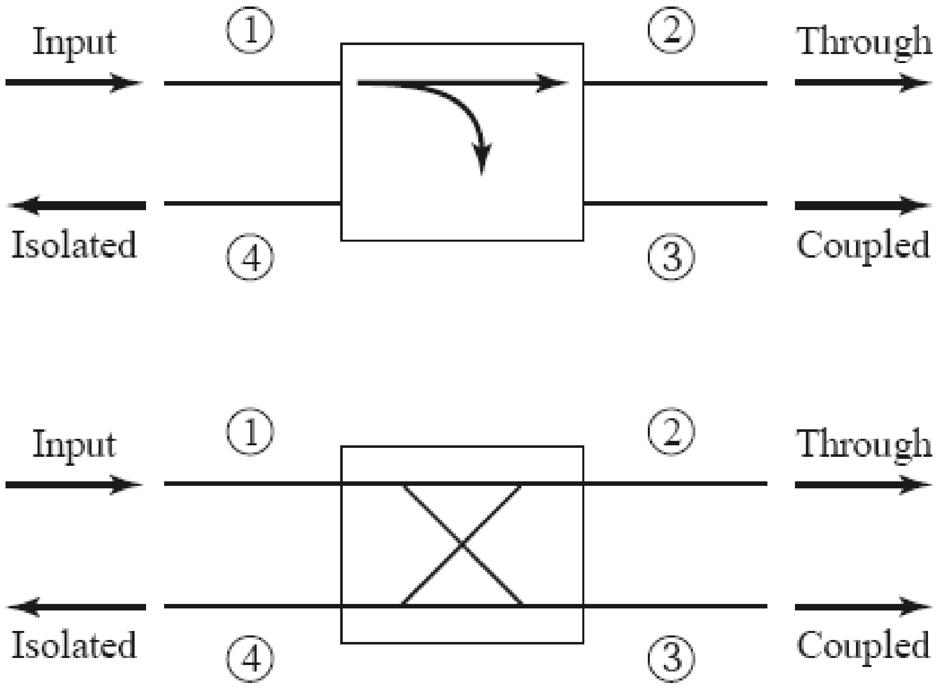
1. *A Symmetric Coupler: θ = φ = π/2.* The phases of the terms having amplitude β are chosen equal. Then the scattering matrix has the following form:

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}.$$

2. *An Antisymmetric Coupler: θ = 0, φ = π.*

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}.$$

$$|S_{12}|^2 + |S_{13}|^2 = 1, \quad \Rightarrow \quad \alpha^2 + \beta^2 = 1.$$



Two commonly used symbols for directional couplers, and power flow conventions.

Power supplied to port 1 is coupled to port 3 (the *coupled* port) with the coupling factor $|S_{13}|^2 = \beta^2$, while the remainder of the input power is delivered to port 2 (the *through* port) with the coefficient $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$. In an ideal directional coupler, no power is delivered to port 4 (the *isolated* port).

The following quantities are commonly used to characterize a directional coupler:

$$\text{Coupling} = C = 10 \log \frac{P_1}{P_3} = -20 \log \beta \text{ dB},$$

$$\text{Directivity} = D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \text{ dB},$$

$$\text{Isolation} = I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| \text{ dB},$$

$$\text{Insertion loss} = L = 10 \log \frac{P_1}{P_2} = -20 \log |S_{12}| \text{ dB}.$$

$$I = D + C \text{ dB}.$$

The coupling factor （耦合度） indicates the fraction of the input power that is coupled to the output port.

The directivity （方向性）is a measure of the coupler’s ability to isolate forward and backward waves (or the coupled and uncoupled ports).

The isolation （隔离度） is a measure of the power delivered to the uncoupled port.

The insertion loss accounts for the input power delivered to the through port, diminished by power delivered to the coupled and isolated ports.

Hybrid couplers （混合网络耦合器） are special cases of directional couplers, where the coupling factor is 3 dB, which implies that

$$\alpha = \beta = 1/\sqrt{2}.$$

There are two types of hybrids.

The quadrature （正交的） hybrid has a 90◦ phase shift between ports 2 and 3 ($\theta = \varphi = \pi/2$) when fed at port 1, and is an example of a symmetric coupler. Its scattering matrix has the following form:

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}.$$

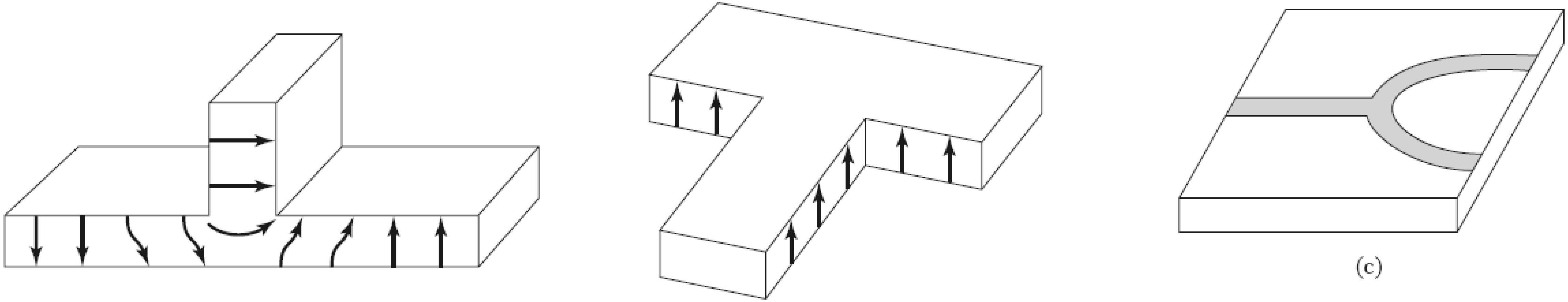
The *magic-T hybrid and the rat-race hybrid have a 180◦ phase difference between ports* 2 and 3 when fed at port 4, and are examples of an antisymmetric coupler. Its scattering matrix has the following form:

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}.$$

7.2 THE T-JUNCTION POWER DIVIDER

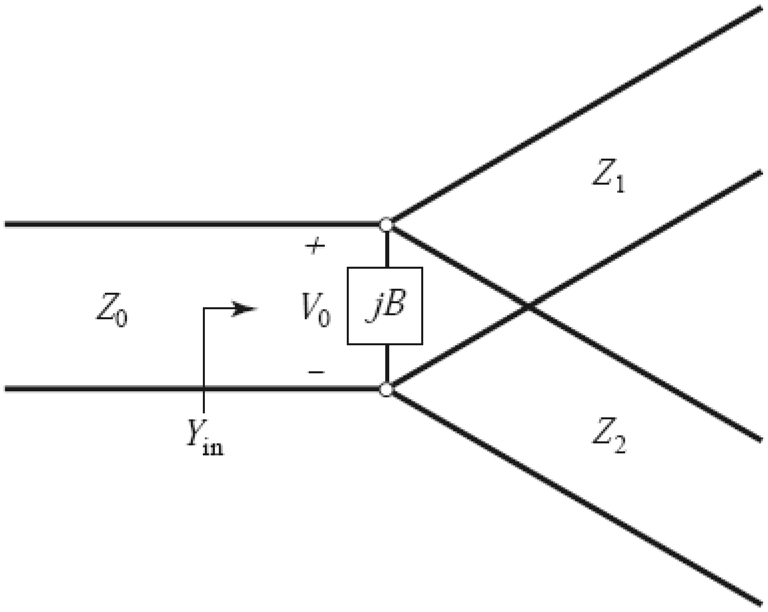
Lossless Divider（无耗分配器）

Such junctions cannot be matched simultaneously at all ports.



(a) (b) (c)

Various T-junction power dividers. (a) *E-plane waveguide T. (b) H-plane waveguide* T. (c) Microstrip line T-junction divider.



There may be fringing fields and higher order modes associated with the **discontinuity** at such a junction, leading to stored energy that can be accounted for by a lumped **susceptance**, *B*.

In order for the divider to be matched to the input line of characteristic impedance Z0, we must have

Transmission line model of a lossless T-junction divider.

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}.$$

If the transmission lines are assumed to be lossless (or of low loss), then the characteristic impedances are real. If we also assume *B = 0, then*

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}.$$

The output line impedances, Z1 and Z2, can be selected to provide various power division ratios. Thus, for a 50 input line, a 3 dB (equal split) power divider can be made by using two 100 output lines.
If necessary, quarter-wave transformers can be used to bring the output line impedances back to the desired levels.

EXAMPLE 7.1 THE T-JUNCTION POWER DIVIDER

A lossless T-junction power divider has a source impedance of 50 . Find the output characteristic impedances so that the output powers are in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.

Solution
If the voltage at the junction is *V0, the input power to the* matched divider is

$$P_{\mathrm{in}} = \frac{1}{2}\frac{V_0^2}{Z_0},$$

while the output powers are

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{in},$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{in}.$$

These results yield the characteristic impedances as

$$Z_1 = 3Z_0 = 150\ \Omega,$$

$$Z_2 = \frac{3Z_0}{2} = 75\ \Omega.$$

The input impedance to the junction is

$$Z_{in} = 75||150 = 50\ \Omega,$$

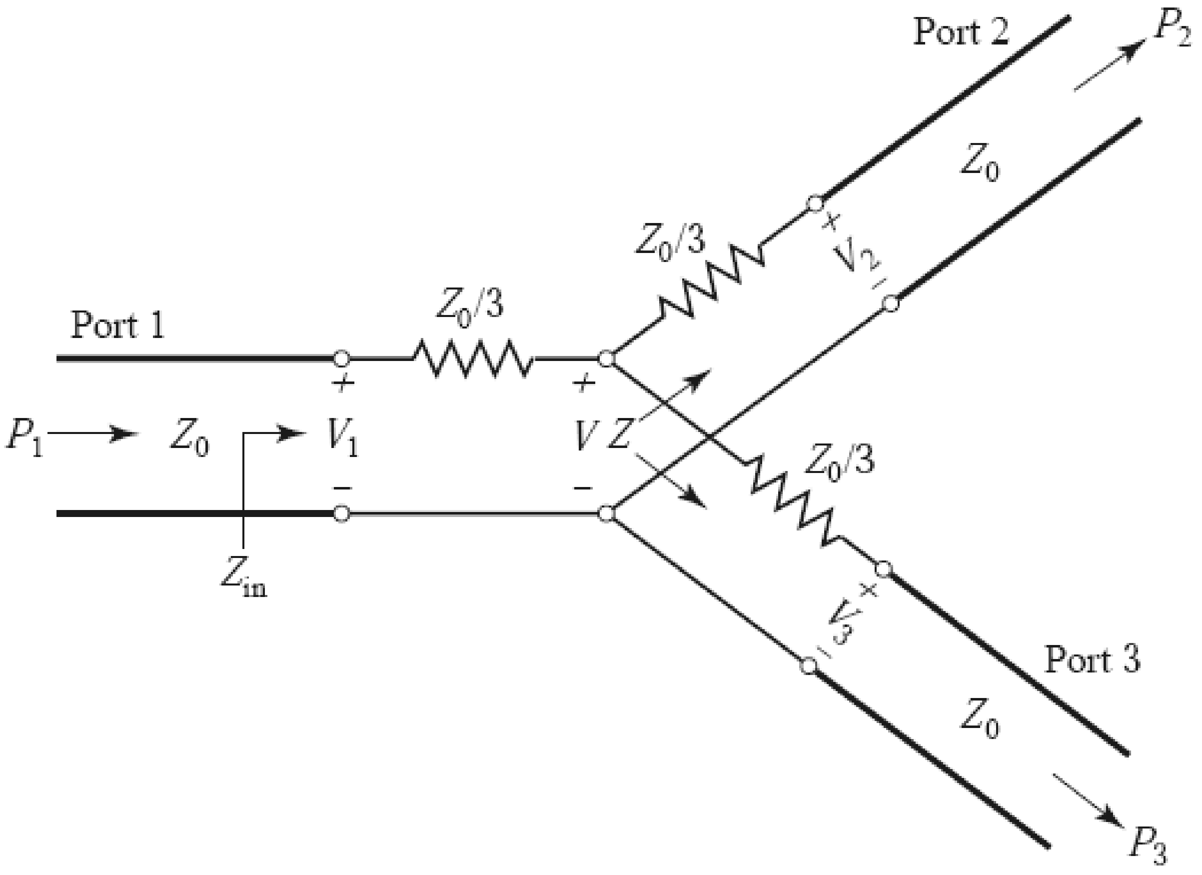
Looking into the 150 output line, we see an impedance of 50 || 75 = 30 , while at the 75 output line we see an impedance of 50 || 150 = 37.5 . The reflection coefficients seen looking into these ports are

$$\Gamma_1 = \frac{30 - 150}{30 + 150} = -0.666,$$

$$\Gamma_2 = \frac{37.5 - 75}{37.5 + 75} = -0.333.$$

Resistive Divider

If a three-port divider contains lossy components, it can be made to be matched at all ports, although the two output ports may not be isolated.



An equal-split three-port resistive power divider.

Assuming that all ports are terminated in the characteristic impedance Z0, the impedance Z, seen looking into the Z0/3 resistor followed by a terminated output line, is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}.$$

Then the input impedance of the divider is

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0,$$

which shows that the input is matched to the feed line.

Because the network is symmetric from all three ports, the output ports are also matched. Thus, $S11 = S22 = S33 = 0$.

If the voltage at port 1 is $V1$, then by voltage division the voltage V at the center of the junction is

$$V = V_1 \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} = \frac{2}{3} V_1,$$

and the output voltages are, again by voltage division,

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4} V = \frac{1}{2} V_1.$$

Thus, $S21 = S31 = S23 = 1/2$, so the output powers are 6 dB below the input power level.

The network is reciprocal, so the scattering matrix is symmetric, and it is

$$[S] = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

The power delivered to the input of the divider is

$$P_{\text{in}} = \frac{1}{2}\frac{V_1^2}{Z_0},$$

while the output powers are

$$P_2 = P_3 = \frac{1}{2}\frac{(1/2\,V_1)^2}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{1}{4}P_{\text{in}},$$

which shows that half of the supplied power is dissipated in the resistors.

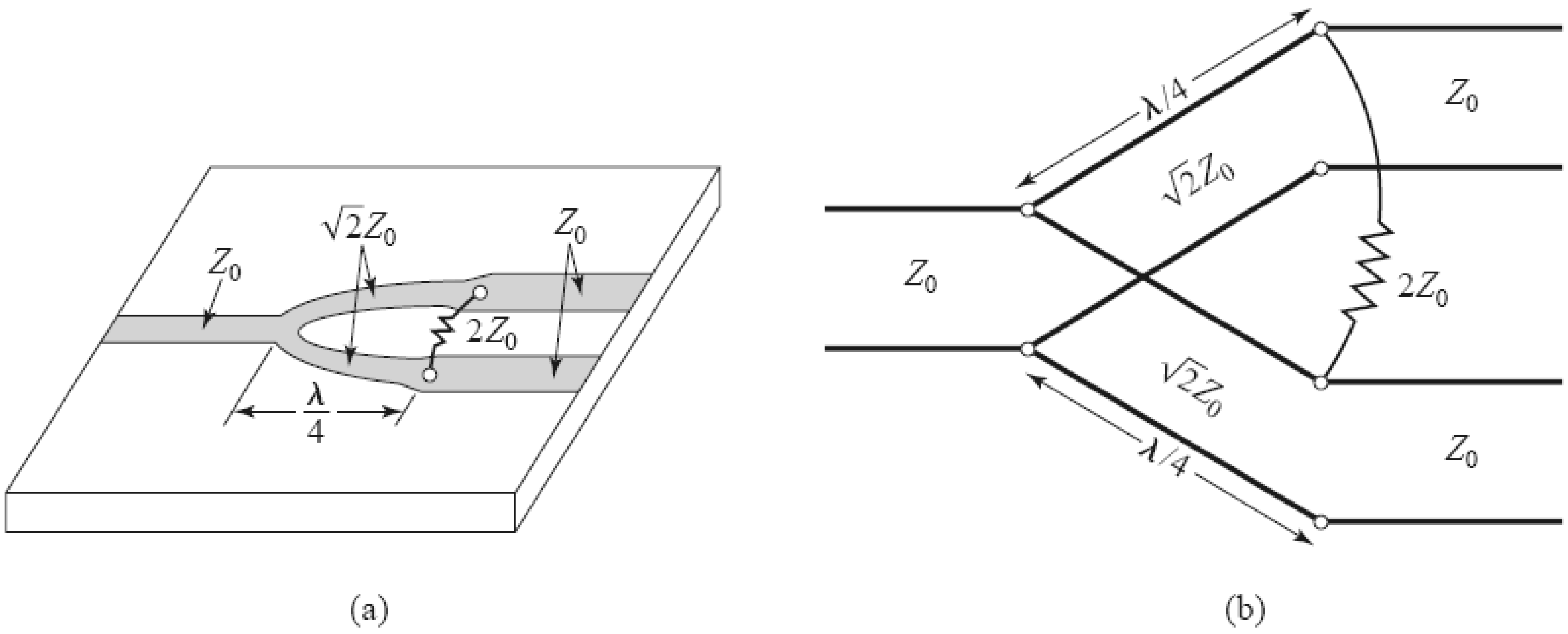
7.3 THE WILKINSON POWER DIVIDER

The lossless T-junction divider suffers from the disadvantage of **not being matched** at all ports, and it does **not have isolation** between output ports.

The resistive divider can be matched at all ports, but even though it is **not lossless**, **isolation is still not** achieved.

The Wilkinson power divider is such a network, with the useful property of appearing lossless when the output ports are matched; that is, only **reflected power from the output ports is dissipated**.

Even-Odd Mode Analysis

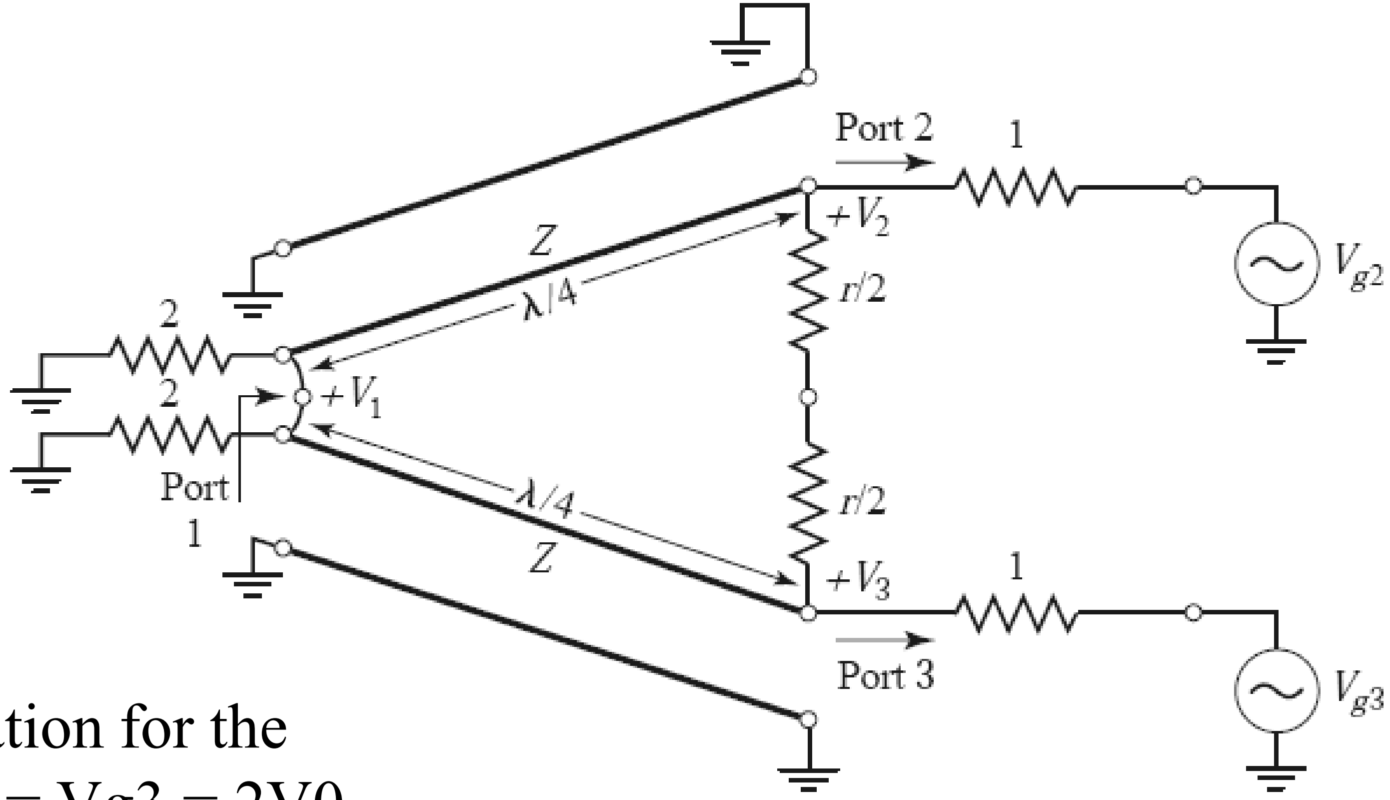


The Wilkinson power divider. (a) An equal-split Wilkinson power divider in microstrip line form. (b) Equivalent transmission line circuit.

The Wilkinson power divider circuit in normalized and symmetric form.

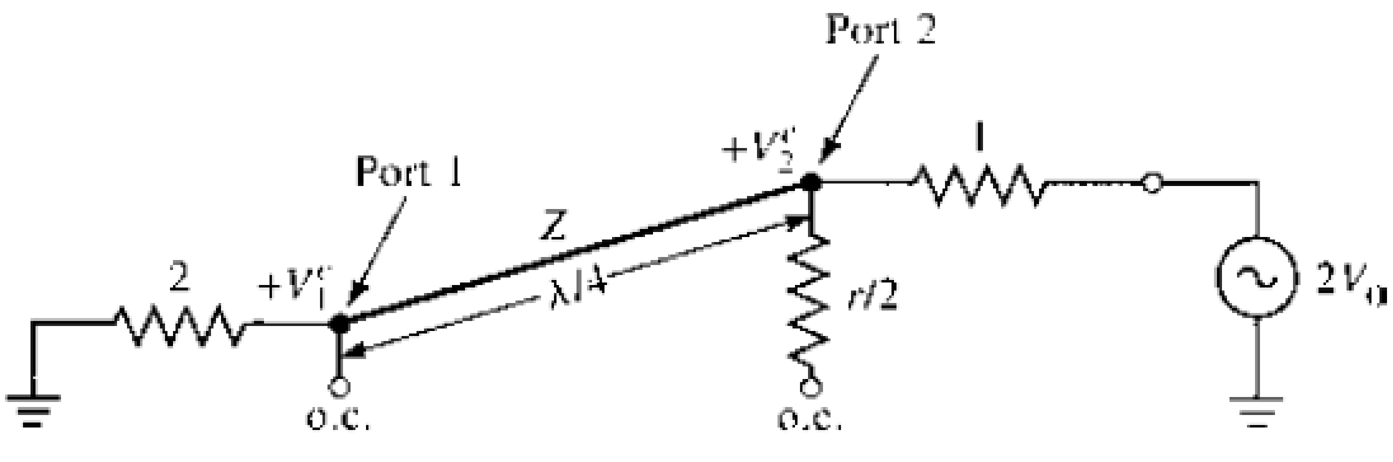
the two source resistors of normalized value 2 combine in parallel to give a resistor of normalized value 1, representing the impedance of a matched source.

$$Z = \sqrt{2} \text{ and } r = 2$$



Define two separate modes of excitation for the circuit : **the even mode**, where Vg2 = Vg3 = 2V0, and **the odd mode**, where Vg2 = −Vg3 = 2V0. **Superposition** of these two modes effectively produces an excitation of Vg2 = 4V0 and Vg3 = 0,

Even mode:



(a)

For even-mode excitation, $V_{g2} = V_{g3} = 2V_0$, so $V_2^e = V_3^e$,

No current flows through the *r/2 resistors*

Looking into port 2, $Z_{in}^e = \frac{Z^2}{2}$,

If Z = √2, port 2 will be matched for even-mode excitation; then Ve2 = V0 since $Z_{ein} = 1$.

Next, we find V_1^e from the transmission line equations.

If we let $x = 0$ at port 1 *and* $x = -\lambda/4$ at port 2, we can write the voltage on the transmission line section as

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x}).$$

Then

$$V_2^e = V(-\lambda/4) = jV^+(1-\Gamma) = V_0,$$

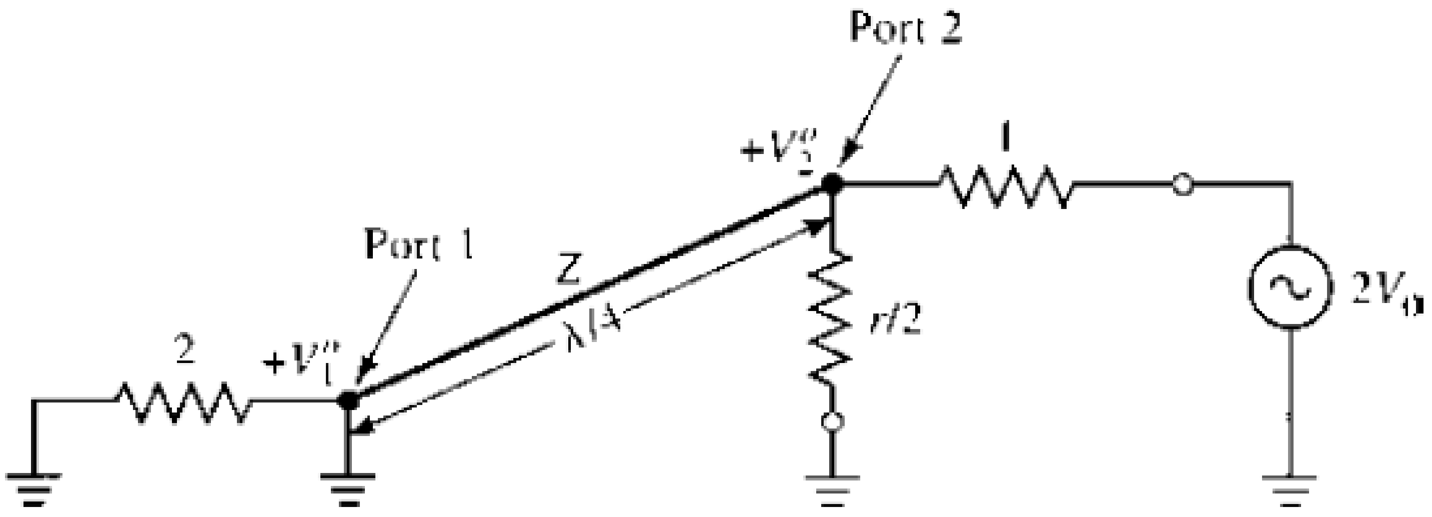
$$V_1^e = V(0) = V^+(1+\Gamma) = jV_0\frac{\Gamma+1}{\Gamma-1}.$$

The reflection coefficient is that seen at port 1 looking toward the resistor of normalized value 2, so

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}},$$

$$V_1^e = -jV_0\sqrt{2}.$$

Odd mode:



(b)

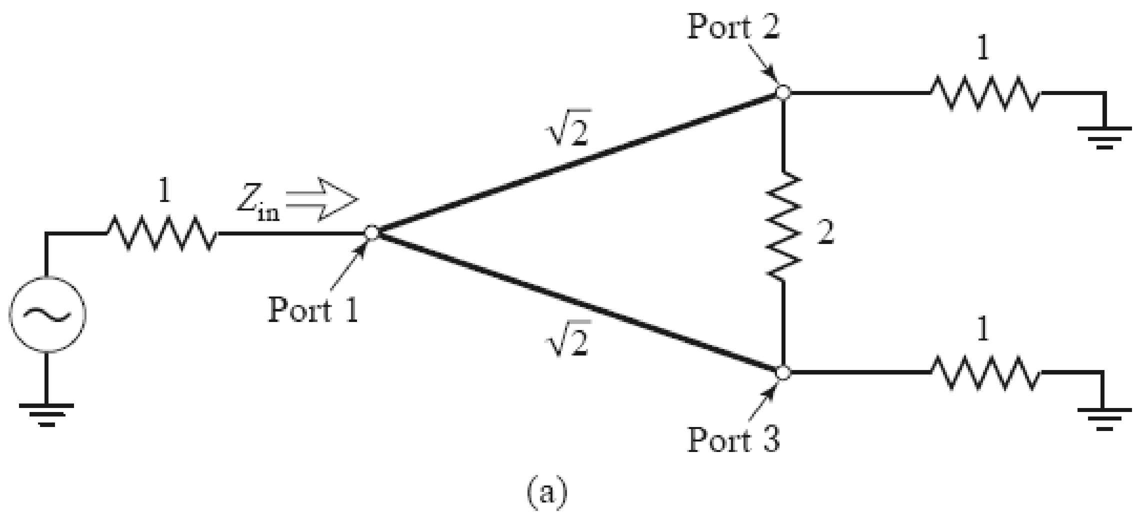
$V_{g2} = -V_{g3} = 2V_0$, and so $V_2^o = -V_3^o$,

and there is a voltage null along the middle of the circuit

Looking into port 2, we see an impedance of *r/2* since the parallel-connected transmission line is λ/4 long and shorted at port 1, and so looks like an open circuit at port 2. Port 2 will be matched for odd-mode excitation if we select *r = 2.*

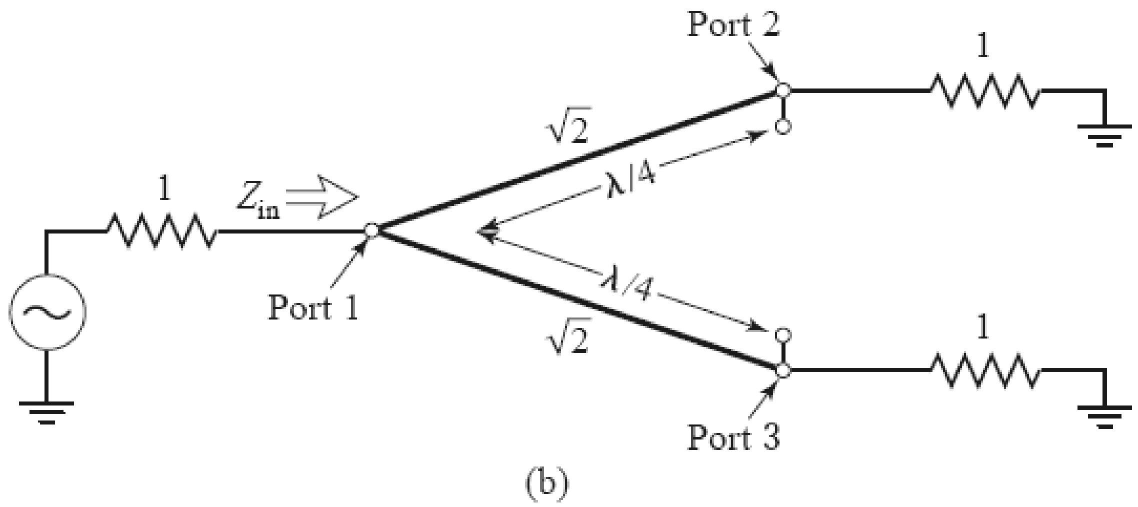
$V_2^o = V_0$ and $V_1^o = 0$; This mode of excitation all power is delivered to the *r/2 resistors, with none going to port 1*.

Finally, we must find the input impedance at port 1 of the Wilkinson divider when ports 2 and 3 are terminated in matched loads.



since $V_2 = V_3$.

No current flows through the resistor of normalized value 2, so it can be removed.



The input impedance at port 1 is

$$Z_{in} = \frac{1}{2}(\sqrt{2})^2 = 1.$$

In summary,

$$S_{11} = 0 \qquad (Z_{\text{in}} = 1 \text{ at port } 1)$$
$$S_{22} = S_{33} = 0 \qquad (\text{ports 2 and 3 matched for even and odd modes})$$
$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2} \qquad (\text{symmetry due to reciprocity})$$
$$S_{13} = S_{31} = -j/\sqrt{2} \qquad (\text{symmetry of ports 2 and 3})$$
$$S_{23} = S_{32} = 0 \qquad (\text{due to short or open at bisection})$$

Note that when the divider is driven at port 1 and the outputs are matched, no power is dissipated in the resistor. Thus the divider is lossless when the outputs are matched; only reflected power from ports 2 or 3 is dissipated in the resistor. Because *S23 = S32 = 0, ports 2 and 3 are isolated.*

EXAMPLE 7.2 DESIGN AND PERFORMANCE OF A WILKINSON DIVIDER

Design an equal-split Wilkinson power divider for a 50 Ω system impedance at frequency f_0, and plot the return loss (S_{11}), insertion loss ($S_{21} = S_{31}$), and isolation ($S_{23} = S_{32}$) versus frequency from 0.5 f_0 to 1.5 f_0.

Solution

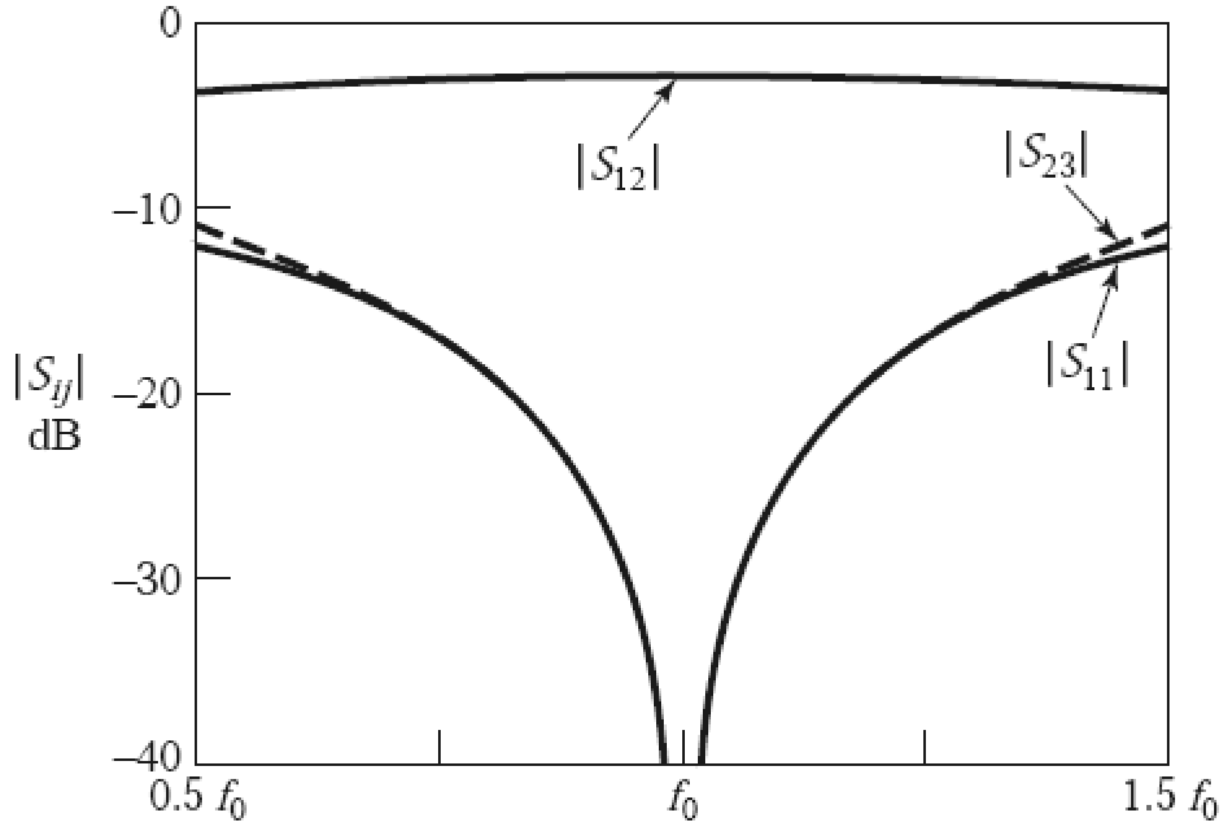
From Figure 7.8 and the above derivation, we have that the quarter-wave transmission lines in the divider should have a characteristic impedance of

$$Z = \sqrt{2} Z_0 = 70.7\ \Omega,$$

and the shunt resistor a value of

$$R = 2Z_0 = 100\ \Omega.$$

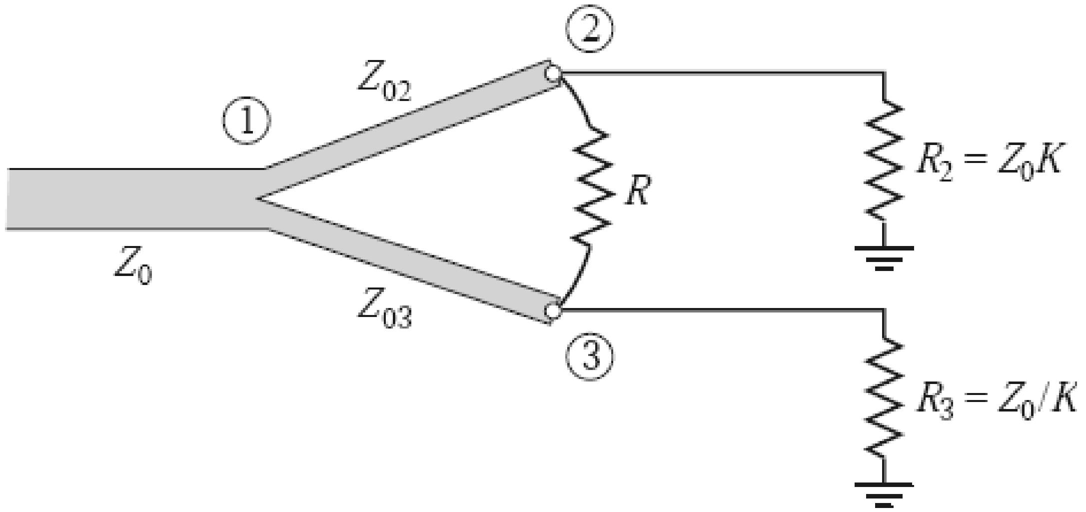
The transmission lines are *λ/4 long at the frequency f0.*



Using a computer-aided design tool for the analysis of microwave circuits, the scattering parameter magnitudes were calculated and plotted.

Unequal Power Division and N -Way Wilkinson Dividers

Wilkinson-type power dividers can also be made with unequal power splits;



If the power ratio between ports 2 and 3 is $K^2 = P3/P2$,

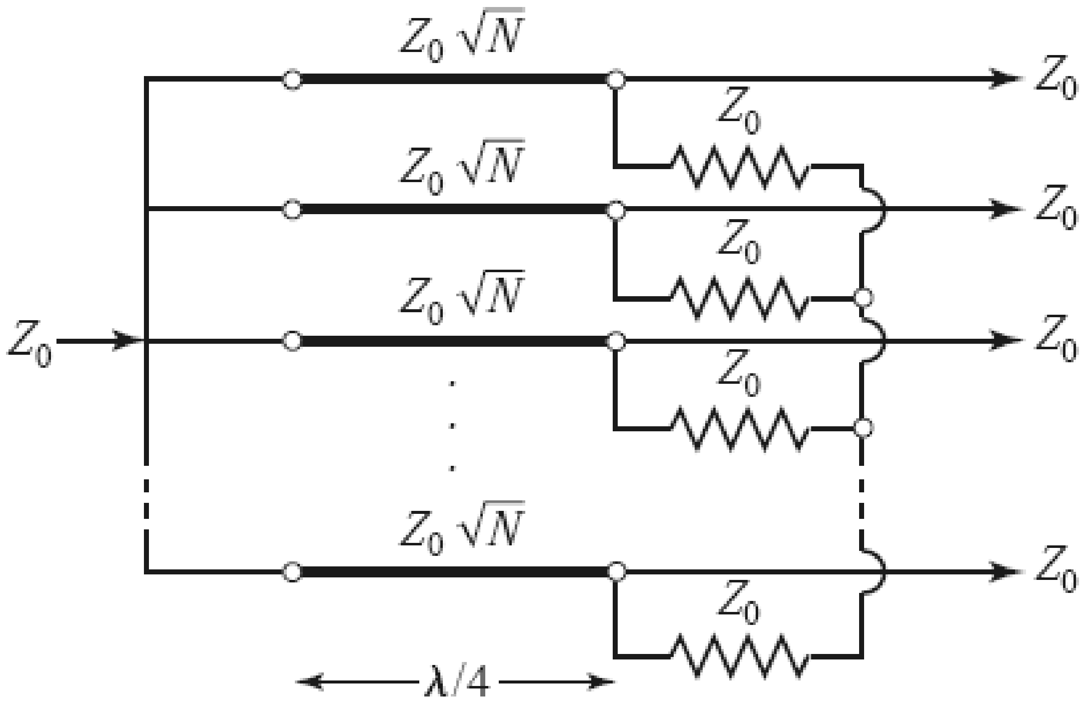
$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}},$$

$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)},$$

$$R = Z_0\left(K + \frac{1}{K}\right).$$

Note that the above results reduce to the equal-split case for $K = 1$.

The Wilkinson divider can also be generalized to an *N-way divider or combiner*



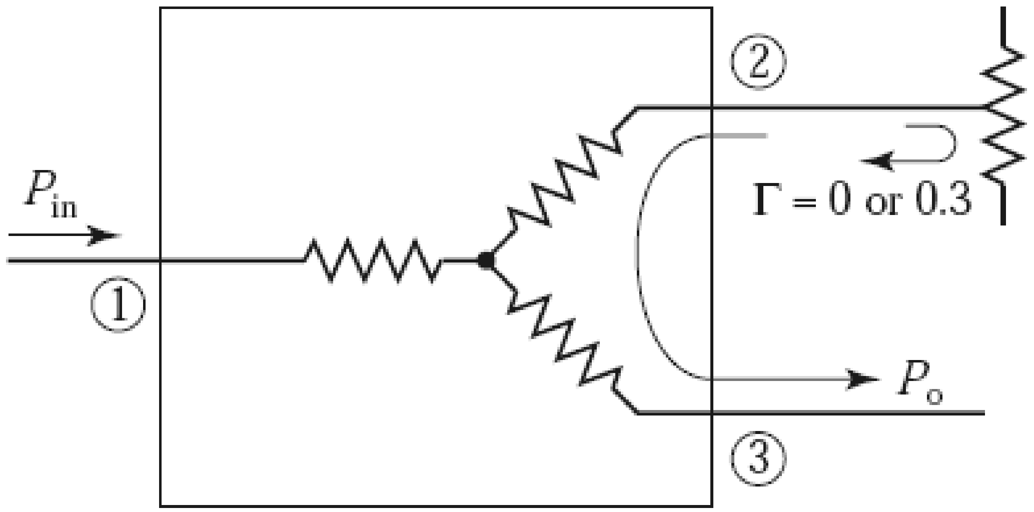
An N-way, equal-split Wilkinson power divider.

Homework

7.3 A directional coupler has the scattering matrix given below. Find the return loss, coupling factor, directivity, and insertion loss. Assume that the ports are terminated in matched loads.

$$[S] = \begin{bmatrix} 0.1\angle 40^\circ & 0.944\angle 90^\circ & 0.178\angle 180^\circ & 0.0056\angle 90^\circ \\ 0.944\angle 90^\circ & 0.1\angle 40^\circ & 0.0056\angle 90^\circ & 0.178\angle 180^\circ \\ 0.178\angle 180^\circ & 0.0056\angle 90^\circ & 0.1\angle 40^\circ & 0.944\angle 90^\circ \\ 0.0056\angle 90^\circ & 0.178\angle 180^\circ & 0.944\angle 90^\circ & 0.1\angle 40^\circ \end{bmatrix}$$

7.8 Design a three-port resistive divider for an equal power split and a 100 Ω system impedance. If port 3 is matched, calculate the change in output power at port 3 (in dB) when port 2 is connected first to a matched load, and then to a load having a mismatch of $\Gamma = 0.3$. See the figure below.



7.10 Design a Wilkinson power divider with a power division ratio of $P_3/P_2 = 1/3$ and a source impedance of 50 Ω.