Lec 4 Transmission line theory (II)

4. THE SMITH CHART

SMITH圆图的作用是什么？

The Smith chart is a graphical aid that can be very useful for solving transmission line problems. It was developed in 1939 by P. Smith at the Bell Telephone Laboratories.

it is based on a polar plot of the voltage reflection coefficient,

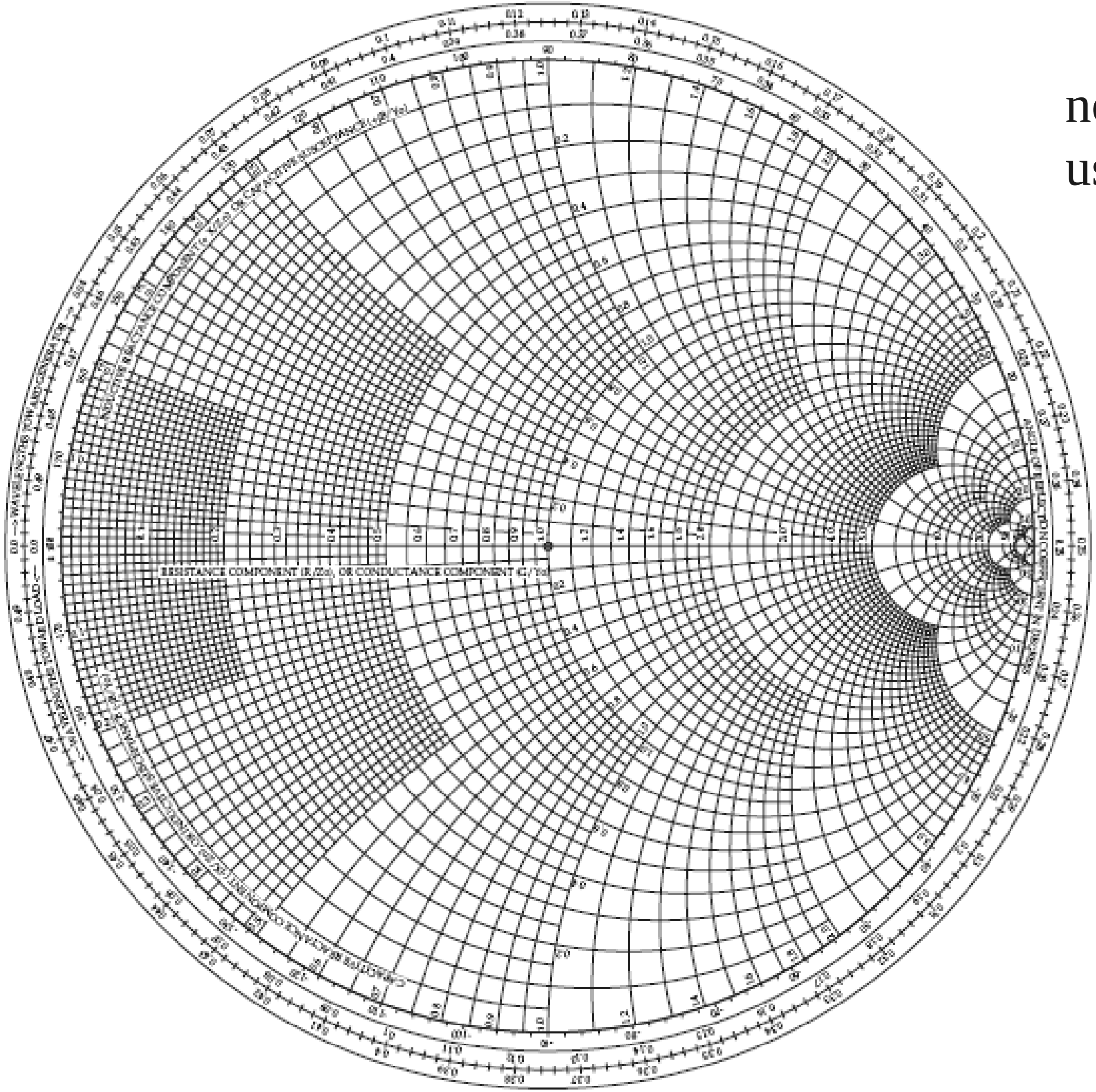
Let the reflection coefficient be expressed in magnitude and phase (polar) form as

$$\Gamma = |\Gamma| e^{j\theta}.$$

the magnitude is plotted as a radius $(|\Gamma| \leq 1)$ from the center of the chart,

the angle θ $(-180° \leqslant \theta \leqslant 180°)$ is measured counterclockwise from the right-hand side of the horizontal diameter.

it can be used to convert from reflection coefficients to normalized impedances (or admittances) and vice versa by using the impedance (or admittance) circles printed on the chart.



normalized quantities are generally used in Smith chart.

$$z = Z / Z_0$$

$$z_L = Z_L / Z_0$$

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta},$$

$$z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}.$$

$$\Gamma = \Gamma_r + j\Gamma_i,$$

$$z_L = r_L + j x_L$$

Reflection circles (等反射系数圆族)

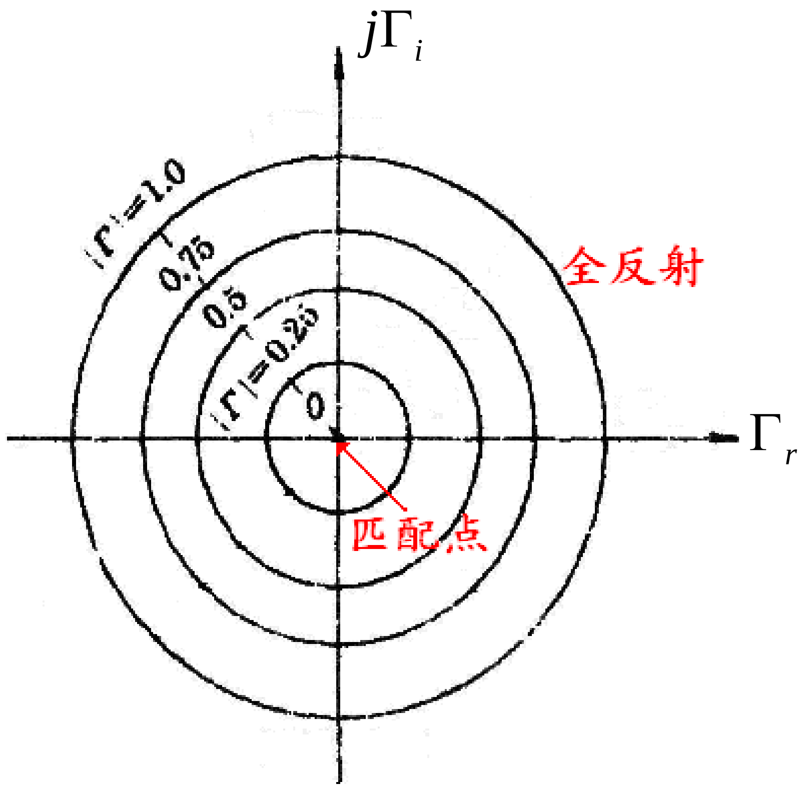


图2.9 等反射系数圆的半径

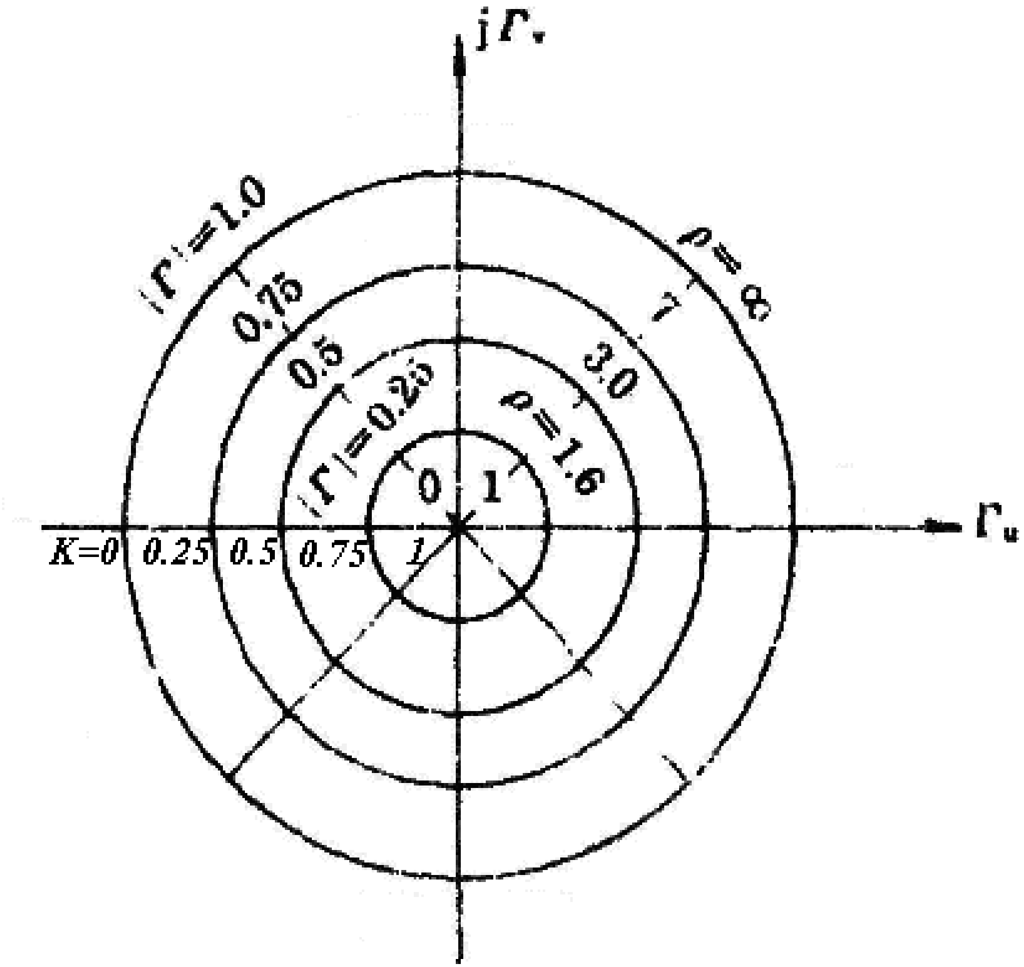
圆心为坐标原点（0,0），半径为反射系数的模

$$|\Gamma| = |\Gamma_l|$$

最小圆半径是零，此点为“**匹配点**”，落在复平面坐标原点o上。

最大圆半径是1，代表**全反射系数**的轨迹。

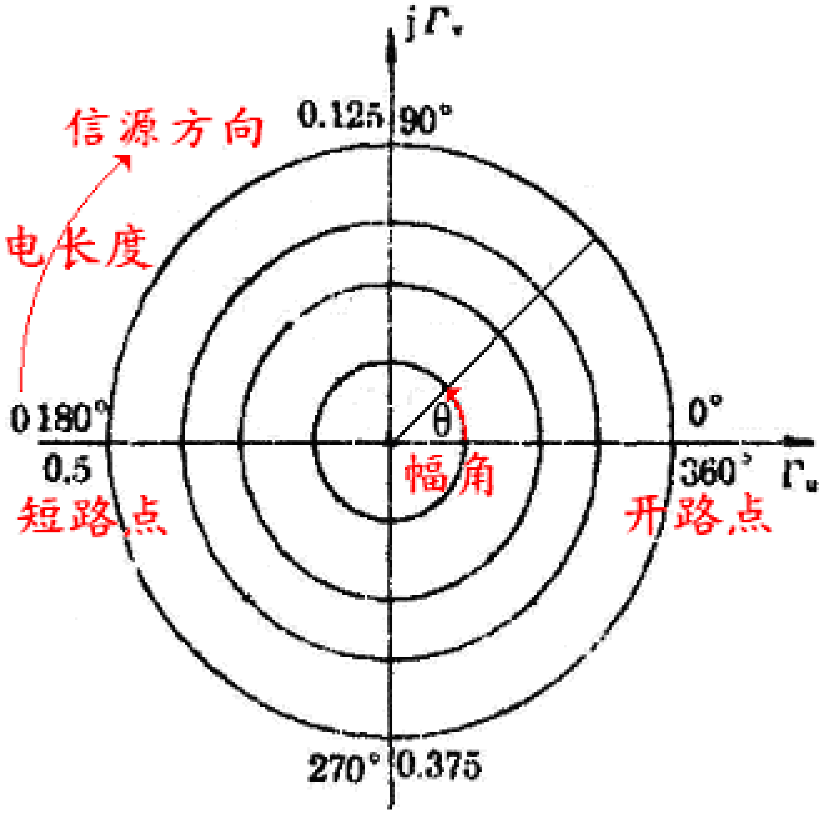
VSWR circles and K circles (等驻波系数圆、等行波系数圆)



等驻波系数圆、等行波系数圆

$$\rho = \frac{1+|\Gamma_l|}{1-|\Gamma_l|}$$

$$K = \frac{1-|\Gamma_l|}{1+|\Gamma_l|}$$



等反射系数圆的幅角、电长度

反射系数的幅角 θ 为常数的等值线是一族从坐标原点出发、终止于单位圆的射线

通常 θ 值标在单位圆外，并规定单位圆与正实轴的交点(即**开路点**)为 $\theta = 0°$

反射系数幅角的变化与传输线上两点间的电长度有关，可用电长度表示幅角，其值标在单位圆外侧(或外面一个圆上)。

等反射圆的半个圆周相当于 0.25λ
整个圆周相当于 0.5λ

传输线上两点间的距离与旋转的电长度相对应。故电长度的起始点可任意选取，通常把$\theta=180^\circ$ (即**短路点**)处作为电长度的零点。

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}. \quad \Rightarrow \quad r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}.$$

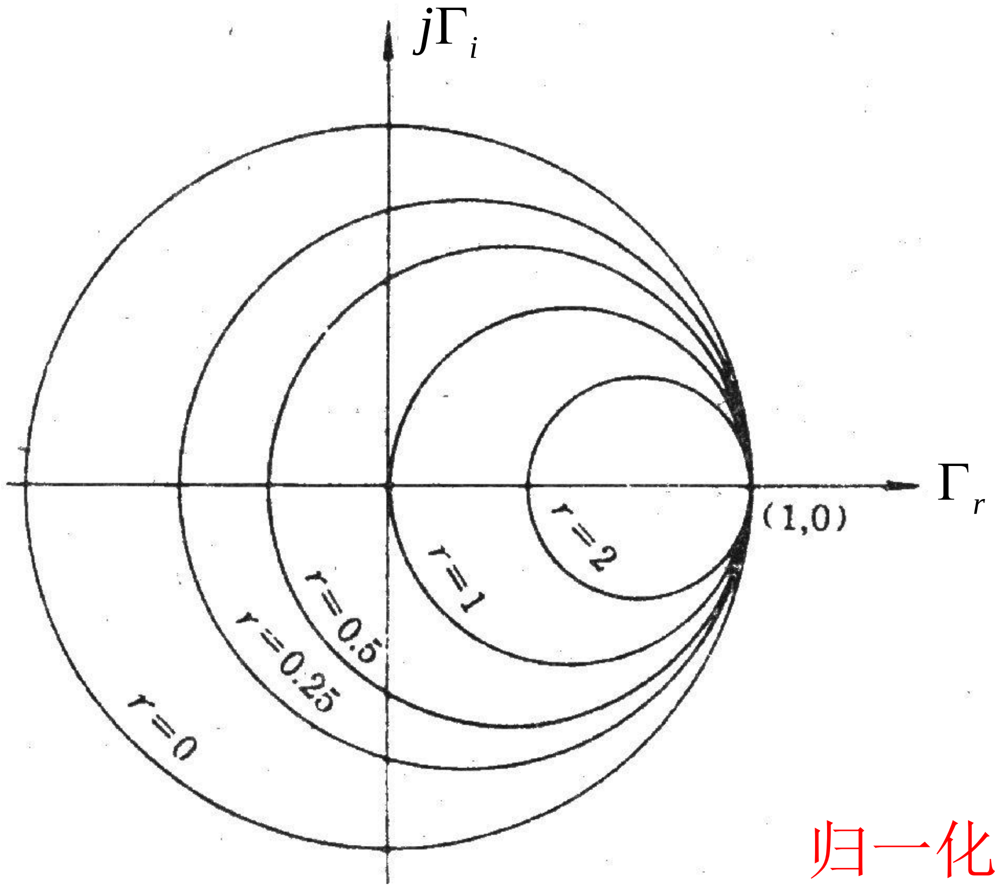
$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}, \qquad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}.$$

Resistance circles
$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2,$$

Reactance circles
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2,$$

For example, the $r_L = 1$ circle has its center at $\Gamma_r = 0.5,\ \Gamma_i = 0,$ and has a radius of 0.5, and so it passes through the center of the Smith chart.

Resistance circles



$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2,$$

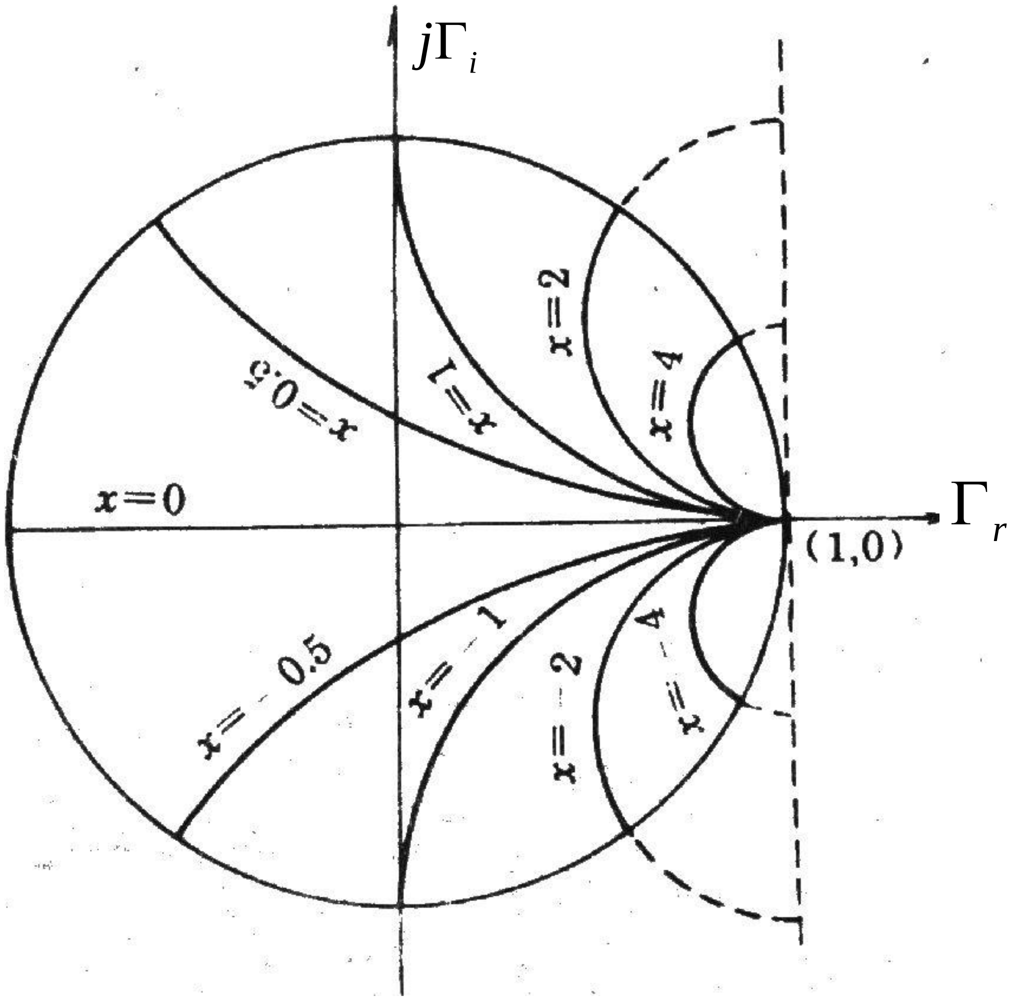
All of the resistance circles have centers on the horizontal $\Gamma_i = 0$ axis and pass through the （1, 0） point on the right-hand side of the chart.

归一化电阻圆是一组公共切点为(1，0)的**内切圆族**。

For example, the $r_L = 1$ circle has its center at $\Gamma_r = 0.5,\ \Gamma_i = 0,$, and has a radius of 0.5, and so it passes through the center of the Smith chart.

Reactance circles

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2,$$



The centers of all of the reactance circles lie on the vertical $\Gamma_r = 1$ line (off the chart), and these circles also pass through the (1,0) point.
The resistance and reactance circles are orthogonal.

正归一化电抗圆族，圆心落在$\Gamma_r=1$上半虚轴上，半径随电抗X的增大而缩小，是一组公共切点为(1，0)的**内切圆**。（感性负载）

负归一化电抗圆族，圆心落在$\Gamma_r=1$下半虚轴上，是一组公共切点为(1，0)的内切圆。

上述两组内切圆间互为以(1，0)公共切点的外切圆。（容性负载）

1. Smith圆图的中心点(0，0)对应匹配点。
2. 实轴上所有点(两端点除外)表示纯归一化负载电阻。因为 x=0时，等x圆的半径为∞，等x圆退化成实轴；
3. 实轴左端点对应短路点 $\Gamma=-1$ $Z_L=0$
4. 实轴右端点对应开路点 $\Gamma=1$ $Z_L=\infty$
5. 圆图的单位圆(外圆)是纯归一化电抗圆，对应于
   $K=0,\quad |\Gamma|=1,\quad r=0,\quad z=jx$
6. 实轴以上半圆的等半径圆曲线对应x>0.故上半圆中各点代表各种不同数值的感性负载阻抗的归一化值；
7. 实轴以下半圆的等圆曲线对应x<0，故下半圆中各点代表各种不同数值的容性负载阻抗的归一化值。
8. 右半实轴上的点是电压波腹点(电流波节点)，r值为驻波系数ρ的值。
9. 左半实轴上的点是电流波腹点(电压波节点)，r值为行波系数K的值。

The Smith chart can also be used to graphically solve the transmission line impedance equation since this can be written in terms of the generalized reflection coefficient as

$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_o^+ \left(e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right)}{V_o^+ \left(e^{j\beta\ell} - \Gamma e^{-j\beta\ell}\right)} Z_0 = \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}} Z_0,$$

$$z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}.$$

Z_{in} and Z_L have the similar form.

if we have plotted the reflection coefficient at the load, the normalized input impedance seen looking into a length l of transmission line terminated with z_L can be found by rotating the point clockwise by an amount $2\beta l$ ($\theta - 2\beta l$) from around the center of the chart. The radius stays the same since the magnitude of does not change with position along the line (assuming a lossless line).

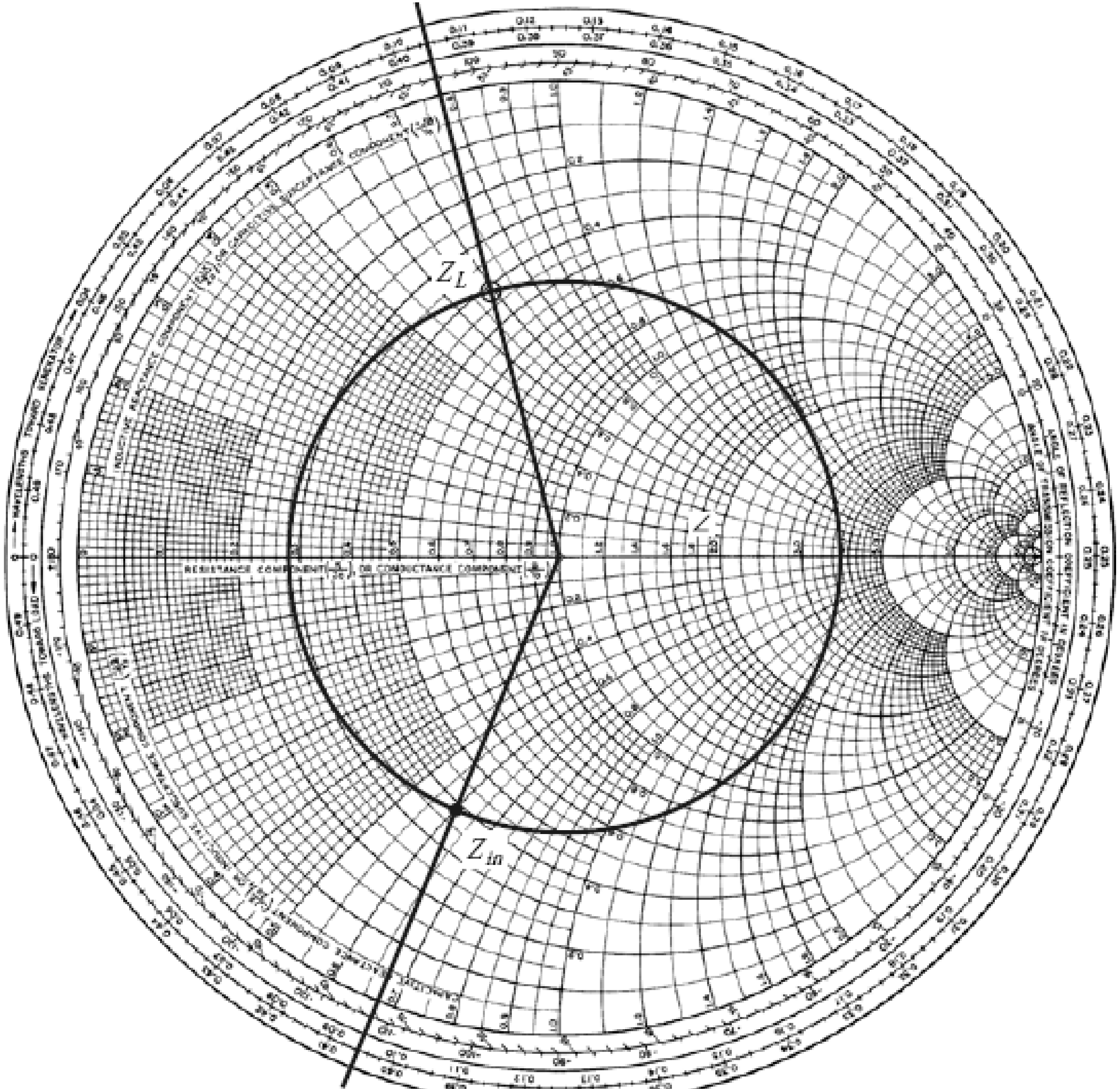
EXAMPLE 2.2 BASIC SMITH CHART OPERATIONS

A load impedance of $40 + j70$ terminates a 100ohm transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the input to the line, the input impedance, the standing wave ratio on the line, and the return loss.

Solution :

The normalized load impedance is $$z_L = \frac{Z_L}{Z_0} = 0.4 + j0.7,$$

It is the cross point of the resistance circle r_L=0.4 and the reactance circle x_L=0.7.
It is also the point in the reflection circle, $|\Gamma|=0.59$ and the phase=104° .
Then the VSWR=3.87, and the return loss RL=4.6 dB.
Reading the reference position of the load on the wavelengths-toward-generator (WTG) scale gives a value of 0.106λ. Moving down the line 0.3λ toward the generator brings us to 0.406λ on the WTG scale.



$z_{\text{in}} = 0.365 - j0.611.$

$Z_{in} = Z_0 z_{in} = 36.5 - j61.1$

The phase is read from the radial line at the phase scale as 248°

The Combined Impedance–Admittance Smith Chart

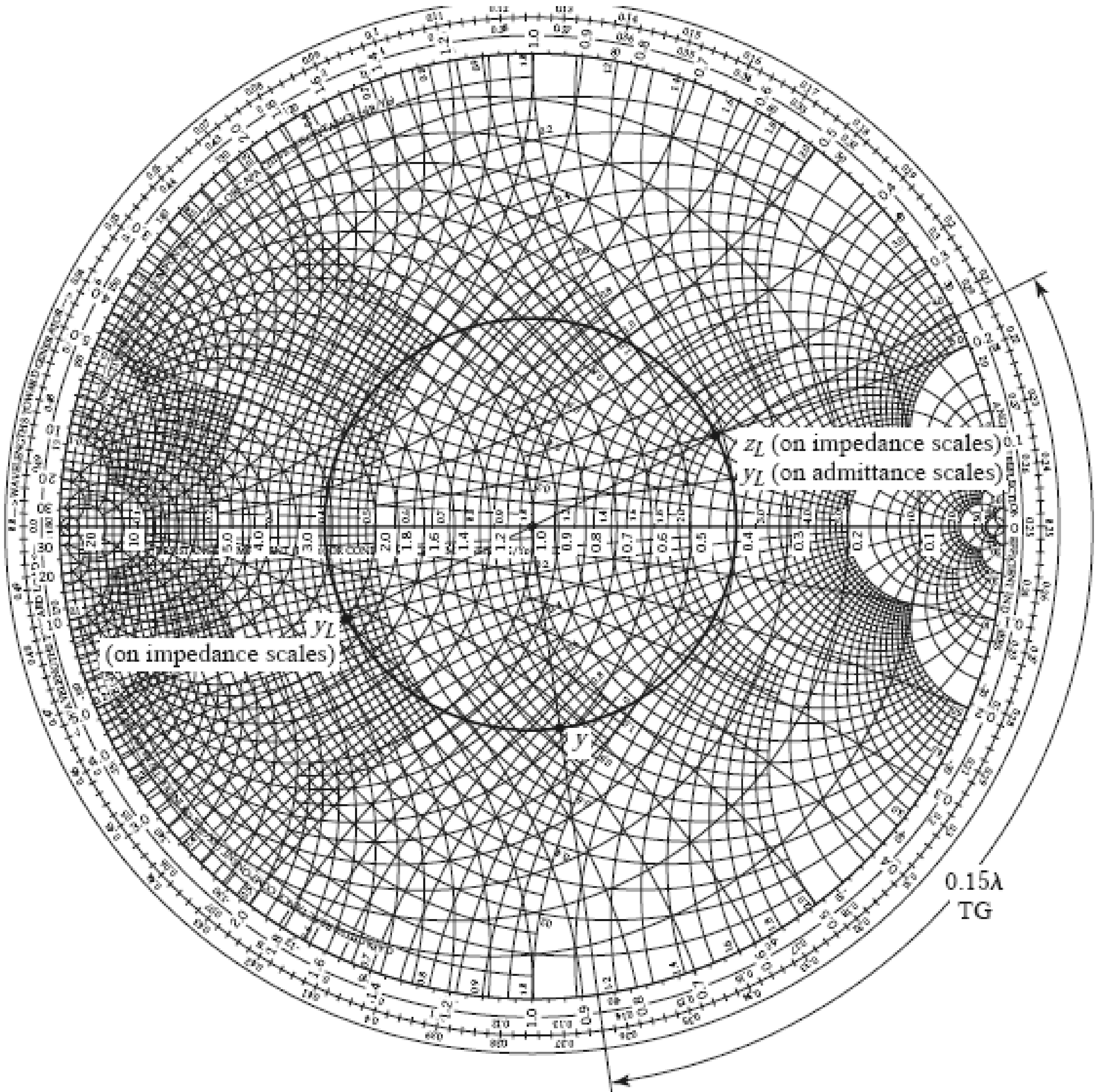
The Smith chart can be used for normalized admittance in the same way

the input impedance of a load z_L connected to a $\lambda/4$ line is,

$$z_{\text{in}} = 1/z_L ,$$

converting a normalized impedance to a normalized admittance.

a $\lambda/4$ transformation is equivalent to a 180° rotation;



Such a chart is referred to as an *impedance and admittance Smith chart* and usually has different-colored scales for impedance and admittance.

EXAMPLE 2.3 SMITH CHART OPERATIONS USING ADMITTANCES

A load of $Z_L = 100 + j50$ terminates a 50 line. What are the load admittance and input admittance if the line is 0.15λ long?

Solution:

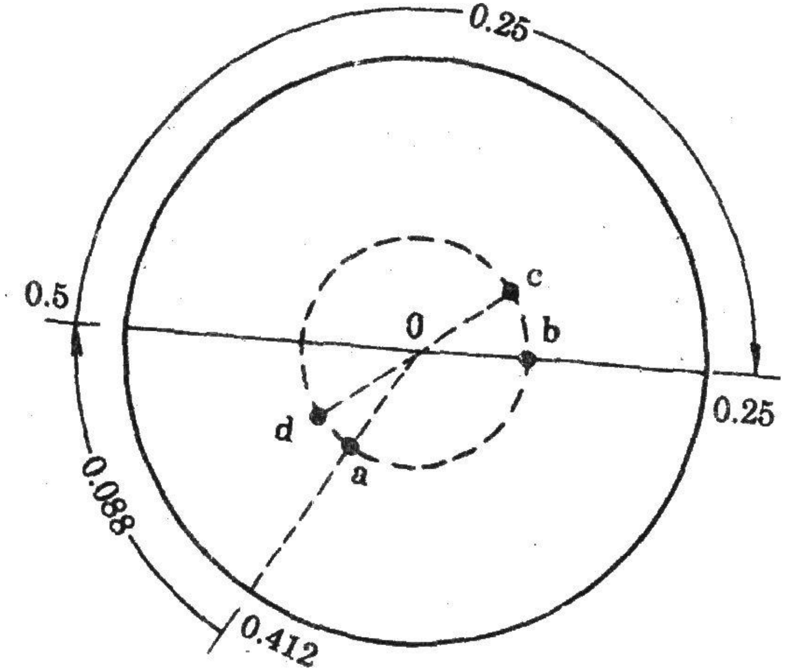
The normalized load impedance is $z_L = 2 + j1$. A standard Smith chart can be used for this problem by initially considering it as an impedance chart and plotting z_L and the SWR circle. Conversion to admittance can be accomplished with a $\lambda/4$ rotation of z_L (easily obtained by drawing a straight line through z_L and the center of the chart to intersect the other side of the SWR circle). The chart can now be considered as an admittance chart, and the input admittance can be found by rotating 0.15λ from y_L.

Alternatively, we can use the combined *zy* chart, where conversion between impedance and admittance is accomplished merely by reading the appropriate scales. Plotting z_L on the impedance scales and reading the admittance scales at this same point gives $y_L = 0.40 - j0.20$. The actual load admittance is then

$$Y_L = y_L Y_0 = \frac{y_L}{Z_0} = 0.0080 - j0.0040 \text{ S}.$$

Then, on the WTG scale, the load admittance is seen to have a reference position of 0.214 λ. Moving 0.15 λ past this point brings us to 0.364 λ. A radial line at this point on the WTG scale intersects the SWR circle at an admittance of $y = 0.61 + j0.66$. The actual input admittance is then $Y = 0.0122 + j0.0132$ S.

例 2 一特性阻抗$Z_c = 50\Omega$的同轴线，终端负载$Z_l = 32.5 - j20\Omega$。试求：①线上的驻波比；②第一个电压波节点和波腹点距终端的距离l_{min1}和l_{max1}；③距终端4.8λ入阻抗和输入导纳。



解：

①求线上的驻波比

因归一化负载阻抗为 $z_l = \dfrac{Z_l}{Z_c} = 0.65 - j0.4$

在阻抗圆图上找到 z_l 的位置，即a点。以o为圆心，oa为半径画圆，交右半实轴于b点，读得 $\rho = 1.9$

②求l_{min1}和l_{max1}

a点对应的电刻度为0.412，由a点沿等反射系数圆顺时针转至左半实轴，得第一电压波节点，其电刻度为0.5

$$l_{min1} = (0.5 - 0.412)\lambda = 0.088\lambda$$

再由a点沿等反射系数圆顺时针转至右半实轴，得

$$l_{max1} = [(0.5 - 0.412) + 0.25]\lambda = 0.338\lambda$$

③求输入阻抗Z_{in}和输入导纳Y_{in}

由 a 点沿等反射系数圆顺时针转 4.8 电刻度(只须转 0.3 电刻度)至 c 点，读得归一化输入阻抗z_{in}为

$$z_{in} = 1.66 + j0.52$$

输入阻抗Z_{in}为

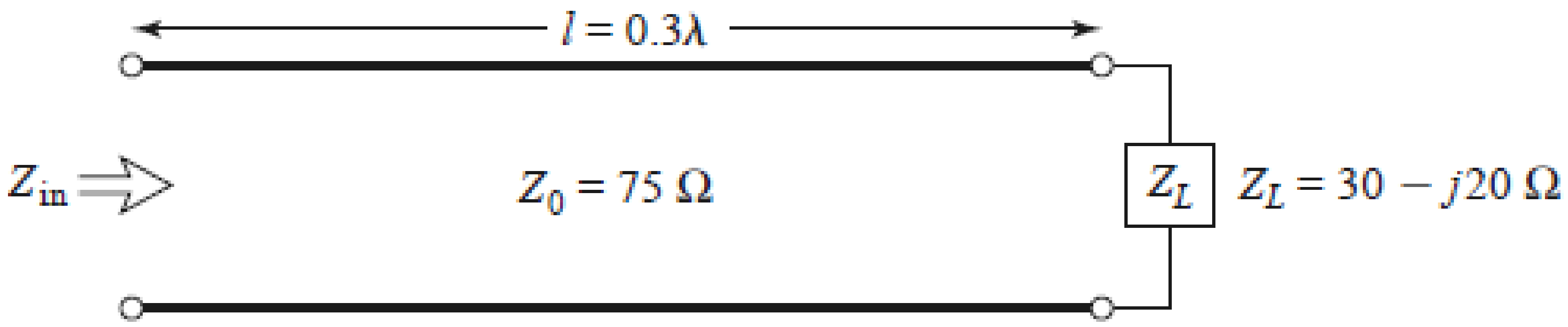
$$Z_{in} = z_{in} Z_c = 83 + j26$$

再由 c 点沿等反射系数圆旋转180°度至 d 点，读得归一化输入导纳y_{in}为$y_{in} = 0.55 - j0.17$, 输入导纳Y_{in}为

$$Y_{in} = y_{in} Y_c = 0.011 - j0.0034 \quad (S)$$

Homework

2.8 A lossless transmission line of electrical length $= 0.3\lambda$ *is terminated with a complex load impedance* as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.



2.9 A 75 Ω coaxial transmission line has a length of 2.0 cm and is terminated with a load impedance of $37.5 + j75\ \Omega$. If the relative permittivity of the line is 2.56 and the frequency is 3.0 GHz, find the input impedance to the line, the reflection coefficient at the load, the reflection coefficient at the input, and the SWR on the line.