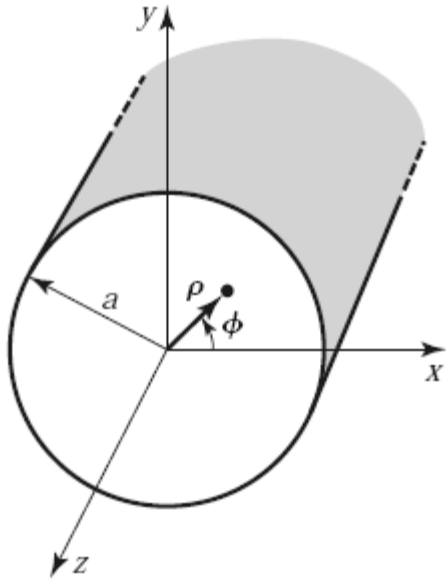


# Lec7 Transmission Lines and waveguides (II)

## 3.4 CIRCULAR WAVEGUIDE

A hollow, round metal pipe also supports TE and TM waveguide modes.

we can derive the cylindrical components of the transverse fields from the longitudinal components as



$$E_{\rho} = \frac{-j}{k_c^2} \left( \beta \frac{\partial E_z}{\partial \rho} + \frac{\omega \mu}{\rho} \frac{\partial H_z}{\partial \phi} \right),$$

$$E_{\phi} = \frac{-j}{k_c^2} \left( \frac{\beta}{\rho} \frac{\partial E_z}{\partial \phi} - \omega \mu \frac{\partial H_z}{\partial \rho} \right),$$

$$H_{\rho} = \frac{j}{k_c^2} \left( \frac{\omega \epsilon}{\rho} \frac{\partial E_z}{\partial \phi} - \beta \frac{\partial H_z}{\partial \rho} \right),$$

$$H_{\phi} = \frac{-j}{k_c^2} \left( \omega \epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho} \frac{\partial H_z}{\partial \phi} \right),$$

## TE Modes

For TE modes,  $E_z = 0$ , and  $H_z$  is a solution to the wave equation,

$$\nabla^2 H_z + k^2 H_z = 0.$$

If  $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-j\beta z}$ ,

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right) h_z(\rho, \phi) = 0.$$

apply the method of separation of variables.

$$h_z(\rho, \phi) = R(\rho)P(\phi),$$

$$\frac{1}{R} \frac{d^2 R}{d\rho^2} + \frac{1}{\rho R} \frac{dR}{d\rho} + \frac{1}{\rho^2 P} \frac{d^2 P}{d\phi^2} + k_c^2 = 0,$$

$$\frac{-1}{P} \frac{d^2 P}{d\phi^2} = k_\phi^2,$$

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + (\rho^2 k_c^2 - k_\phi^2) R = 0.$$

$$P(\phi) = A \sin k_\phi \phi + B \cos k_\phi \phi.$$

Because the solution to  $h_z$  must be periodic in  $\phi$  [i.e.,  $h_z(\rho, \phi) = h_z(\rho, \phi \pm 2m\pi)$ ],

$k_\phi$  must be an integer,  $n$ .

$$P(\phi) = A \sin n\phi + B \cos n\phi,$$

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + (\rho^2 k_c^2 - n^2) R = 0,$$

which is recognized as **Bessel's differential equation**.

The solution is

$$R(\rho) = C J_n(k_c \rho) + D Y_n(k_c \rho),$$

where  $J_n(x)$  and  $Y_n(x)$  are the Bessel functions of first and second kinds, respectively.

$Y_n(k_c \rho)$  becomes infinite at  $\rho = 0$ , so  $D = 0$ .

The solution for  $h_z$  can then be simplified to

$$h_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k_c \rho),$$

We must still determine the cutoff wave number  $k_c$ , which we can do by enforcing the boundary condition that  $E_{\phi} = 0$  on the waveguide wall.

$$E_{\phi}(\rho, \phi) = 0 \quad \text{at } \rho = a.$$

$$E_{\phi}(\rho, \phi, z) = \frac{j\omega\mu}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z},$$

$J'_n(k_c \rho)$  refers to the derivative of  $J_n$

For  $E_{\phi}$  to vanish at  $\rho = a$ , we must have  $J'_n(k_c a) = 0$ .

If the roots of  $J'_n(x)$  are defined as  $p'_{nm}$ ,

$$k_{c_{nm}} = \frac{p'_{nm}}{a}.$$

Values of  $p'_{nm}$  are given in mathematical tables;

**TABLE 3.3** Values of  $p'_{nm}$  for TE Modes of a Circular Waveguide

$n$	$p'_{n1}$	$p'_{n2}$	$p'_{n3}$
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

the cutoff wave number  $k_{c_{nm}} = p'_{nm}/a$ ,

where  $n$  refers to the number of circumferential ( $\phi$ ) variations and  $m$  refers to the number of radial ( $\rho$ ) variations.

The propagation constant of the  $TE_{nm}$  mode is

$$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2},$$

with a cutoff frequency of

$$f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p'_{nm}}{2\pi a\sqrt{\mu\epsilon}}.$$

**The first TE mode is the TE<sub>11</sub> mode** since it has the smallest  $\rho'_{nm}$ .  
Because  $m \geq 1$ , there is no TE<sub>10</sub> mode, but there is a TE<sub>01</sub> mode.

The transverse field components are,

$$E_\rho = \frac{-j\omega\mu n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z},$$

$$E_\phi = \frac{j\omega\mu}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z},$$

$$H_\rho = \frac{-j\beta}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z},$$

$$H_\phi = \frac{-j\beta n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z}.$$

The wave impedance is

$$Z_{\text{TE}} = \frac{E_\rho}{H_\phi} = \frac{-E_\phi}{H_\rho} = \frac{\eta k}{\beta}.$$

$A$  and  $B$  control the amplitude of the  $\sin n\phi$  and  $\cos n\phi$  terms, which are independent. The actual amplitudes of these terms will depend on the excitation of the waveguide.

Now consider the dominant TE<sub>11</sub> mode with an excitation such that  $B = 0$ . The fields can be written as

$$\begin{aligned}H_z &= A \sin \phi J_1(k_c \rho) e^{-j\beta z}, \\E_\rho &= \frac{-j\omega\mu}{k_c^2 \rho} A \cos \phi J_1(k_c \rho) e^{-j\beta z}, \\E_\phi &= \frac{j\omega\mu}{k_c} A \sin \phi J_1'(k_c \rho) e^{-j\beta z}, \\H_\rho &= \frac{-j\beta}{k_c} A \sin \phi J_1'(k_c \rho) e^{-j\beta z}, \\H_\phi &= \frac{-j\beta}{k_c^2 \rho} A \cos \phi J_1(k_c \rho) e^{-j\beta z}, \\E_z &= 0.\end{aligned}$$



The power flow down the guide can be computed as

$$\begin{aligned}
 P_o &= \frac{1}{2} \operatorname{Re} \int_{\rho=0}^a \int_{\phi=0}^{2\pi} \bar{\mathbf{E}} \times \bar{\mathbf{H}}^* \cdot \hat{\mathbf{z}} \rho \, d\phi \, d\rho \\
 &= \frac{1}{2} \operatorname{Re} \int_{\rho=0}^a \int_{\phi=0}^{2\pi} \left( E_\rho H_\phi^* - E_\phi H_\rho^* \right) \rho \, d\phi \, d\rho \\
 &= \frac{\omega \mu |A|^2 \operatorname{Re}(\beta)}{2k_c^4} \int_{\rho=0}^a \int_{\phi=0}^{2\pi} \left[ \frac{1}{\rho^2} \cos^2 \phi J_1^2(k_c \rho) + k_c^2 \sin^2 \phi J_1'^2(k_c \rho) \right] \rho \, d\phi \, d\rho \\
 &= \frac{\pi \omega \mu |A|^2 \operatorname{Re}(\beta)}{2k_c^4} \int_{\rho=0}^a \left[ \frac{1}{\rho} J_1^2(k_c \rho) + \rho k_c^2 J_1'^2(k_c \rho) \right] d\rho \\
 &= \frac{\pi \omega \mu |A|^2 \operatorname{Re}(\beta)}{4k_c^4} \left( P_{11}'^2 - 1 \right) J_1^2(k_c a), \tag{3.131}
 \end{aligned}$$

which is seen to be nonzero only when  $\beta$  is *real*, corresponding to a propagating mode.

Attenuation due to dielectric loss is

$$\alpha_d = \frac{k^2 \tan \delta}{2\beta} \text{ Np/m (TE or TM waves).}$$

The attenuation due to a lossy waveguide conductor can be found by computing the power loss per unit length of guide:

$$\begin{aligned} P_\ell &= \frac{R_s}{2} \int_{\phi=0}^{2\pi} |\bar{J}_s|^2 a \, d\phi \\ &= \frac{R_s}{2} \int_{\phi=0}^{2\pi} (|H_\phi|^2 + |H_z|^2) a \, d\phi \\ &= \frac{|A|^2 R_s}{2} \int_{\phi=0}^{2\pi} \left( \frac{\beta^2}{k_c^4 a^2} \cos^2 \phi + \sin^2 \phi \right) J_1^2(k_c a) a \, d\phi \\ &= \frac{\pi |A|^2 R_s a}{2} \left( 1 + \frac{\beta^2}{k_c^4 a^2} \right) J_1^2(k_c a). \end{aligned}$$

$$\begin{aligned} \alpha_c &= \frac{P_\ell}{2P_o} = \frac{R_s (k_c^4 a^2 + \beta^2)}{\eta k \beta a (p_{11}'^2 - 1)} \\ &= \frac{R_s}{a k \eta \beta} \left( k_c^2 + \frac{k^2}{p_{11}'^2 - 1} \right) \text{ Np/m.} \end{aligned}$$

The attenuation constant is then

# TM Modes

we must solve for  $E_z$  from the wave equation in cylindrical coordinates:

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right) e_z = 0,$$

where  $E_z(\rho, \phi, z) = e_z(\rho, \phi) e^{-j\beta z}$ , and  $k_c^2 = k^2 - \beta^2$ .

the general solutions  $e_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k_c \rho)$ .

the boundary conditions  $E_z(\rho, \phi) = 0$  at  $\rho = a$ .

$$J_n(k_c a) = 0,$$

then

$$k_c = p_{nm}/a,$$

where  $p_{nm}$  is the  $m$ th root of  $J_n(x)$ , that is,  $J_n(p_{nm}) = 0$ .

**TABLE 3.4** Values of  $p_{nm}$  for TM Modes of a Circular Waveguide

$n$	$p_{n1}$	$p_{n2}$	$p_{n3}$
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

The propagation constant of the  $\text{TM}_{nm}$  mode is

$$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (p_{nm}/a)^2},$$

the cutoff frequency is

$$f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p_{nm}}{2\pi a\sqrt{\mu\epsilon}}.$$

the first TM mode to propagate is the  $\text{TM}_{01}$  mode, with  $p_{01} = 2.405$ .

Since  $p_{01} = 2.405$  is greater than  $p'_{11} = 1.841$

**the TE<sub>11</sub> mode is the dominant mode of the circular waveguide.**

$m \geq 1$ , so there is no TM<sub>10</sub> mode.

the transverse fields can be derived as

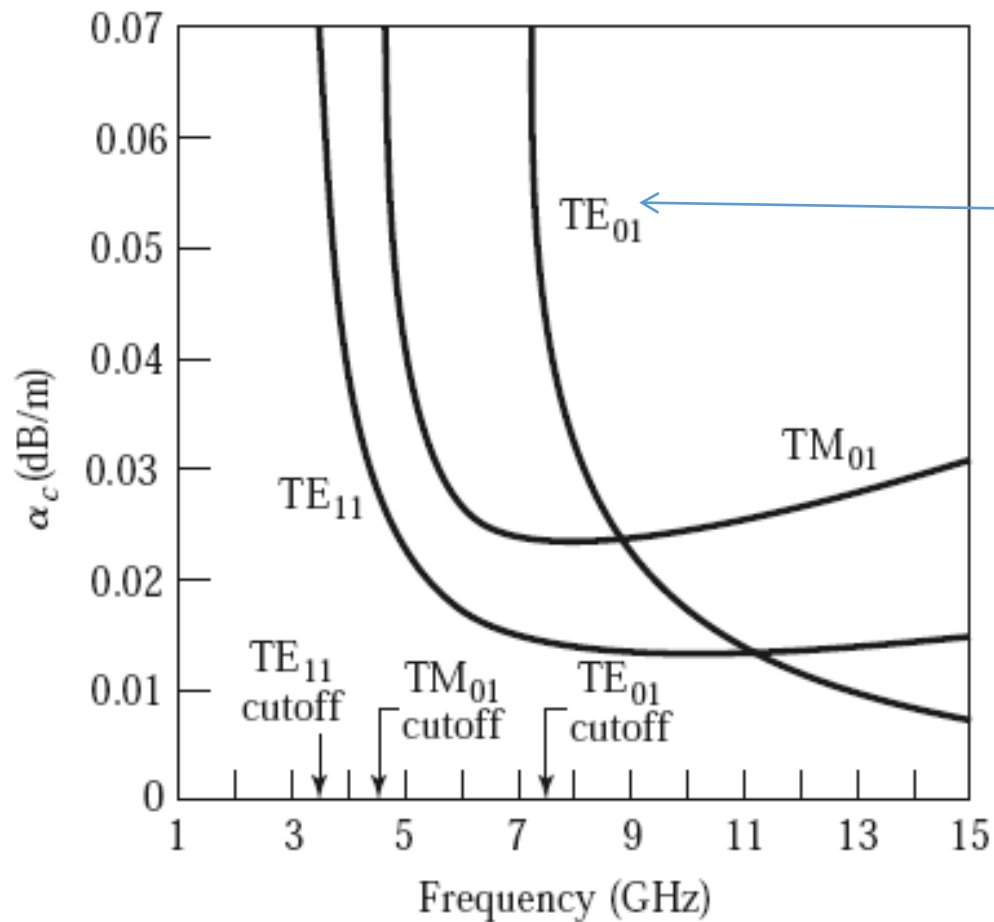
$$E_{\rho} = \frac{-j\beta}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z},$$

$$E_{\phi} = \frac{-j\beta n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z},$$

$$H_{\rho} = \frac{j\omega\epsilon n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z},$$

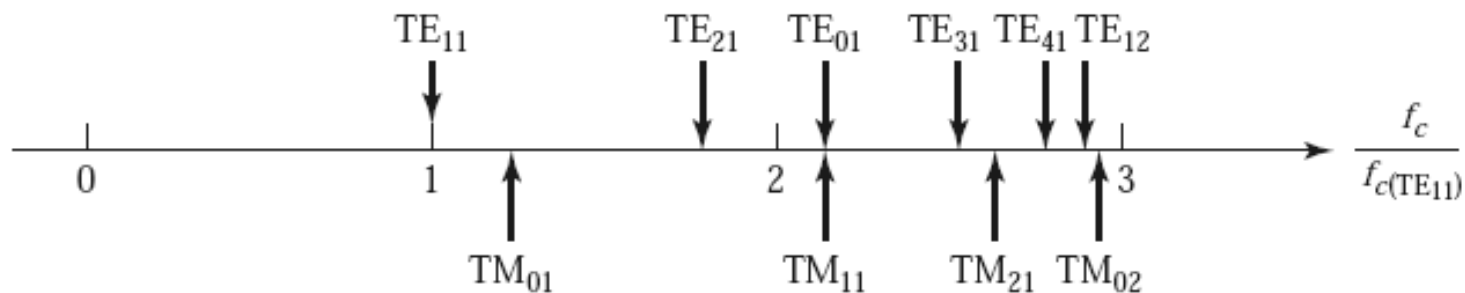
$$H_{\phi} = \frac{-j\omega\epsilon}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z}.$$

The wave impedance is  $Z_{\text{TM}} = \frac{E_{\rho}}{H_{\phi}} = \frac{-E_{\phi}}{H_{\rho}} = \frac{\eta\beta}{k}$ .



the attenuation of the  $TE_{01}$  mode decreases to a very small value with increasing frequency.

This property makes the  $TE_{01}$  mode of interest for low-loss transmission over long distances. Unfortunately, this mode is not the dominant mode of the circular waveguide, so in practice power can be lost from the  $TE_{01}$  mode to lower order propagating modes.

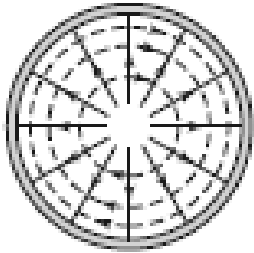
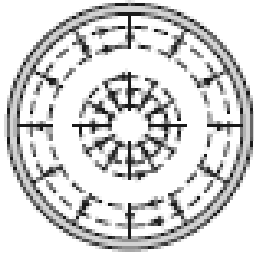
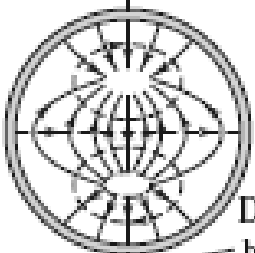
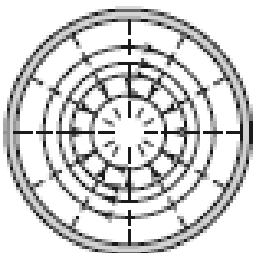
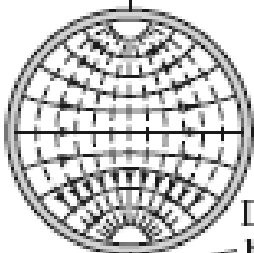
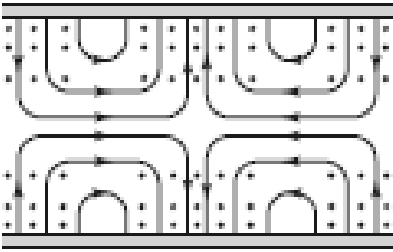
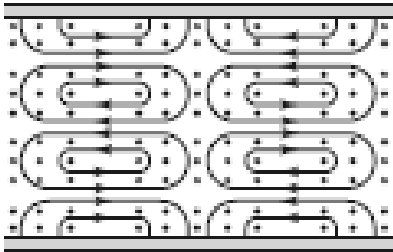
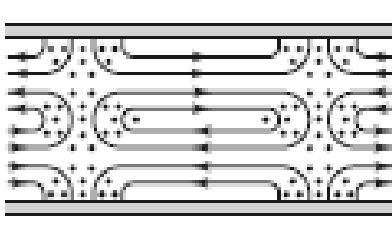
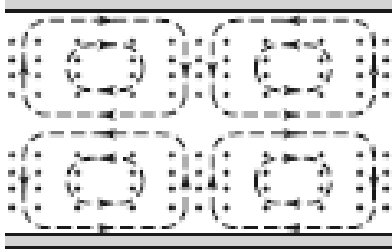
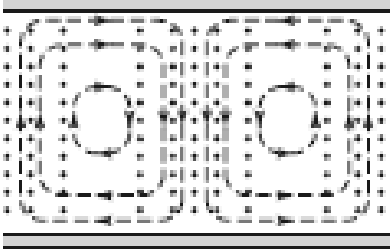


**TABLE 3.5** Summary of Results for Circular Waveguide

Quantity	TE <sub><i>nm</i></sub> Mode	TM <sub><i>nm</i></sub> Mode
$k$	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
$k_c$	$\frac{P'_{nm}}{a}$	$\frac{P_{nm}}{a}$
$\beta$	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$
$\lambda_c$	$\frac{2\pi}{k_c}$	$\frac{2\pi}{k_c}$
$\lambda_g$	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$
$v_p$	$\frac{\omega}{\beta}$	$\frac{\omega}{\beta}$
$\alpha_d$	$\frac{k^2 \tan \delta}{2\beta}$	$\frac{k^2 \tan \delta}{2\beta}$

Quantity	TE <sub>nm</sub> Mode	TM <sub>nm</sub> Mode
$E_z$	0	$(A \sin n\phi + B \cos n\phi) J_n(k_c \rho) e^{-j\beta z}$
$H_z$	$(A \sin n\phi + B \cos n\phi) J_n(k_c \rho) e^{-j\beta z}$	0
$E_\rho$	$\frac{-j\omega\mu n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z}$	$\frac{-j\beta}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z}$
$E_\phi$	$\frac{j\omega\mu}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z}$	$\frac{-j\beta n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z}$
$H_\rho$	$\frac{-j\beta}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z}$	$\frac{j\omega\epsilon n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z}$
$H_\phi$	$\frac{-j\beta n}{k_c^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_c \rho) e^{-j\beta z}$	$\frac{-j\omega\epsilon}{k_c} (A \sin n\phi + B \cos n\phi) J'_n(k_c \rho) e^{-j\beta z}$
$Z$	$Z_{\text{TE}} = \frac{k\eta}{\beta}$	$Z_{\text{TM}} = \frac{\beta\eta}{k}$



$TM_{01}$	$TM_{02}$	$TM_{11}$	$TE_{01}$	$TE_{11}$
		 <p data-bbox="1345 496 1523 611">Distributions below along this plane</p>		 <p data-bbox="2331 496 2509 611">Distributions below along this plane</p>
				

## EXAMPLE 3.2 CHARACTERISTICS OF A CIRCULAR WAVEGUIDE

Find the cutoff frequencies of the first two propagating modes of a Teflon-filled circular waveguide with  $a = 0.5 \text{ cm}$ . *If the interior of the guide is gold plated,* calculate the overall loss in dB for a 30 cm length operating at 14 GHz.

### *Solution*

The first two propagating modes of a circular waveguide are the TE<sub>11</sub> and TM<sub>01</sub> modes. The cutoff frequencies can be found as

$$\text{TE}_{11}: \quad f_c = \frac{p'_{11}c}{2\pi a\sqrt{\epsilon_r}} = \frac{1.841(3 \times 10^8)}{2\pi(0.005)\sqrt{2.08}} = 12.19 \text{ GHz},$$

$$\text{TM}_{01}: \quad f_c = \frac{p_{01}c}{2\pi a\sqrt{\epsilon_r}} = \frac{2.405(3 \times 10^8)}{2\pi(0.005)\sqrt{2.08}} = 15.92 \text{ GHz}.$$

So only the TE<sub>11</sub> mode is propagating at 14 GHz. The wave number is

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = \frac{2\pi(14 \times 10^9)\sqrt{2.08}}{3 \times 10^8} = 422.9 \text{ m}^{-1},$$

and the propagation constant of the TE<sub>11</sub> mode is

$$\beta = \sqrt{k^2 - \left(\frac{P'_{11}}{a}\right)^2} = \sqrt{(422.9)^2 - \left(\frac{1.841}{0.005}\right)^2} = 208.0 \text{ m}^{-1}.$$

The attenuation due to dielectric loss is

$$\alpha_d = \frac{k^2 \tan \delta}{2\beta} = \frac{(422.9)^2 (0.0004)}{2(208.0)} = 0.172 \text{ Np/m} = 1.49 \text{ dB/m}.$$

The conductivity of gold is  $\sigma = 4.1 \times 10^7 \text{ S/m}$ , so the surface resistance is

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0367 \ \Omega.$$

the attenuation due to conductor loss is  $\alpha_c = \frac{R_s}{ak\eta\beta} \left( k_c^2 + \frac{k^2}{P'_{11}{}^2 - 1} \right) = 0.0672 \text{ Np/m} = 0.583 \text{ dB/m}.$

The total attenuation is  $\alpha = \alpha_d + \alpha_c = 2.07 \text{ dB/m}$ , and the loss in the 30 cm length of guide is

$$\text{attenuation (dB)} = \alpha(\text{dB/m}) \times L \text{ (m)} = (2.07)(0.3) = 0.62 \text{ dB.} \quad \blacksquare$$

## 3.5 COAXIAL LINE

### TEM Modes

the fields can be derived from a scalar potential function, which is a solution to Laplace's equation

In cylindrical coordinates Laplace's equation

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \Phi(\rho, \phi)}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \Phi(\rho, \phi)}{\partial \phi^2} = 0.$$

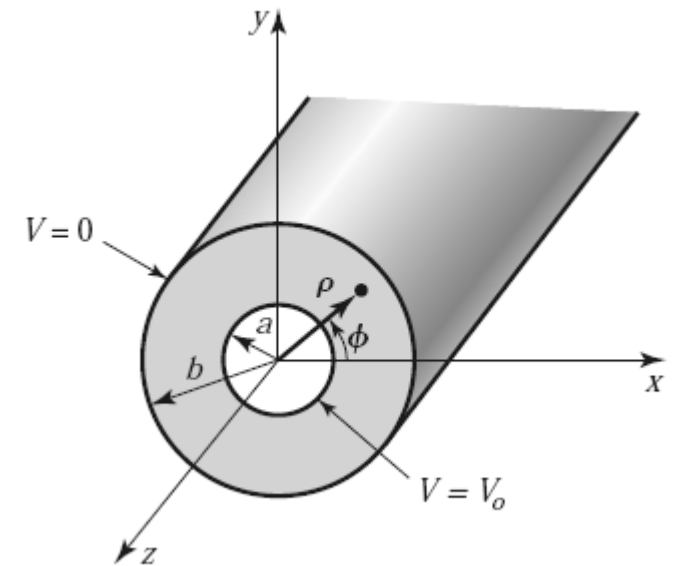
With the boundary conditions

$$\Phi(a, \phi) = V_o,$$

$$\Phi(b, \phi) = 0.$$

By the method of separation of variables,

$$\Phi(\rho, \phi) = R(\rho)P(\phi).$$



$$\frac{\rho}{R} \frac{\partial}{\partial \rho} \left( \rho \frac{dR}{d\rho} \right) + \frac{1}{P} \frac{d^2 P}{d\phi^2} = 0.$$

By the usual separation-of-variables argument,

$$\frac{\rho}{R} \frac{\partial}{\partial \rho} \left( \rho \frac{dR}{d\rho} \right) = -k_\rho^2,$$

$$\frac{1}{P} \frac{d^2 P}{d\phi^2} = -k_\phi^2,$$

$$k_\rho^2 + k_\phi^2 = 0.$$

The general solution  $P(\phi) = A \cos n\phi + B \sin n\phi, \quad k_\phi = n$

$\Phi(\rho, \phi)$  should not vary with  $\phi. \quad k_\phi = 0$

$$\frac{\partial}{\partial \rho} \left( \rho \frac{dR}{d\rho} \right) = 0. \quad R(\rho) = C \ln \rho + D, \quad \Phi(\rho, \phi) = C \ln \rho + D.$$

Applying the boundary conditions

$$\Phi(a, \phi) = V_o = C \ln a + D,$$

$$\Phi(b, \phi) = 0 = C \ln b + D.$$

After solving for C and D, we get the final solution

$$\Phi(\rho, \phi) = \frac{V_o \ln b/\rho}{\ln b/a}.$$

The E and H fields can now be found using

$$\bar{e}(x, y) = -\nabla_t \Phi(x, y), \quad \bar{h}(x, y) = \frac{1}{Z_{\text{TEM}}} \hat{z} \times \bar{e}(x, y).$$

$$\bar{E}(x, y, z) = [\bar{e}(x, y) + \hat{z}e_z(x, y)]e^{-j\beta z},$$

$$\bar{H}(x, y, z) = [\bar{h}(x, y) + \hat{z}h_z(x, y)]e^{-j\beta z},$$

# Higher Order Modes

The coaxial line, like the parallel plate waveguide, can also support TE and TM waveguide modes in addition to the TEM mode.

**In practice, these modes are usually cut off (evanescent), and so have only a reactive effect near discontinuities or sources, where they may be excited.**

It is important in practice, however, to be aware of the cutoff frequency of the lowest order waveguide-type modes to avoid the propagation of these modes.

Undesirable effects can occur if two or more modes with different propagation constants are propagating at the same time.

如何确定传输线的工作带宽？

For TE modes,  $E_z = 0$ , and  $H_z$  satisfies the wave equation of

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right) h_z(\rho, \phi) = 0,$$

where  $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-j\beta z}$ , and  $k_c^2 = k^2 - \beta^2$ .

$$h_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi)(C J_n(k_c \rho) + D Y_n(k_c \rho)).$$

$$E_\phi = \frac{j\omega\mu}{k_c} (A \sin n\phi + B \cos n\phi)[C J'_n(k_c \rho) + D Y'_n(k_c \rho)]e^{-j\beta z}.$$

The boundary conditions are

$$E_\phi(\rho, \phi, z) = 0 \text{ for } \rho = a, b.$$

Then

$$C J'_n(k_c a) + D Y'_n(k_c a) = 0,$$

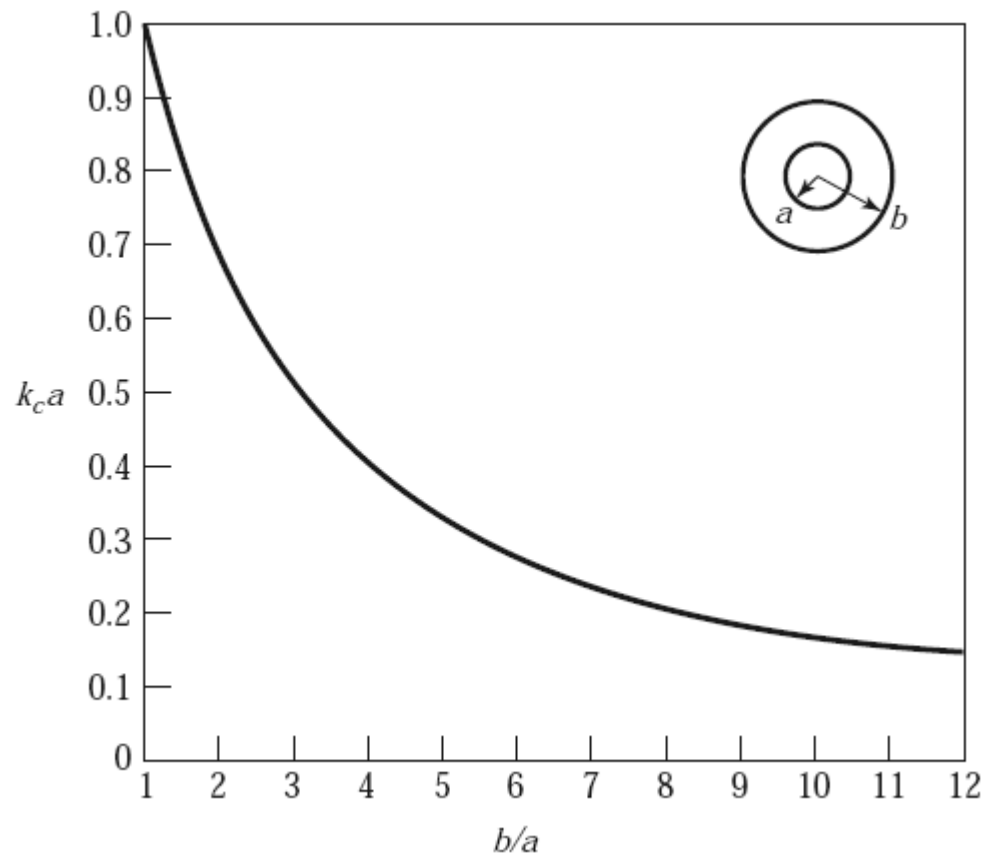
$$C J'_n(k_c b) + D Y'_n(k_c b) = 0.$$



Because this is a homogeneous set of equations, the only nontrivial ( $C \neq 0$ ,  $D \neq 0$ ) solution occurs when the determinant is zero. Thus we must have

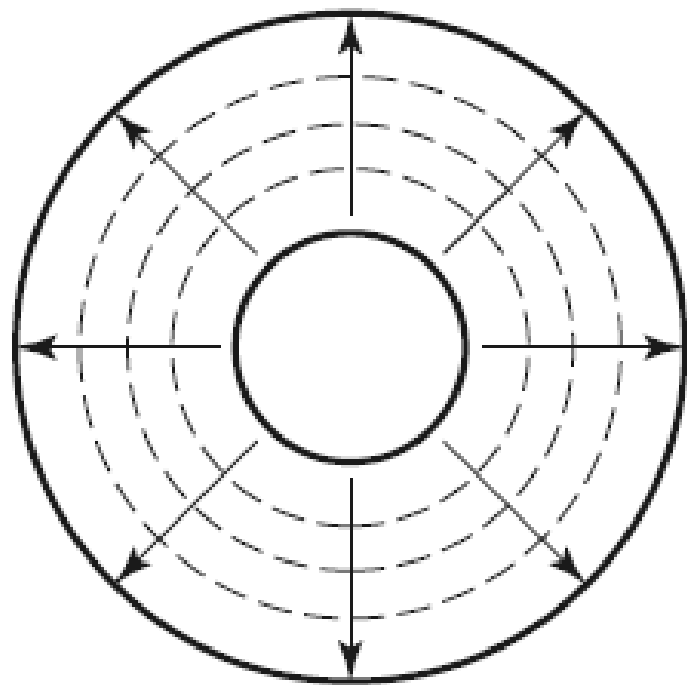
$$J'_n(k_c a) Y'_n(k_c b) = J'_n(k_c b) Y'_n(k_c a).$$

This is a characteristic (or eigenvalue) equation for  $k_c$ .

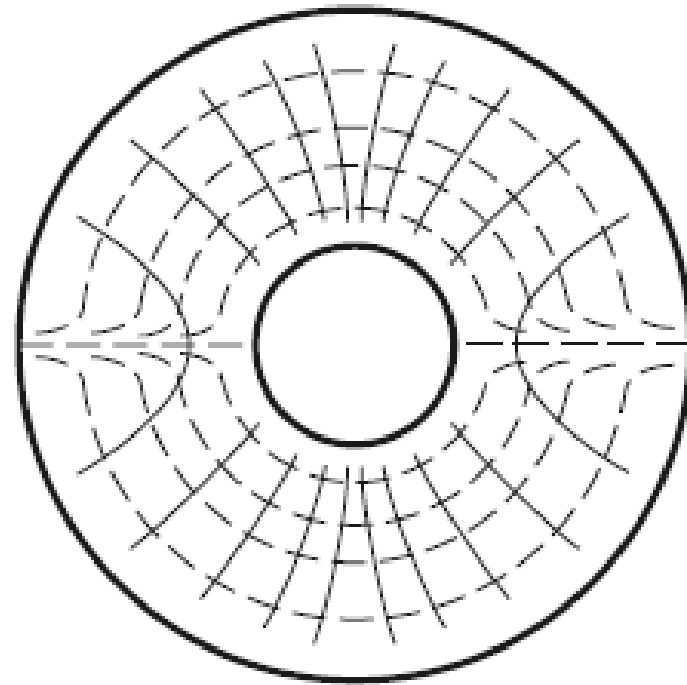


An approximate solution of the cutoff number for the **TE<sub>11</sub> mode** that is often used in practice is

$$k_c = \frac{2}{a + b}.$$



(a)



(b)

Field lines for the (a) TEM and (b)  $TE_{11}$  modes of a coaxial line.

### EXAMPLE 3.3 HIGHER ORDER MODE OF A COAXIAL LINE

Consider a RG-401U semirigid coaxial cable, with inner and outer conductor diameters of 0.0645 in. and 0.215 in., and a Teflon dielectric 2.2. *What is the highest usable frequency before the TE<sub>11</sub> waveguide mode starts to propagate?*

*Solution*

We have

$$\frac{b}{a} = \frac{2b}{2a} = \frac{0.215}{0.0645} = 3.33.$$

From Figure 3.16 this value of  $b/a$  gives  $k_c a = 0.45$  [the approximate result is  $k_c a = 2/(1 + b/a) = 0.462$ ]. Thus,  $k_c = 549.4 \text{ m}^{-1}$ , and the cutoff frequency of the  $TE_{11}$  mode is

$$f_c = \frac{ck_c}{2\pi\sqrt{\epsilon_r}} = 17.7 \text{ GHz.}$$

In practice, a 5% safety margin is usually recommended, so

$$f_{\max} = (0.95)(17.7 \text{ GHz}) = 16.8 \text{ GHz.}$$





## Coaxial Connectors

Most coaxial cables and connectors in common use have a  $50\ \Omega$  characteristic impedance, with an exception being the  $75\ \Omega$  cable used in television systems. The reasoning behind these choices is that an air-filled coaxial line has minimum attenuation for a characteristic impedance of about  $77\ \Omega$  (Problem 2.27), while maximum power capacity occurs for a characteristic impedance of about  $30\ \Omega$  (Problem 3.28). A  $50\ \Omega$  characteristic impedance thus represents a compromise between minimum attenuation and maximum power capacity.

Connectors are used in pairs, with a male end and a female end (or plug and jack).

**SMA:** The need for smaller and lighter connectors led to the development of this connector in the 1960s. The outer diameter of the female end is about 0.25 in. It can be used up to frequencies in the range of 18–25 GHz and is probably the most commonly used microwave connector today.

# Homework

- 3.13** A circular copper waveguide has a radius of 0.4 cm and is filled with a dielectric material having  $\epsilon_r = 1.5$  and  $\tan \delta = 0.0002$ . Identify the first four propagating modes and their cutoff frequencies. For the dominant mode, calculate the total attenuation at 20 GHz.