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Lecture7

Semiconductor physics V

Carrier Transport Phenomenon

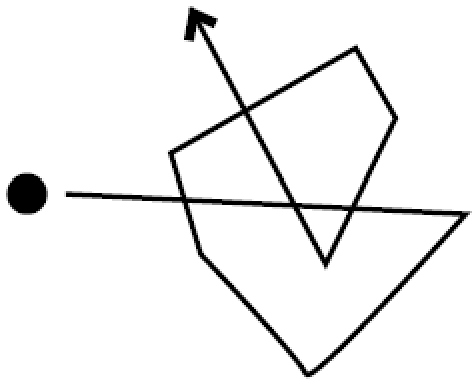
Carrier Transport

- The net flow of the electrons and holes in a semiconductor will generate currents.
- Transport：The process by which these charged particles move
- Three transport mechanisms
- Thermal Motion: the movement of charge due to temperature gradients. It decreases with technology scaling.
- Carrier drift(漂移电流): the movement of charge due to electric fields
- Carrier diffusion（扩散电流）：the flow of charge due to density gradients

Thermal Motion

In thermal equilibrium, carriers are not sitting still:

- ✓ Undergo collisions with vibrating Si atoms (Brownian motion)
- ✓ Electrostatically interact with each other and with ionized (charged) dopants



Characteristic time constant of thermal motion:
⇒ mean free time between collisions

$\tau_c \equiv$ collision time $[s]$

In between collisions, carriers acquire high velocity: $v_{th} \equiv$ **thermal velocity** $[cms^{-1}]$

Characteristic length of thermal motion:

$\lambda \equiv$ **mean free path [*cm*]**

$$\lambda = v_{th}\tau_c$$

Put numbers for Si at room temperature:

$$\tau_c \approx 10^{-13}\ \mathrm{s}$$

$$v_{th} = 10^7\,\mathrm{cms^{-1}}$$

$$\Rightarrow\ \lambda \approx 0.01\mu\mathrm{m}$$

For reference, state-of-the-art production MOSFET: $L_g \approx 0.1\mu\mathrm{m}$

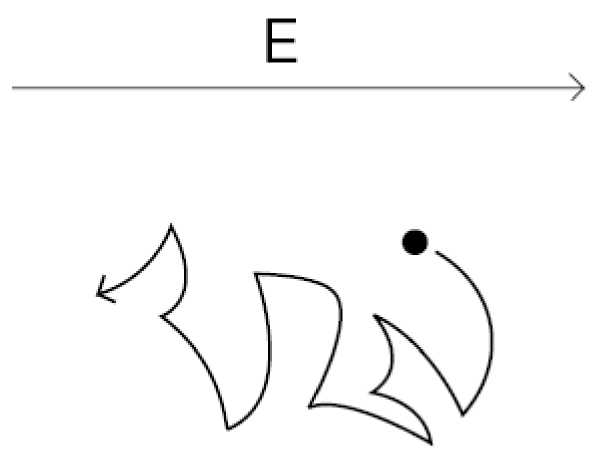
⇒ Carriers undergo many collisions as they travel through devices

Carrier Drift

Apply electric field to semiconductor:

$$E \equiv \text{electric field } [V\, cm^{-1}]$$

$\Rightarrow$ net force on carrier $F = \pm qE$



Between collisions, carriers accelerate in the direction of the electrostatic field:

$$v(t) = a \cdot t = \pm \frac{qE}{m_{n,p}} t$$

Newton's laws of motion-Second law

a- acceleration rate; *m*-mass of the body

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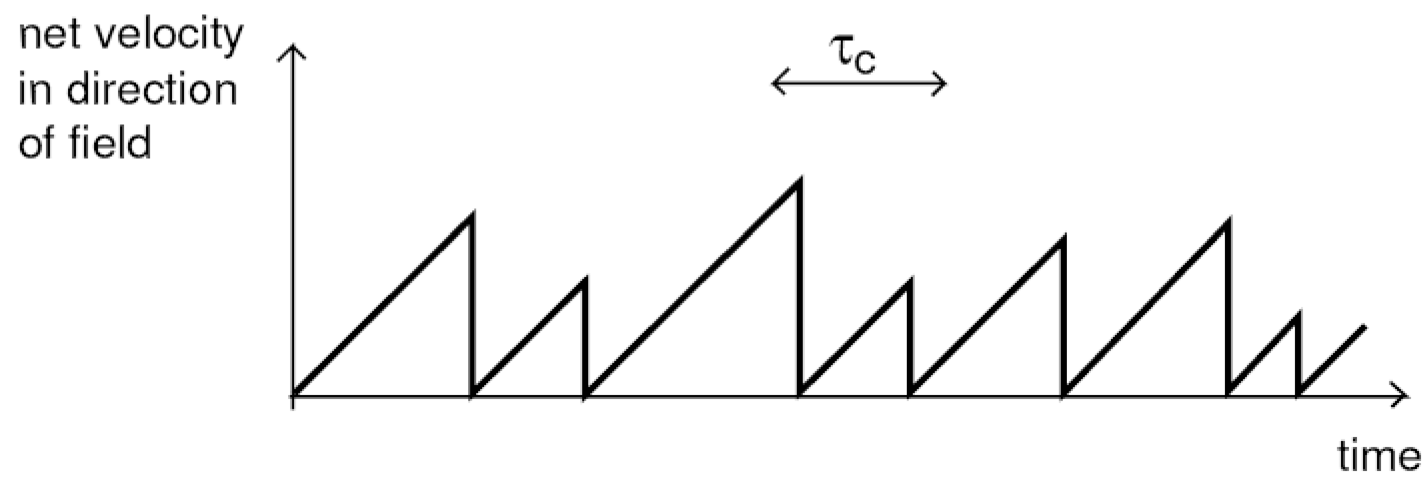
But there is (on the average) a collision every τ_c and the velocity is randomized:



The average net velocity in direction of the field:

$$\overline{v} = v_d = \pm\frac{qE}{2m_{n,p}}\tau_c = \pm\frac{q\tau_c}{2m_{n,p}}E$$

This is called **drift velocity [*cm s*$^{-1}$]**

Define the **mobility** of electrons or holes:

$$\mu_{n,p} = \pm \frac{q\tau_c}{2m_{n,p}} \equiv \text{mobility}[\text{cm}^2\text{V}^{-1}\text{s}^{-1}]$$

$$v_{\text{d}n} = -\mu_n E \;(\text{for electrons})$$

$$v_{\text{d}p} = \mu_p E \;(\text{for holes})$$

$$\frac{1}{\mu} = \frac{1}{\mu_\text{I}} + \frac{1}{\mu_\text{L}}$$

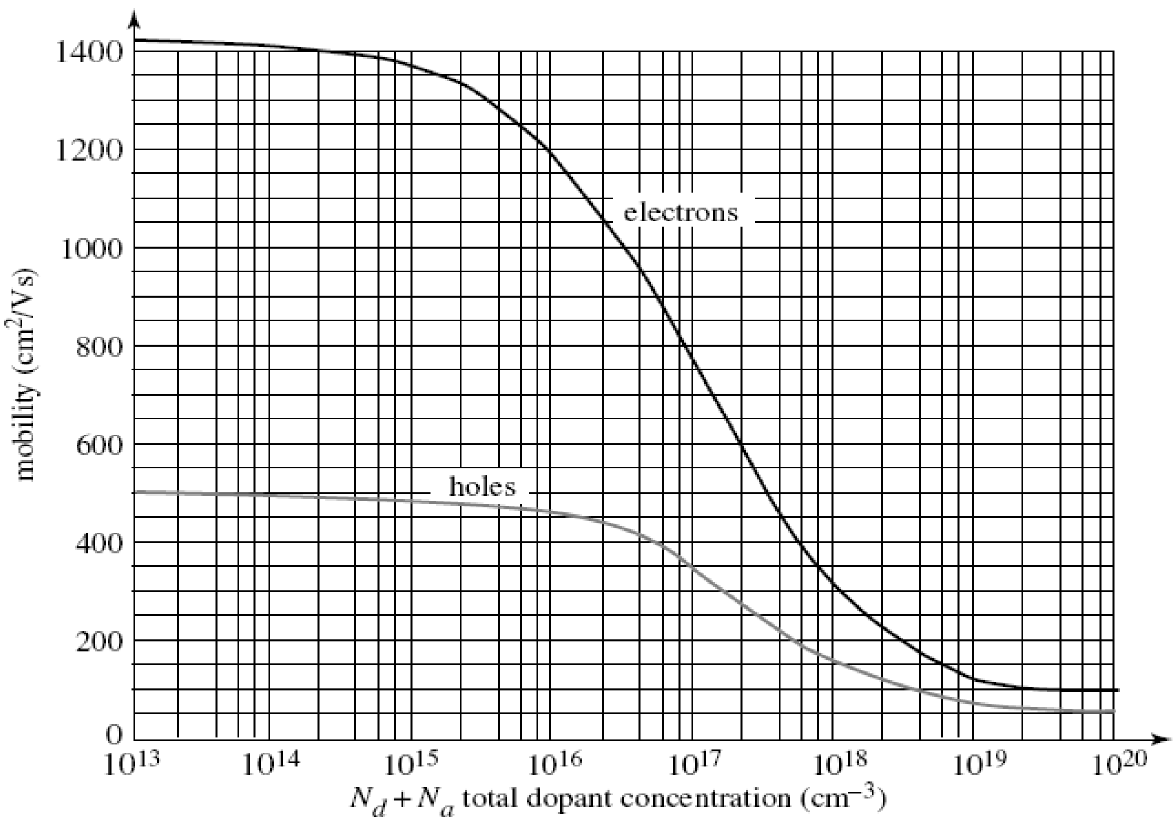
μ_I :the mobility due to the ionized impurity scattering process.

μ_L :the mobility due to the lattice scattering process.

Mobility is a measure of ease of carrier drift

- If $\tau_c\uparrow$, longer time between collisions $\Rightarrow \mu\uparrow$
- If $m\downarrow$, "lighter" particle $\Rightarrow \mu\uparrow$

At room temperature, mobility in Si depends on doping:



$$\mu_I \propto \frac{T^{3/2}}{N_I}$$

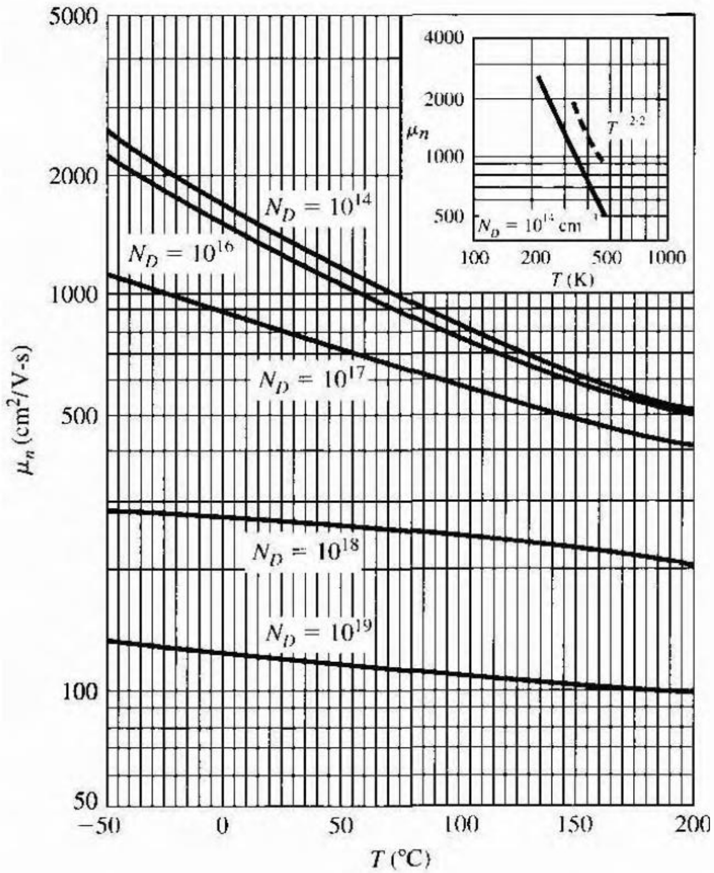
$$N_I = N_d^+ + N_a^-$$

The total ionized impurity concentration in the semiconductor.

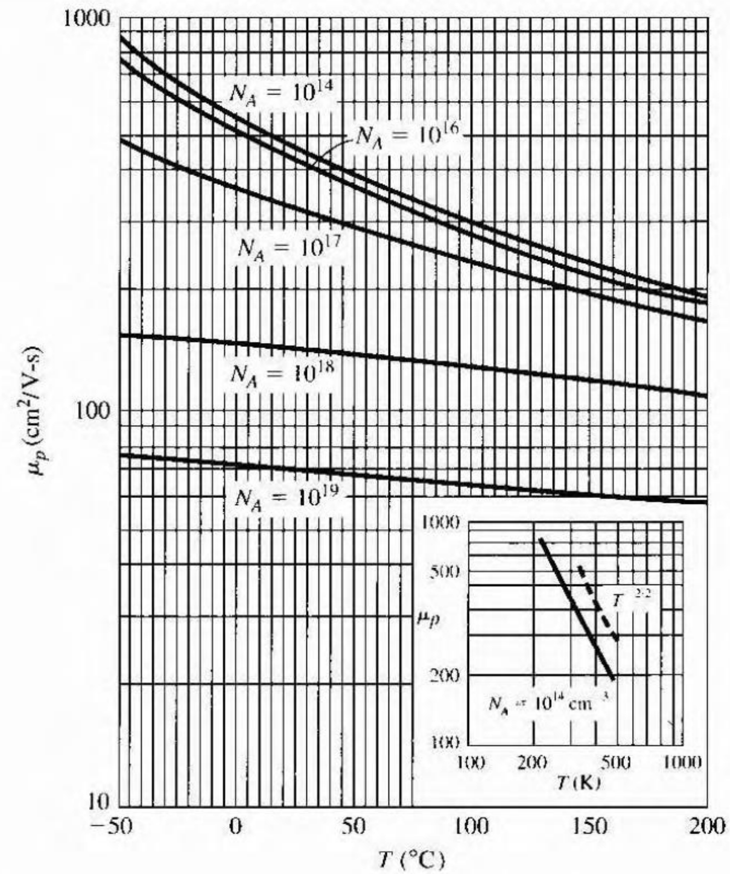
Electron and hole motilities are functions of temperature.

- For low doping level, the mobility is limited by collisions with lattice. As Temp ->**INCREASES;** mobility **-> DECREASES**

$$\mu_L \propto T^{-3/2}$$



(a) n-type



p-type (b)

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- For medium doping and high doping level, mobility limited by collisions with ionized impurities
- Holes " heavier" than electrons

– For same doping level, $\mu_n > \mu_p$

Velocity saturation:

The linear relationship between drift velocity and electric field breaks down when the electric field is high enough that the drift velocity approaches

$$v_{sat} = 10^7 \text{ cm/s}$$

-an important phenomenon for accurate modeling of VLSI devices characteristics .

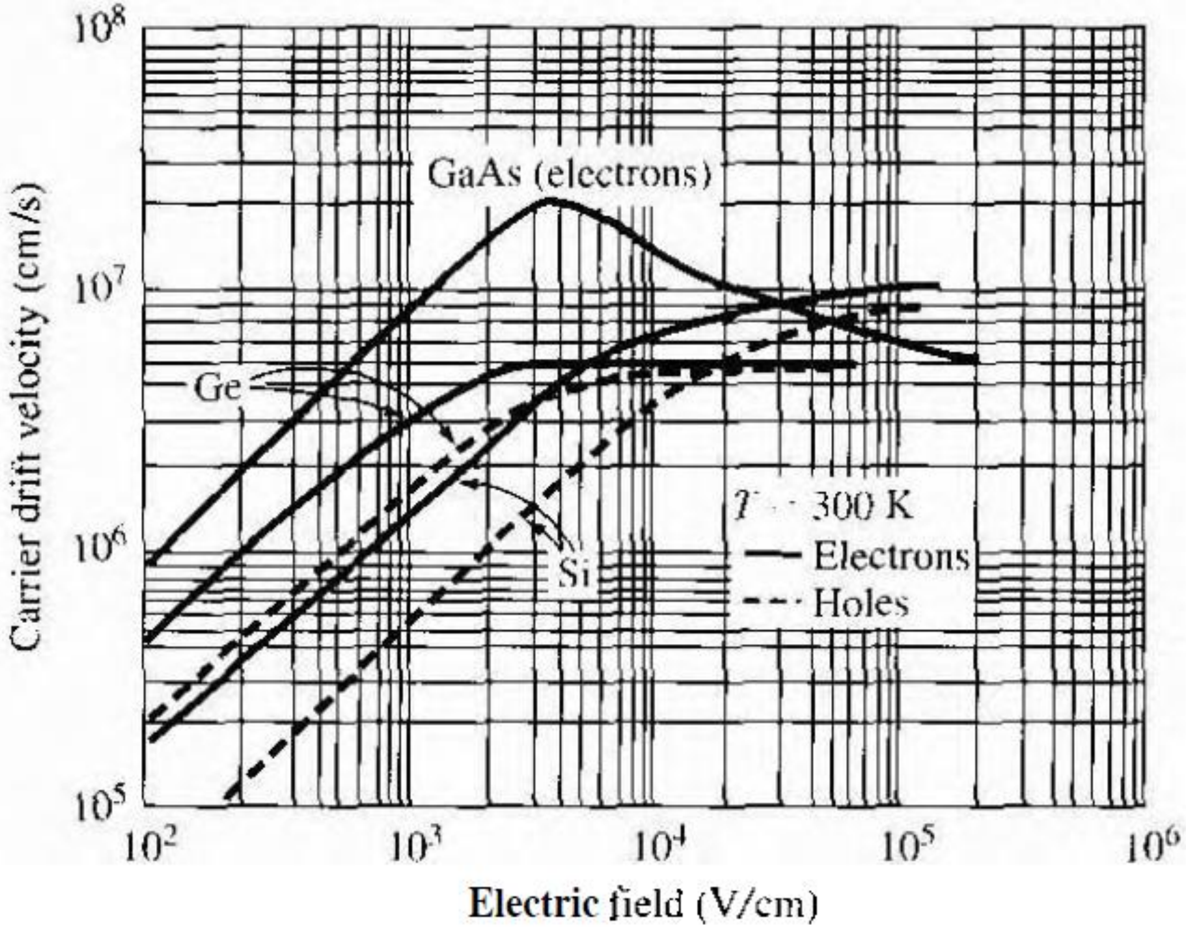

Fig Carrier drift velocity versus electric field for high-impurity silicon. germanium, and gallium arsenide.

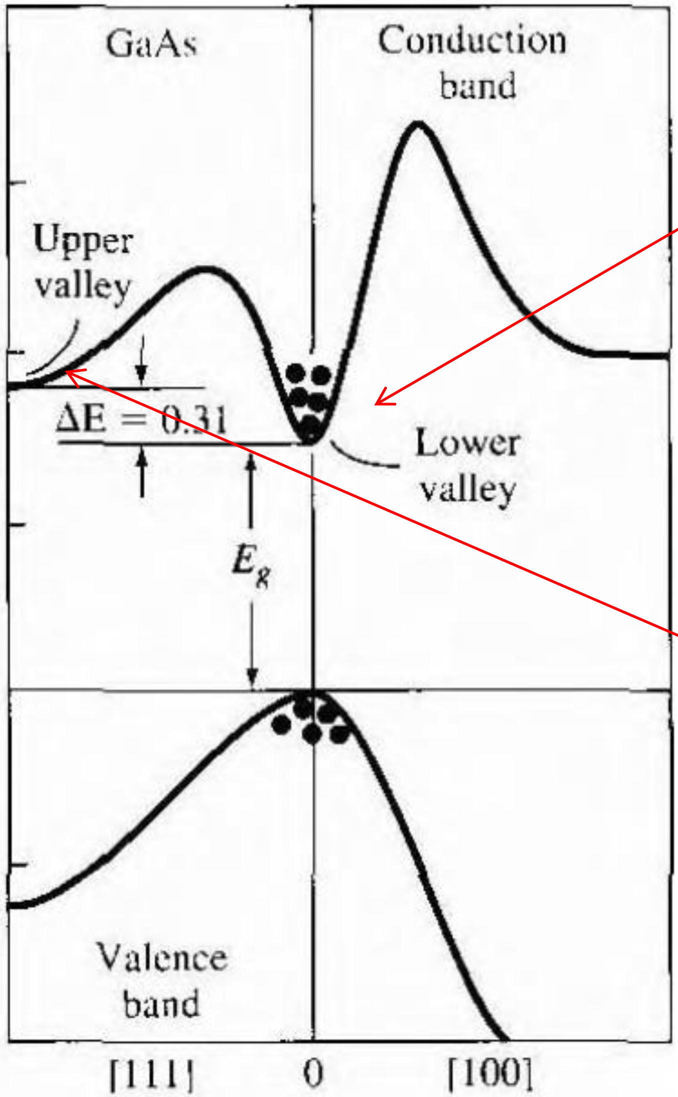

Fig Energy-band structure for gallium arsenide showing the upper valley and lower valley in the conduction band.

In the lower valley, the small effective mass leads to a large mobility.

As the E-field increases. the energy of the electron increases and the electron can be scattered into the upper valley

The larger effective mass in the upper valley yields a smaller mobility.

This intervalley transfer mechanism results in a decreasing average drift velocity of electrons with electric field, or the negative differential mobility characteristic.

Drift Current

Net velocity of charged particles

⇒ electric current:

Drift current denstity $\propto$ ***carrier drift velocity***

$\propto$ ***carrier concentration***

$\propto$ ***carrier charge***

$$J_{p-drift} = qp\mu_p E$$

$$J_{n-drift} = qn\mu_n E$$

for $|v_{dn,p} << v_{sat}|$

The total drift current density is obtained by combining the drift current density of positive carriers and negative carriers.

$$J_{drift} = q(p\mu_p + n\mu_n)E$$

It is a form of **Ohm's law**

$$J = \sigma E = \frac{E}{\rho}$$

$\sigma \equiv$ conductivity $[J^{-1} \bullet cm^{-1}]$

$\rho \equiv$ resistivity $[J \bullet cm]$

with the resistivity $$\rho = \frac{1}{q(p\mu_p + n\mu_n)}$$

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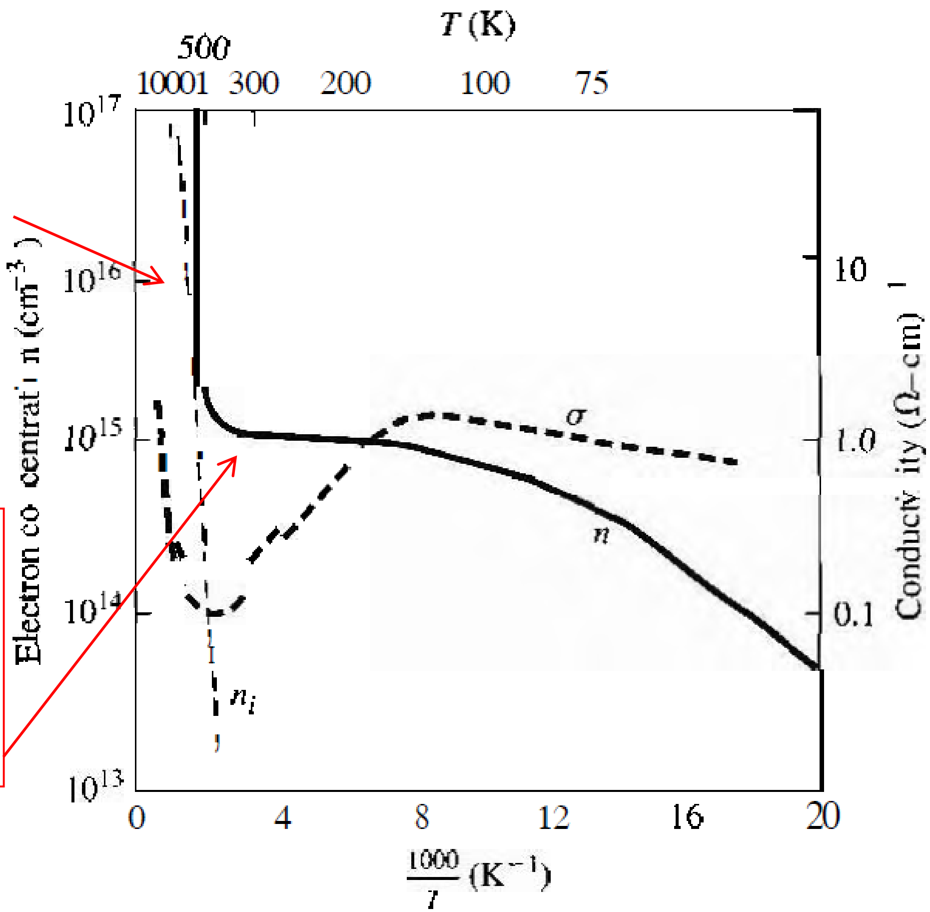
$$n_i^2 = N_c N_\upsilon \exp\left[\frac{-(E_c - E_\upsilon)}{kT}\right] = N_c N_\upsilon \exp\left[\frac{-E_g}{kT}\right]$$

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

At higher T, n_i increases and begins to dominate n and conductivity.

In the mid T range, or extrinsic range, complete ionization means the constant

$$n \approx N_d$$



In the lower T, freeze-out begins to occur, n decrease with $1/T$.

Fig Electron concentration and conductivity versus inverse temperature for silicon with $N_d = 10^{15}\ cm^{-3}$

Resistivity is commonly used to specify the doping level

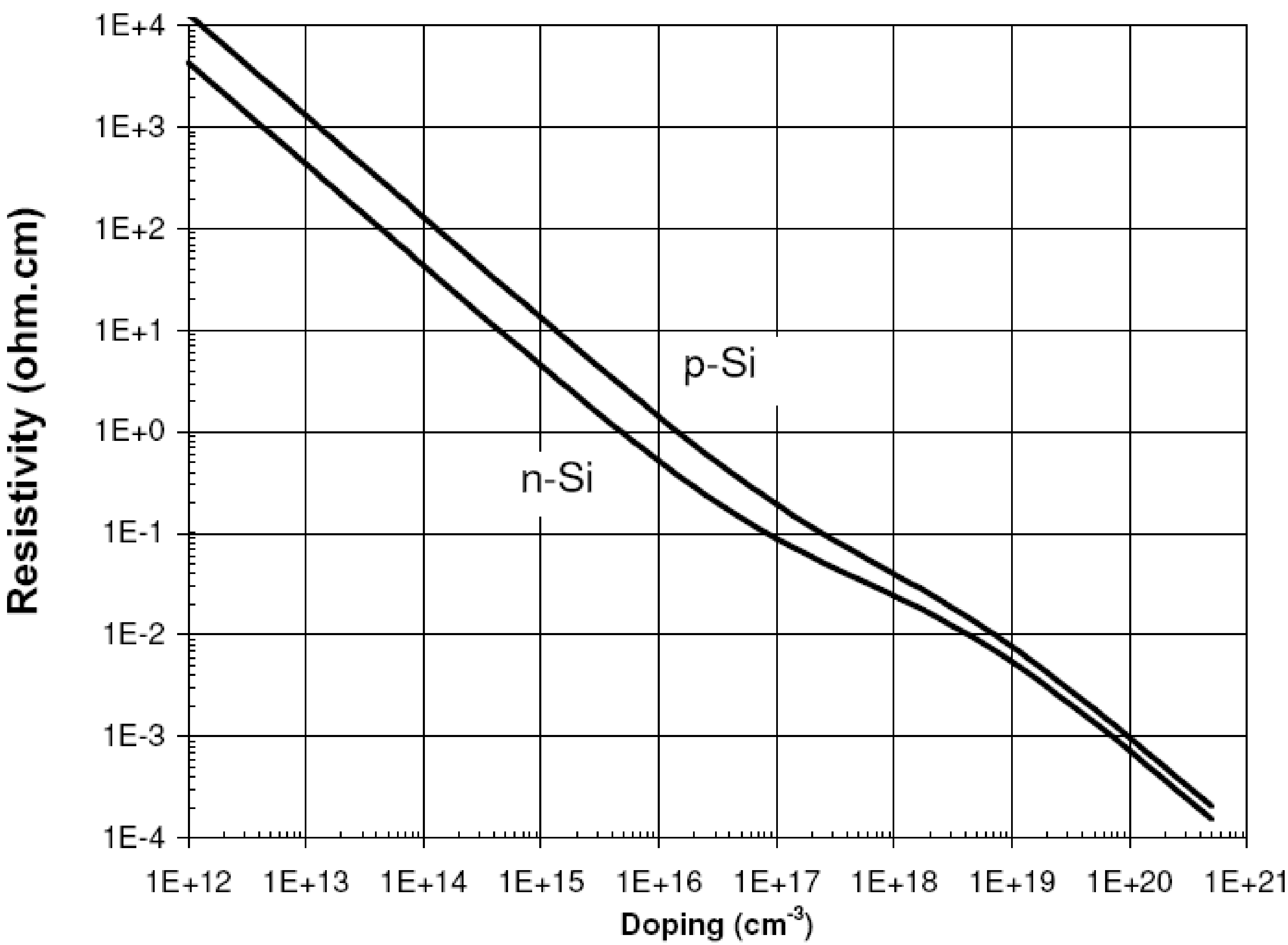
$$\rho_n \approx \frac{1}{qN_d \mu_n}$$, n-type semiconductor

$$\rho_p \approx \frac{1}{qN_a \mu_p}$$, p-type semiconductor

If impurity atoms are completely ionized

Sheet resistance $$R_{\square} = \left(\frac{1}{qN_d \mu_n t} \right)$$ Technology-dependent

$$R = \rho_n \left(\frac{L}{Wt} \right) = \left(\frac{1}{qN_d \mu_n t} \right) \left(\frac{L}{W} \right) = R_{\square} \left(\frac{L}{W} \right)$$


Resistivity (ohm.cm)
1E+4
1E+3
1E+2
1E+1
1E+0
1E-1
1E-2
1E-3
1E-4
p-Si
n-Si
1E+12
1E+13
1E+14
1E+15
1E+16
1E+17
1E+18
1E+19
1E+20
1E+21
Doping (cm-3)

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Example:

Si with $N_d = 3\times10^{16}\text{cm}^{-3}$ at room temperature

$\mu_n \approx 1000\text{cm}^2/\text{V}\cdot\text{s}$

$\rho_n \approx 0.21\Omega\cdot\text{cm}$

$n \approx 3\times10^{16}\text{cm}^{-3}$

Apply E=1kV/cm

$v_{\text{dn}} \approx -10^6\,\text{cm/s} \ll v_{th}$

$$J_n^{drift} \approx qnv_{\text{dn}} = qn\mu_n E = \sigma E = \frac{E}{\rho}$$

$$J_n^{drift} \approx 4\times10^3\,\text{A/cm}^2$$

Time to drift through the length L=0.1 um

$$t_d = \frac{L}{v_{\text{dn}}} = 10\text{ps}$$

Fast!

Example:

Consider a gallium arsenide sample at $T = 300\ K$ with doping concentrations of $N_a = 0$ and $N_d = 10^{16}\ cm^{-3}$. Assume complete ionization and assume electron and hole mobility are 1350 $cm^2/v\text{-}s$ and 480 $cm^2/v\text{-}s$, respectively.

Calculate the drift current density if the applied electric field is $E = 10\text{V/cm}$

Solution

Since $N_d > N_a$, the semiconductor is n type and the majority carrier electron concentration is

$$n = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \approx 10^{16}\,\text{cm}^{-3}$$

The minority carrier hole concentration is $\dfrac{n_i^2}{n} = \dfrac{\left(1.8\times10^{6}\right)^2}{10^{16}} = 3.24\times10^{-4}\,\text{cm}^{-3}$

For this extrinsic n-type semiconductor the drift current density is

$$J_{drf} = e\left(\mu_n n + \mu_p p\right)E \approx e\mu_n N_d E \approx 136\,\text{A/cm}^2$$

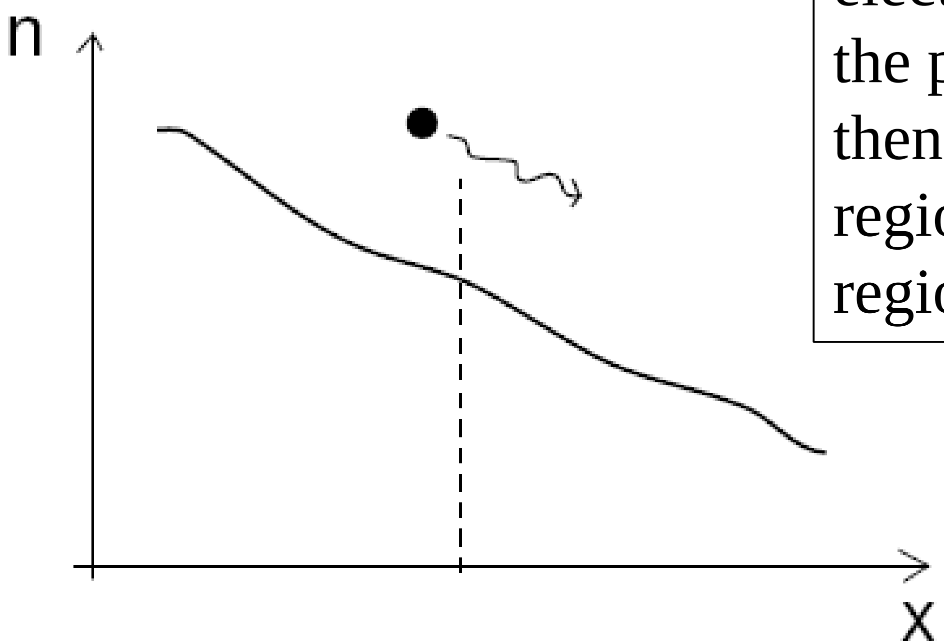
Homework6

Given that a dose $Q_d = 10^{12}\,\mathrm{cm}^{-2}$ is implanted into p-type silicon, which a junction depth of $x_j = 400\mathrm{nm}$, estimate the sheet resistance of this n-type layer.

Carrier Diffusion

Diffusion = particle movement (flux) in response to concentration gradient

If the **concentration** of free electrons is higher in one part of the piece of silicon than in another, then electrons will diffuse from the regions of high concentration to the regions of low concentration.



Elements of diffusion:

- A medium (Si Crystal)
- A gradient of particles (electrons and holes) inside the medium
- Collisions between particles and medium send particles off in random directions

– **Overall result is to erase gradient**

Fick's first law- Key diffusion relationship

Diffusion flux $\propto$ - concentration gradient

Flux ≡ number of particles crossing a unit area per unit time [$cm^{-2} \cdot s^{-1}$]

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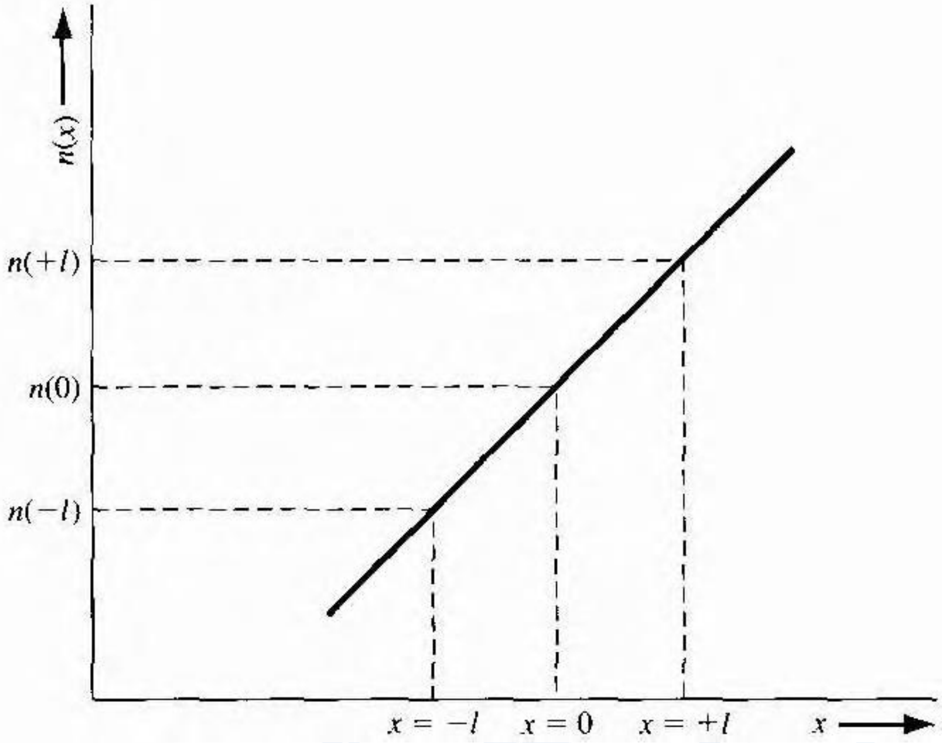
The mean-free path of an electron, that is, the average distance an electron travels between collisions is $l = v_{th}\tau_{cn}$



At any given time, one half of the electrons at $x=-l$ will be traveling to the right and one half of the electrons at $x=+l$ will be traveling to the left.

The diffusion flux

$$F_n = \frac{1}{2}n(-l)\upsilon_{th} - \frac{1}{2}n(+l)\upsilon_{th} = \frac{1}{2}\upsilon_{th}\left[n(-l)-n(+l)\right]$$

By Taylor expansion

$$F_n = \frac{1}{2}\upsilon_{th}\left\{\left[n(0)-l\frac{\mathrm{d}n}{\mathrm{d}x}\right]-\left[n(0)+l\frac{\mathrm{d}n}{\mathrm{d}x}\right]\right\} \Rightarrow F_n = -\upsilon_{th} l \frac{\mathrm{d}n}{\mathrm{d}x}$$

$$F_n = -D_n \frac{\mathrm{d}n}{\mathrm{d}x}, \text{for electrons}$$

$$F_p = -D_p \frac{\mathrm{d}p}{\mathrm{d}x}, \text{for holes}$$

The negative sign reflects the fact that particles diffuse from regions of high concentration toward regions of low concentration.

D_p is **diffusion coefficient** or **diffusivity** of holes
D_n is **diffusion coefficient** or **diffusivity** of electrons

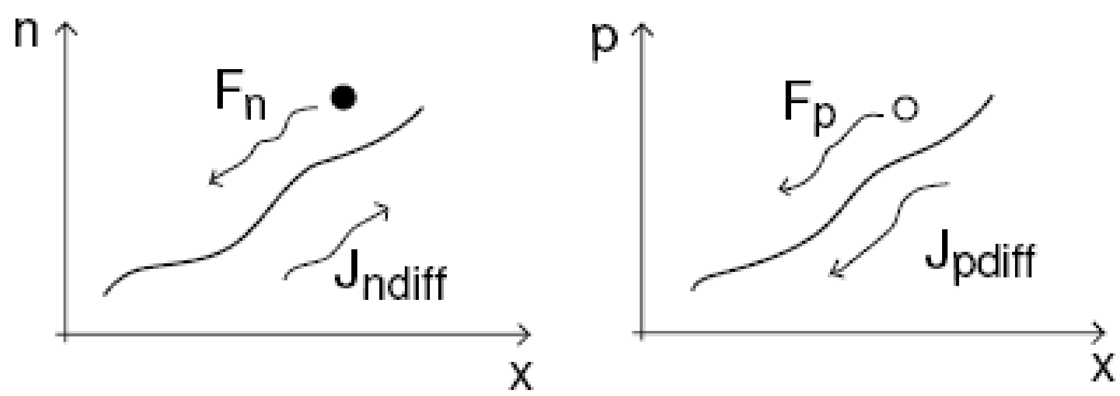
D measures the **ease of carrier diffusion in response to a concentration gradient:** $D \uparrow \Rightarrow F^{\text{diff}} \uparrow$
D is limited by vibration of lattice atoms and ionized dopants.

Diffusion Current

Driffusion current destity = charge × carrier flux

$$J_n = qD_n \frac{dn}{dx} \qquad J_p = -qD_p \frac{dp}{dx}$$

Check signs



Total Current Density

- In general, total current can flow by drift and diffusion separately.
- Total current density:

$$J_n = J_n^{drift} + J_n^{diff} = qn\mu_n E + qD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

$$J_p = J_p^{drift} + J_p^{diff} = qp\mu_p E - qD_p \frac{\mathrm{d}p}{\mathrm{d}x}$$

$$J_{total} = J_n + J_p$$

GRADED IMPURITY DISTRIBUTION

How a nonuniformly doped semiconductor reaches thermal equilibrium

□Induced Electric Field

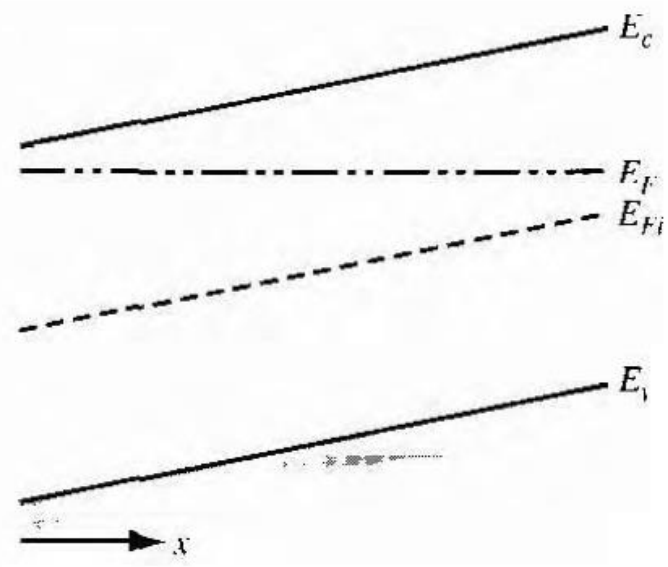


Fig Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration

The doping concentration in n-type material decreases as x increases.

⇩

There will be a diffusion of majority carrier electrons with positively charged donor ions left.

⇩

The separation of positive and negative charge induces an electric field which prevents the diffusion of electrons.
=> Balance

Recall

$$E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right) \qquad E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right)$$

The electric potential

$$\phi = +\frac{1}{e}\left(E_F - E_{Fi}\right)$$

The induced electric field for the one-dimensional situation is defined as

$$E_x = -\frac{\mathrm{d}\phi}{\mathrm{d}x} = \frac{1}{e}\frac{\mathrm{d}E_{Fi}}{\mathrm{d}x}$$

If we assume a quasi-neutrality condition

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$$

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d(x)}{n_i}\right)$$

$$-\frac{\mathrm{d}E_{Fi}}{\mathrm{d}x} = \frac{kT}{N_d(x)}\frac{\mathrm{d}N_d(x)}{\mathrm{d}x}$$

$$\rightarrow E_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{\mathrm{d}N_d(x)}{\mathrm{d}x}$$

In thermal equilibrium, the net current is zero.

$$J_n = 0 = en\mu_n E_x + eD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

If we assume quasi-neutrality so that

$$J_n = 0 = e\mu_n N_d(x) E_x + eD_n \frac{\mathrm{d}N_d(x)}{\mathrm{d}x}$$

Substitute $$E_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{\mathrm{d}N_d(x)}{\mathrm{d}x}$$

$$0 = -e\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{\mathrm{d}N_d(x)}{\mathrm{d}x} + eD_n \frac{\mathrm{d}N_d(x)}{\mathrm{d}x}$$

$$\Rightarrow \frac{D_n}{\mu_n} = \frac{kT}{e}$$

e is also denoted as *q*

Einstein relation

In semiconductors

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \frac{kT}{q}$$

Einstein relationship

$$\frac{D}{\mu} = \frac{kT}{q}$$

$kT/q \equiv$ **thermal voltage** V_T (25 mV at room temperature)

Summary of Key Concepts

Electrons and holes in semiconductors are mobile and charged

– ⇒ Carriers of electrical current!

- **Drift current:** produced by electric field

$$J_n^{drift} \propto E \qquad J_n^{drift} \propto \frac{\mathrm{d}\phi}{\mathrm{d}x}$$

- **Diffusion current:** produced by concentration gradient

$$J_n^{diffusion} \propto \frac{\mathrm{d}n}{\mathrm{d}x} \frac{\mathrm{d}p}{\mathrm{d}x}$$

- Diffusion and drift currents are sizeable in modern devices
- Carriers move fast in response to fields and gradients