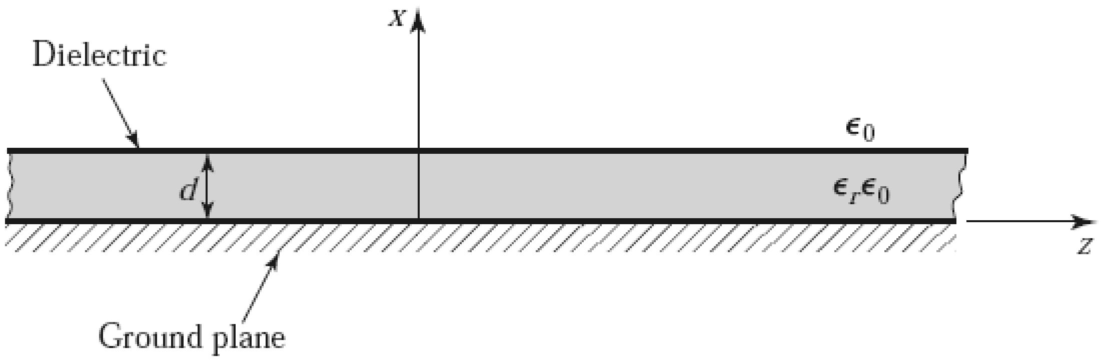
Lec8 Transmission Lines and waveguides (III)

3.6 SURFACE WAVES ON A GROUNDED DIELECTRIC SHEET

TM Modes



The dielectric sheet, of thickness *d* and relative permittivity ϵ_r, s assumed to be of infinite extent in the y and z directions

We will assume propagation in the +z *direction with an* $e^{-j\beta z}$ propagation factor and no variation in the y direction ($\partial/\partial y = 0$).

Ez must satisfy the wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \epsilon_r k_0^2 - \beta^2\right) e_z(x, y) = 0, \qquad \text{for } 0 \leq x \leq d,$$

$$\left(\frac{\partial^2}{\partial x^2} + k_0^2 - \beta^2\right) e_z(x, y) = 0, \qquad \text{for } d \leq x < \infty,$$

where $E_z(x, y, z) = e_z(x, y)e^{-j\beta z}$.

We define the cutoff wave numbers for the two regions as

$$k_c^2 = \epsilon_r k_0^2 - \beta^2,$$
$$h^2 = \beta^2 - k_0^2,$$

where the sign on h^2 has been selected in anticipation of an exponentially decaying result for $x > d$. Both kc and h are real.

β, has been used for both regions to achieve phase matching of the tangential fields at the $x = d$ interface for all values of z.

The general solutions are

$$e_z(x, y) = A \sin k_c x + B \cos k_c x, \qquad \text{for } 0 \le x \le d$$
$$e_z(x, y) = Ce^{hx} + De^{-hx}, \qquad \text{for } d \le x < \infty$$

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y},$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}.$$

$$\Rightarrow \quad H_x = E_y = H_z = 0.$$

The boundary conditions that must be satisfied are

$$E_z(x, y, z) = 0, \qquad \text{at } x = 0, \quad \Rightarrow \quad B = 0$$

$$E_z(x, y, z) < \infty, \qquad \text{as } x \to \infty, \quad \Rightarrow \quad C = 0.$$

$$E_z(x, y, z) \text{ continuous at } x = d, \quad \Rightarrow \quad A \sin k_c d = D e^{-hd},$$

$$H_y(x, y, z) \text{ continuous at } x = d. \quad \Rightarrow \quad \frac{\epsilon_r A}{k_c}\cos k_c d = \frac{D}{h} e^{-hd}.$$

For a nontrivial solution, the determinant of the two equations must vanish, leading to

$$k_c \tan k_c d = \epsilon_r h.$$

Eliminating β from the above equations gives

$$k_c^2 + h^2 = (\epsilon_r - 1)k_0^2.$$

the propagation constants *kc and h* can be solved, given *ko and r .*

The cutoff frequency of the TMn mode can then be derived as

$$f_c = \frac{nc}{2d\sqrt{\epsilon_r - 1}}, \qquad n = 0,\ 1,\ 2, \ldots .$$

For any nonzero-thickness sheet with a relative permittivity greater than unity, there is at least one propagating TM mode, which we will call the TM0 mode.

TM0 mode is the dominant mode of the dielectric slab waveguide, and it has a zero cutoff frequency.

Once kc and h have been found for a particular surface wave mode, the field expressions can be found as

$$E_z(x, y, z) = \begin{cases} A \sin k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ A \sin k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty, \end{cases}$$

$$E_x(x, y, z) = \begin{cases} \dfrac{-j\beta}{k_c} A \cos k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ \dfrac{-j\beta}{h} A \sin k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty, \end{cases}$$

$$H_y(x, y, z) = \begin{cases} \dfrac{-j\omega\epsilon_0\epsilon_r}{k_c} A \cos k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ \dfrac{-j\omega\epsilon_0}{h} A \sin k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty. \end{cases}$$

TE Modes

TE modes can also be supported by the grounded dielectric sheet.

$$\left(\frac{\partial^2}{\partial x^2} + k_c^2\right) h_z(x, y) = 0, \qquad \text{for } 0 \le x \le d,$$

$$\left(\frac{\partial^2}{\partial x^2} - h^2\right) h_z(x, y) = 0, \qquad \text{for } d \le x < \infty,$$

with $H_z(x, y, z) = h_z(x, y)e^{-j\beta z}$

the general solutions to

$$h_z(x, y) = A \sin k_c x + B \cos k_c x,$$

$$h_z(x, y) = Ce^{hx} + De^{-hx}.$$

To satisfy the radiation condition, *C = 0.*

Ey can be found from Hz and using the boundary conditions leads to A = 0 for Ey = 0 at x = 0 and to the equation

$$\frac{-B}{k_c}\sin k_c d = \frac{D}{h}e^{-hd}$$

for continuity of *Ey at x = d.*

$$B\cos k_c d = De^{-hd}.$$

leads to the determinantal equation

$$-k_c \cot k_c d = h.$$

$$k_c^2 + h^2 = (\epsilon_r - 1)k_0^2.$$

The cutoff frequency of the TE*n* modes can then be found as

$$f_c = \frac{(2n-1)c}{4d\sqrt{\epsilon_r - 1}} \qquad \text{for } n = 1, 2, 3, \ldots.$$

the order of propagation for the TM*n and TEn modes* is TM0, TE1, TM1, TE2, TM2, ...

3.7 STRIPLINE

Stripline is a planar type of transmission line that lends itself well to microwave integrated circuitry, miniaturization, and photolithographic fabrication.

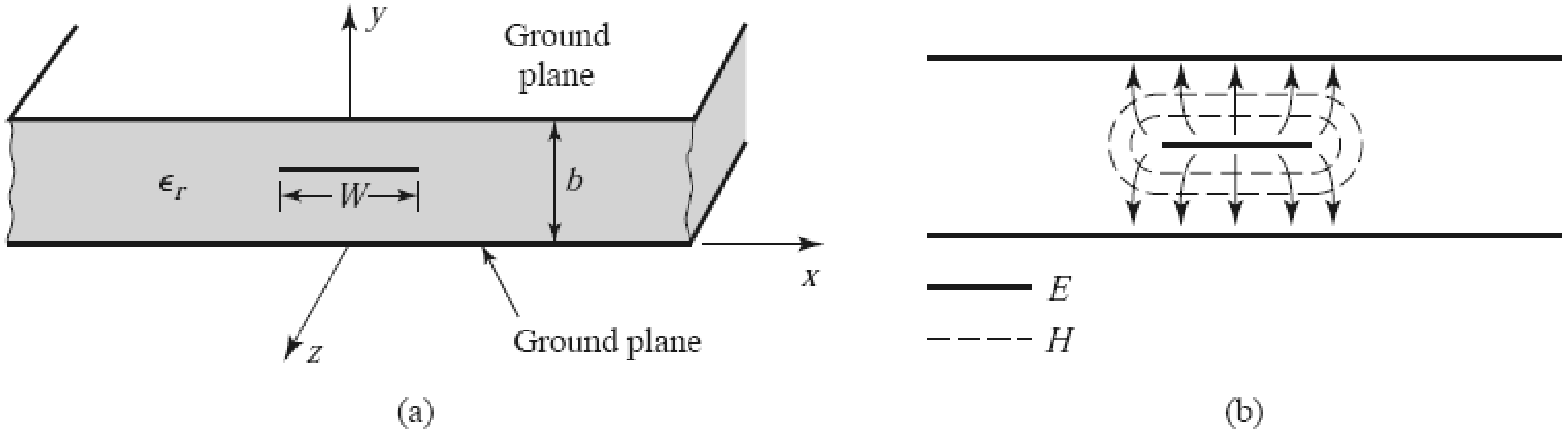


FIGURE 3.22 Stripline transmission line. (a) Geometry. (b) Electric and magnetic field lines.

Variations of the basic geometry include stripline with differing dielectric substrate thicknesses (*asymmetric stripline) or different* dielectric constants (*inhomogeneous stripline).* Air dielectric is sometimes used when it is necessary to minimize loss.

Because stripline has two conductors and a homogeneous dielectric, it supports a TEM wave, and this is the usual mode of operation.
Like parallel plate guide and coaxial line, however, stripline can also support higher order waveguide modes. These can usually be avoided in practice by **restricting both the ground plane spacing and the sidewall width to less than λ_d /2.**

$$k = \frac{2\pi}{\lambda_d} < k_c = \frac{\pi}{\mathrm{b}} \Rightarrow b < \frac{\lambda_d}{2}$$

Shorting vias between the ground planes are often used to enforce this condition relative to the sidewall width.

Stripline can be thought as a sort of “flattened-out” coax—both have a center conductor completely enclosed by an outer conductor and are uniformly filled with a dielectric medium.

Formulas for Propagation Constant, Characteristic Impedance, and Attenuation

An electrostatic analysis is sufficient to give the propagation constant and characteristic impedance for TEM mode of the stripline.

the phase velocity of a TEM mode is $v_p = 1/\sqrt{\mu_0 \epsilon_0 \epsilon_r} = c/\sqrt{\epsilon_r},$

the propagation constant $\beta = \frac{\omega}{v_p} = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r} = \sqrt{\epsilon_r} k_0.$

the characteristic impedance $Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C},$

An exact solution of Laplace’s equation is possible by **a conformal mapping approach**, but the procedure and results are cumbersome. Instead, we will present **closed-form expressions that give good approximations** to the exact results and then discuss an approximate numerical technique for solving Laplace’s equation for a geometry similar to stripline.

By curve fitting method

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b},$$

where *We* is the effective width of the center conductor given by

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \frac{W}{b} > 0.35 \\ (0.35 - W/b)^2 & \text{for } \frac{W}{b} < 0.35. \end{cases}$$

the characteristic impedance decreases as the strip width *W increases.*

When designing stripline circuits one usually needs to find the strip width, given the characteristic impedance (and height *b and relative permittivity)*,

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\epsilon_r}\, Z_0 < 120\ \Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\epsilon_r}\, Z_0 > 120\ \Omega, \end{cases}$$

where

$$x = \frac{30\pi}{\sqrt{\epsilon_r}\, Z_0} - 0.441.$$

The attenuation due to conductor loss can be found by the perturbation method or Wheeler's incremental inductance rule. An approximate result is

$$\alpha_c = \begin{cases} \dfrac{2.7 \times 10^{-3} R_s \epsilon_r Z_0}{30\pi(b-t)} A & \text{for } \sqrt{\epsilon_r} Z_0 < 120\ \Omega \\ \dfrac{0.16 R_s}{Z_0 b} B & \text{for } \sqrt{\epsilon_r} Z_0 > 120\ \Omega \end{cases} \quad \text{Np/m},$$

with

$$A = 1 + \frac{2W}{b-t} + \frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right),$$

$$B = 1 + \frac{b}{(0.5W + 0.7t)}\left(0.5 + \frac{0.414t}{W} + \frac{1}{2\pi}\ln\frac{4\pi W}{t}\right),$$

where t is the thickness of the strip.

EXAMPLE 3.5 STRIPLINE DESIGN

Find the width for a 50 copper stripline conductor with b = 0.32 cm and ϵ_r = 2.20. If the dielectric loss tangent is 0.001 and the operating frequency is 10 GHz, calculate the attenuation in dB/λ. Assume a conductor thickness of t = 0.01 mm.

Solution

Because $\sqrt{\epsilon_r} Z_0 = \sqrt{2.2}(50) = 74.2 < 120$ and $x = 30\pi/(\sqrt{\epsilon_r} Z_0) - 0.441 =$ 0.830

$$W = bx = (0.32)(0.830) = 0.266 \text{ cm}.$$

At 10 GHz, the wave number is $k = \frac{2\pi f \sqrt{\epsilon_r}}{c} = 310.6 \text{ m}^{-1}.$

the dielectric attenuation is $\alpha_d = \frac{k \tan \delta}{2} = \frac{(310.6)(0.001)}{2} = 0.155 \text{ Np/m}.$

The surface resistance of copper at 10 GHz is *Rs = 0.026* .

the conductor attenuation is $$\alpha_c = \frac{2.7 \times 10^{-3} R_s \epsilon_r Z_0}{30\pi(b-t)} A = \frac{2.7 \times 10^{-3} R_s \epsilon_r Z_0 A}{30\pi(b-t)} = 0.122 \text{ Np/m},$$

The total attenuation constant is $$\alpha = \alpha_d + \alpha_c = 0.277 \text{ Np/m}.$$

$$\alpha(\text{dB}) = 20 \log e^{\alpha} = 2.41 \text{ dB/m}.$$

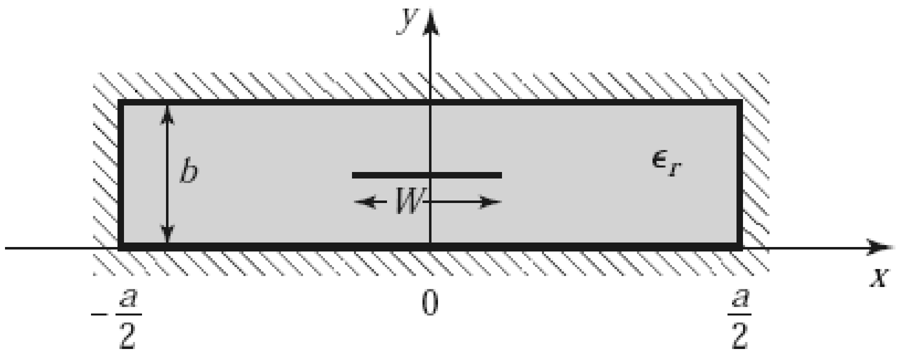
At 10 GHz, the wavelength on the stripline is

$$\lambda = \frac{c}{\sqrt{\epsilon_r} f} = 2.02 \text{ cm},$$

so in terms of wavelength the attenuation is

$$\alpha(\text{dB}) = (2.41)(0.0202) = 0.049 \text{ dB}/\lambda.$$

Because we suspect that the field lines do not extend very far away from the center conductor, we can simplify the geometry by truncating the plates beyond some distance, say $|x| > a/2$, and placing metal walls on the sides.



Geometry of enclosed stripline.

where a>> b, so that the fields around the center conductor are not perturbed by the sidewalls. We then have a closed finite region in which the potential (x, y) satisfies Laplace's equation,

$$\nabla_t^2 \Phi(x, y) = 0 \quad \text{for } |x| \leq a/2,\ 0 \leq y \leq b,$$

with the boundary conditions

$$\Phi(x, y) = 0, \qquad \text{at } x = \pm a/2,$$

$$\Phi(x, y) = 0, \qquad \text{at } y = 0, b.$$

Laplace's equation can be solved by the method of separation of variables.

The general solutions are

$$\Phi(x,y)=\begin{cases}\displaystyle\sum_{\substack{n=1\\ \text{odd}}}^{\infty} A_n \cos\frac{n\pi x}{a}\sinh\frac{n\pi y}{a} & \text{for } 0\le y\le b/2\\ \displaystyle\sum_{\substack{n=1\\ \text{odd}}}^{\infty} B_n \cos\frac{n\pi x}{a}\sinh\frac{n\pi}{a}(b-y) & \text{for } b/2\le y\le b.\end{cases}$$

The potential must be continuous at $y = b/2$, $A_n = B_n.$

Because $E_y = -\partial\Phi/\partial y$, we have

$$E_y=\begin{cases}\displaystyle -\sum_{\substack{n=1\\ \text{odd}}}^{\infty} A_n\left(\frac{n\pi}{a}\right)\cos\frac{n\pi x}{a}\cosh\frac{n\pi y}{a} & \text{for } 0\le y\le b/2\\ \displaystyle\sum_{\substack{n=1\\ \text{odd}}}^{\infty} A_n\left(\frac{n\pi}{a}\right)\cos\frac{n\pi x}{a}\cosh\frac{n\pi}{a}(b-y) & \text{for } b/2\le y\le b.\end{cases}$$

The surface charge density on the strip at $y = b/2$ *is*

$$\rho_s = D_y(x, y = b/2^+) - D_y(x, y = b/2^-)$$
$$= \epsilon_0\epsilon_r[E_y(x, y = b/2^+) - E_y(x, y = b/2^-)]$$
$$= 2\epsilon_0\epsilon_r \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} A_n \left(\frac{n\pi}{a}\right) \cos\frac{n\pi x}{a} \cosh\frac{n\pi b}{2a},$$

the exact surface charge density is unknown but can be approximated as a constant over the width of the strip,

$$\rho_s(x) = \begin{cases} 1 & \text{for } |x| < W/2 \\ 0 & \text{for } |x| > W/2. \end{cases}$$

Then

$$A_n = \frac{2a \sin(n\pi W/2a)}{(n\pi)^2 \epsilon_0\epsilon_r \cosh(n\pi b/2a)}.$$

• The voltage of the strip conductor relative to the bottom conductor can be found by integrating the vertical electric field from *y = 0 to b/2.*

• This voltage is not constant over the width of the strip but varies with position, *x.*

• Rather than choosing the voltage at an arbitrary position, we can obtain an improved result by averaging the voltage over the width of the strip:

$$V_{avg} = \frac{1}{W} \int_{-W/2}^{W/2} \int_0^{b/2} E_y(x, y)\, dy\, dx = \sum_{\substack{n=1 \\ odd}}^{\infty} A_n \left(\frac{2a}{n\pi W} \right) \sin \frac{n\pi W}{2a} \sinh \frac{n\pi b}{2a}.$$

The total charge per unit length on the center conductor is

$$Q = \int_{-W/2}^{W/2} \rho_s(x)\, dx = W \text{ Coul/m},$$

so **the capacitance per unit length of the stripline** is

$$C = \frac{Q}{V_{\text{avg}}} = \frac{W}{\sum\limits_{\substack{n=1\\ \text{odd}}}^{\infty} A_n \left(\frac{2a}{n\pi W}\right) \sin\frac{n\pi W}{2a} \sinh\frac{n\pi b}{2a}} \text{ F/m.}$$

Finally, **the characteristic impedance from the numerical computation is given by**

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C} = \frac{\sqrt{\epsilon_r}}{cC},$$

where $c = 3 \times 10^8$ m/sec.

EXAMPLE 3.6 NUMERICAL CALCULATION OF STRIPLINE IMPEDANCE

Evaluate the numerical expressions for a stripline having dielectric = 2.55 and a = 100b to find the characteristic impedance for *W/b = 0.25 to 5.0.* Compare with the results from the analytical expression (3.179).

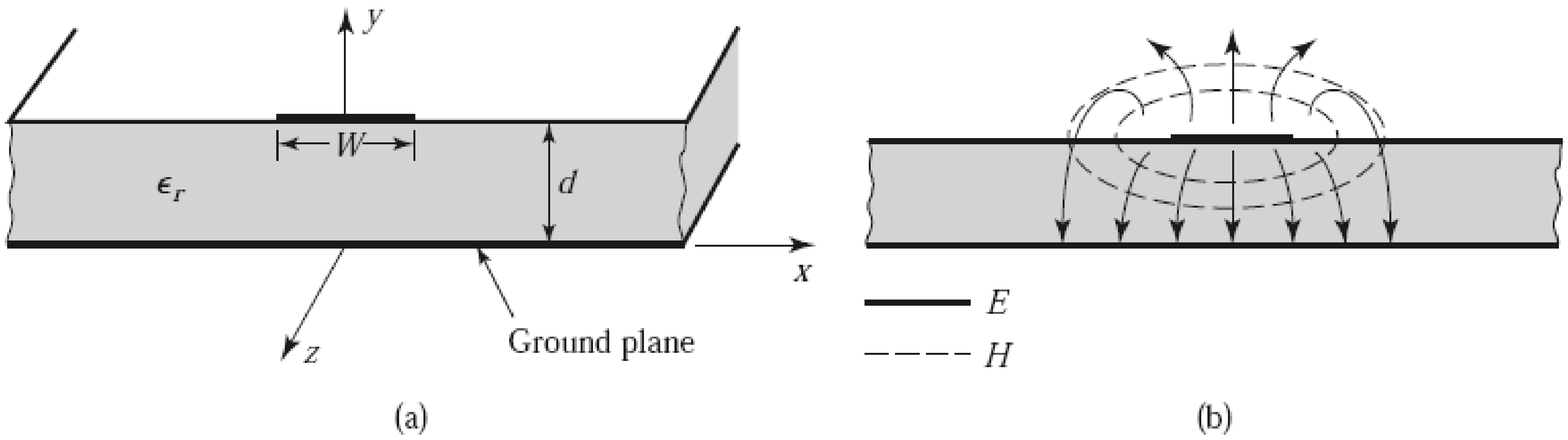
Solution

A computer program was written to evaluate the numerical expression . The series was truncated after 500 terms, and the results for *Z0 are as follows.*

	Z_0, Ω		
W/b	Numerical, Eq. (3.192)	Formula, Eq. (3.179)	Commercial CAD
0.25	90.9	86.6	85.3
0.50	66.4	62.7	61.7
1.0	43.6	41.0	40.2
2.0	25.5	24.2	24.4
5.0	11.1	10.8	11.9

3.8 MICROSTRIP LINE

Microstrip line is one of the most popular types of planar transmission lines primarily because it can be fabricated by photolithographic processes and is easily miniaturized and integrated with both passive and active microwave devices.



Microstrip transmission line. (a) Geometry. (b) Electric and magnetic field lines.

- If the dielectric substrate were not present $(\epsilon_r = 1)$, we would have a two-wire line consisting of a flat strip conductor over a ground plane, embedded in a homogeneous medium (air). This would constitute **a simple TEM transmission line with phase velocity $v_p = c$ and propagation constant $\beta = k_0$.**

- The presence of the dielectric, particularly the fact that the dielectric does not fill the region above the strip (y > d), complicates the behavior and analysis of microstrip line. Microstrip line cannot support a pure TEM wave since **the phase velocity of TEM fields in the dielectric region would be** $c/\sqrt{\epsilon_r}$, while the phase velocity of TEM fields in the air region would be *c*, so **a phase-matching condition at the dielectric–air interface would be impossible to enforce**.

- In actuality, **the exact fields of a microstrip line constitute a hybrid TM-TE wave**.

- In most practical applications, however, the dielectric substrate is electrically very thin *($d << \lambda$),* and so the fields are quasi-TEM.

Then **the phase velocity and propagation constant** can be expressed as

$$v_p = \frac{c}{\sqrt{\epsilon_e}},$$

$$\beta = k_0\sqrt{\epsilon_e},$$

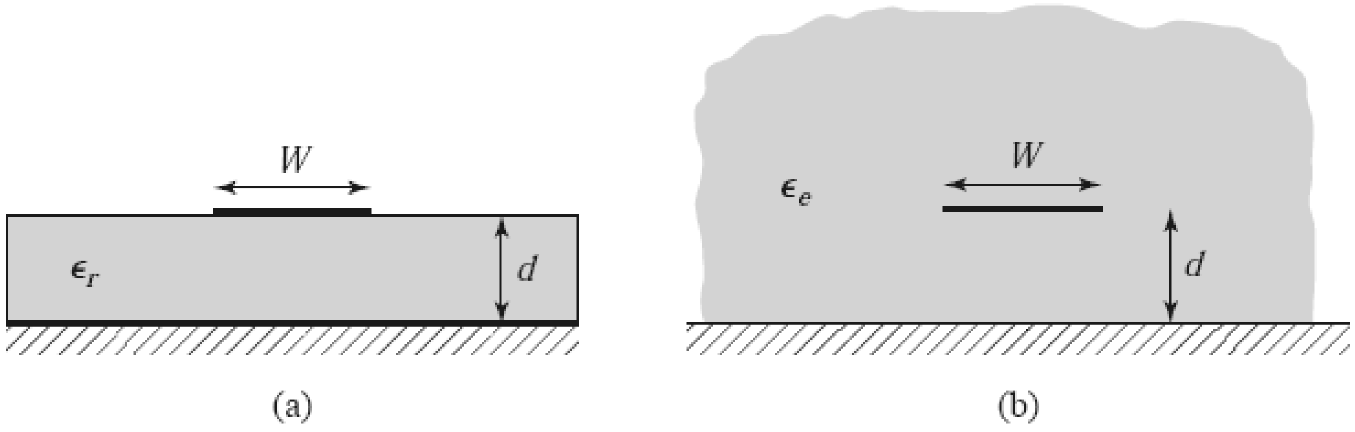
where ϵ_e is the *effective dielectric constant* of the microstrip line.

$$1 < \epsilon_e < \epsilon_r$$

Formulas for the effective dielectric constant, characteristic impedance, and attenuation of microstrip line are curve-fit approximations to rigorous quasi-static solutions

Formulas for Effective Dielectric Constant, Characteristic Impedance, and Attenuation

The effective dielectric constant can be interpreted as the dielectric constant of a homogeneous medium that equivalently replaces the air and dielectric regions of the microstrip line



$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}.$$

Given the dimensions of the microstrip line, the characteristic impedance

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\epsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \le 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_e}\,[W/d + 1.393 + 0.667 \ln(W/d + 1.444)]} & \text{for } W/d \ge 1. \end{cases}$$

For a given characteristic impedance Z_0 and dielectric constant , *the W/d* ratio can be found as

$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \text{for } W/d < 2 \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\epsilon_r - 1}{2\epsilon_r}\left\{\ln(B - 1) + 0.39 - \dfrac{0.61}{\epsilon_r}\right\}\right] & \text{for } W/d > 2, \end{cases}$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}.$$

the attenuation due to dielectric loss is

$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_e - 1) \tan \delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} \text{ Np/m},$$

The attenuation due to conductor loss is

$$\alpha_c = \frac{R_s}{Z_0 W} \text{ Np/m},$$

where $R_s = \sqrt{\omega \mu_0 / 2\sigma}$ is the surface resistivity of the conductor.

For most microstrip substrates, conductor loss is more significant than dielectric loss; exceptions may occur, however, with some semiconductor substrates.

EXAMPLE 3.7 MICROSTRIP LINE DESIGN

Design a microstrip line on a 0.5 mm alumina substrate $(\epsilon_r = 9.9, \tan\delta = 0.001)$ for a 50 characteristic impedance. Find the length of this line required to produce a phase delay of 270◦ at 10 GHz, and compute the total loss on this line, assuming copper conductors.

Solution

First find *W/d for* Z_0 *= 50 , and initially guess that W/d < 2. From (3.197),*

A = 2.142, W/d = 0.9654.

So the condition that *W/d < 2* is satisfied; otherwise we would use the expression for *W/d > 2.*

$$W = 0.9654d = 0.483 \text{ mm}.$$

the effective dielectric constant is $\epsilon_e = 6.665$.

$$\phi = 270° = \beta\ell = \sqrt{\epsilon_e}k_0\ell,$$

$$k_0 = \frac{2\pi f}{c} = 209.4 \text{ m}^{-1},$$

$$\ell = \frac{270°(\pi/180°)}{\sqrt{\epsilon_e}k_0} = 8.72 \text{ mm}.$$

Attenuation due to dielectric loss is

$$\alpha_d = \frac{k_0\epsilon_r(\epsilon_e - 1)\tan\delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} \text{ Np/m} = 0.255 \text{ Np/m}$$

$$\alpha_c = \frac{R_s}{Z_0 W} \text{ Np/m} = 0.0108 \text{ Np/cm}$$

The total loss on the line is then 0.101 dB.

3.10 WAVE VELOCITIES AND DISPERSION

Two types of velocities related to the propagation of electromagnetic waves:

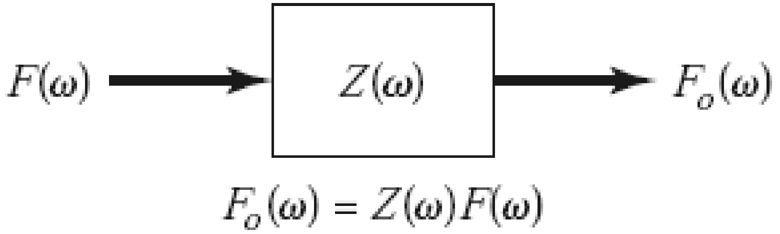
- The speed of light in a medium ($1/\sqrt{\mu\epsilon}$)
- The phase velocity ($v_p = \omega/\beta$)

For a TEM plane wave, these two velocities are identical, but for other types of guided wave propagation the phase velocity may be greater or less than the speed of light.

• If the phase velocity and attenuation of a line or guide are constants that do not change with frequency, then the phase of a signal that contains more than one frequency component will not be distorted.

• If the phase velocity is different for different frequencies, signal distortion will occur.

• However, if the bandwidth of the signal is relatively small or if the dispersion is not too severe, **a group velocity** can be defined in a meaningful way.



A transmission line or waveguide represented as a linear system with transfer function *Z(ω)*.

$$F_o(\omega) = Z(\omega)F(\omega).$$

For a lossless matched transmission line or waveguide,

$$Z(\omega) = Ae^{-j\beta z} = |Z(\omega)|e^{-j\psi},$$

where A is a constant and β is the propagation constant of the line or guide.

The time domain representation of the output signal,

$$f_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)|Z(\omega)|e^{j(\omega t-\psi)}\,d\omega.$$

If $|Z(\omega)| = A$ is a constant and the phase ψ of $Z(\omega)$ is a linear function of ω, say $\psi = a\omega$,

$$f_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} AF(\omega)e^{j\omega(t-a)}\,d\omega = Af(t-a),$$

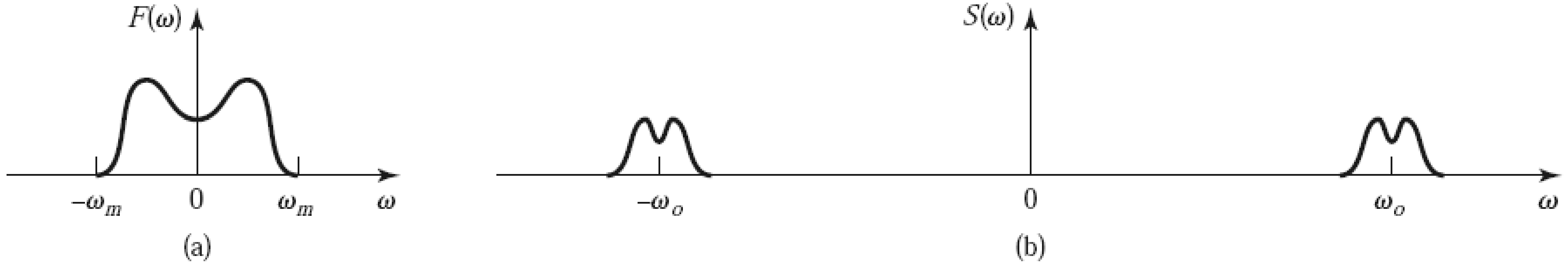
which is seen to be a replica of f (t), except for an amplitude factor *A* and time shift *a*.

- **A TEM line is *dispersionless* and does not lead to signal distortion.**
- **If the TEM line is lossy, the attenuation may be a function of frequency, which could lead to signal distortion.**

Now consider a narrowband input signal of the form

$$s(t) = f(t)\cos\omega_0 t = \mathrm{Re}\left\{ f(t)e^{j\omega_o t}\right\},$$

which represents an amplitude-modulated carrier wave of frequency ω_o.



$$S(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega_o t} e^{j\omega t} dt = F(\omega - \omega_o),$$

Assume that the highest frequency component of *f (t) is* ω_m, where $\omega_m << \omega_o$.

The output signal spectrum is $S_o(\omega) = AF(\omega - \omega_o) e^{-j\beta z}$,

$$s_o(t) = \frac{1}{2\pi} \text{Re} \int_{-\infty}^{\infty} S_o(\omega) e^{j\omega t} d\omega$$
$$= \frac{1}{2\pi} \text{Re} \int_{\omega_o - \omega_m}^{\omega_o + \omega_m} AF(\omega - \omega_o) e^{j(\omega t - \beta z)} d\omega.$$

if $F(\omega)$ is narrowband $(\omega_m << \omega o)$, *then* β *can often be linearized by using a Taylor series* expansion about ω_o:

$$\beta(\omega) = \beta(\omega_o) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_o} (\omega - \omega_o) + \frac{1}{2}\left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_o} (\omega - \omega_o)^2 + \cdots.$$

$$\beta(\omega) \simeq \beta_o + \beta_o'(\omega - \omega_o),$$

where

$$\beta_o = \beta(\omega_o),$$

$$\beta_o' = \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_o}.$$

After a change of variables to $y = \omega - \omega_o$, the expression for *so(t)* becomes

$$\begin{aligned} s_o(t) &= \frac{A}{2\pi} \operatorname{Re}\left\{ e^{j(\omega_o t - \beta_o z)} \int_{-\omega_m}^{\omega_m} F(y) e^{j(t-\beta_o' z)y}\, dy \right\} \\ &= A \operatorname{Re}\left\{ f(t - \beta_o' z) e^{j(\omega_o t - \beta_o z)} \right\} \\ &= A f(t - \beta_o' z) \cos(\omega_o t - \beta_o z), \end{aligned}$$

$$s(t) = f(t) \cos \omega_0 t :$$

which is a time-shifted replica of the original modulation envelope, *f (t),* The velocity of this envelope is the **group velocity**, v_g:

$$v_g = \frac{1}{\beta'_o} = \left(\frac{d\beta}{d\omega}\right)^{-1}\bigg|_{\omega=\omega_o}.$$

EXAMPLE 3.9 WAVEGUIDE WAVE VELOCITIES

Calculate the group velocity for a waveguide mode propagating in an air-filled guide. Compare this velocity to the phase velocity and speed of light.

Solution

The propagation constant for a mode in an air-filled waveguide is

$$\beta = \sqrt{k_0^2 - k_c^2} = \sqrt{(\omega/c)^2 - k_c^2}.$$

$$\frac{d\beta}{d\omega} = \frac{\omega/c^2}{\sqrt{(\omega/c)^2 - k_c^2}} = \frac{k_o}{c\beta},$$

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \frac{c\beta}{k_0}.$$

群速 相速 光速三者关系?

$$v_p = \omega/\beta = ck_0/\beta.$$

Since $\beta < k_0$, $v_g < c < v_p$

The phase velocity of a waveguide mode may be greater than the speed of light, but the group velocity (the velocity of a narrowband signal) will be less than the speed of light.

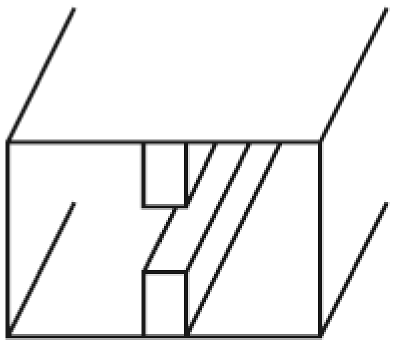
3.11 SUMMARY OF TRANSMISSION LINES AND WAVEGUIDES

Comparison of Common Transmission Lines and Waveguides

Characteristic	Coax	Waveguide	Stripline	Microstrip
Modes: Preferred	TEM	TE_{10}	TEM	Quasi-TEM
Other	TM,TE	TM,TE	TM,TE	Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	High
Loss	Medium	Low	High	High
Power capacity	Medium	High	Low	Low
Physical size	Large	Large	Medium	Small
Ease of fabrication	Medium	Medium	Easy	Easy
Integration with	Hard	Hard	Fair	Easy

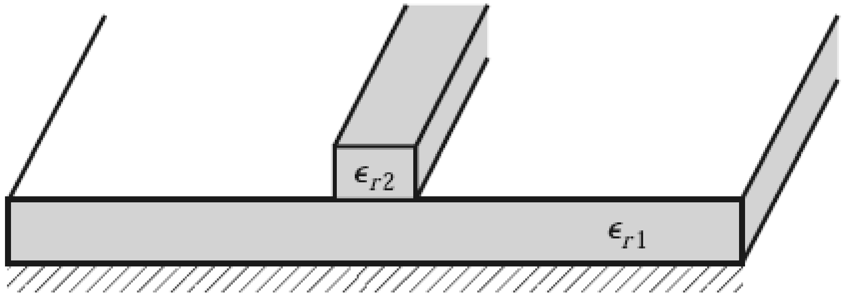
Other Types of Lines and Guides

Ridge waveguide:



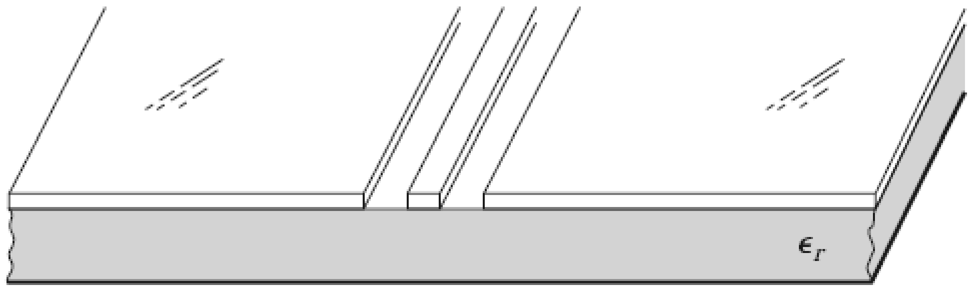
The ridge waveguide, consists of a rectangular waveguide loaded with conducting ridges on the top and/or bottom walls. This loading tends to lower the cutoff frequency of the dominant mode, leading to increased bandwidth and better (more constant) impedance characteristics.

Dielectric waveguide:



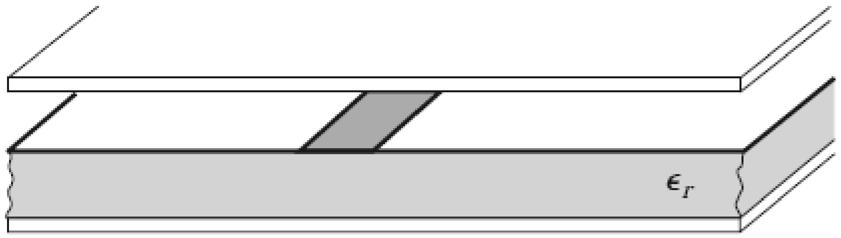
Its small size makes it useful for millimeter wave to optical frequencies.

Coplanar waveguide:



Coplanar waveguides are particularly useful for fabricating active circuitry due to the presence of the center conductor and the close proximity of the ground planes.

Covered microstrip:



The metallic cover plate is often used for electrical shielding and physical protection of the microstrip circuitry.

Homework

3.19 A copper stripline transmission line is to be designed for a 100 Ω characteristic impedance. The ground plane separation is 1.02 mm and the dielectric constant is 2.20, with $\tan\delta = 0.001$. At 5 GHz, find the guide wavelength on the line and the total attenuation. **Assume t=0.01mm**

3.20 A copper microstrip transmission line is to be designed for a 100 Ω characteristic impedance. The substrate is 0.51 mm thick, with $\epsilon_r = 2.20$ and $\tan\delta = 0.001$. At 5 GHz, find the guide wavelength on the line and the total attenuation. Compare these results with those for the similar stripline case of the preceding problem.

3.28 As discussed in the Point of Interest on the power-handling capacity of transmission lines, the maximum power capacity of a coaxial line is limited by voltage breakdown and is given by

$$P_{\text{max}} = \frac{\pi a^2 E_d^2}{\eta_0} \ln \frac{b}{a},$$

where E_d is the field strength at breakdown. Find the value of b/a that maximizes the maximum power capacity and show that the corresponding characteristic impedance is about 30 Ω.