Lec9 Microwave Network Analysis （I）

微波网络分析与电磁场分析的区别与联系。

□Circuits operating at low frequencies

- The circuit dimensions are small enough such that there is negligible phase delay from one point in the circuit to another.
- The fields can be considered as TEM fields， a quasi-static type of solution to Maxwell’s equations lead to the well-known Kirchhoff voltage and current laws and impedance concepts of **circuit theory.**

□Circuits operating at microwave frequencies

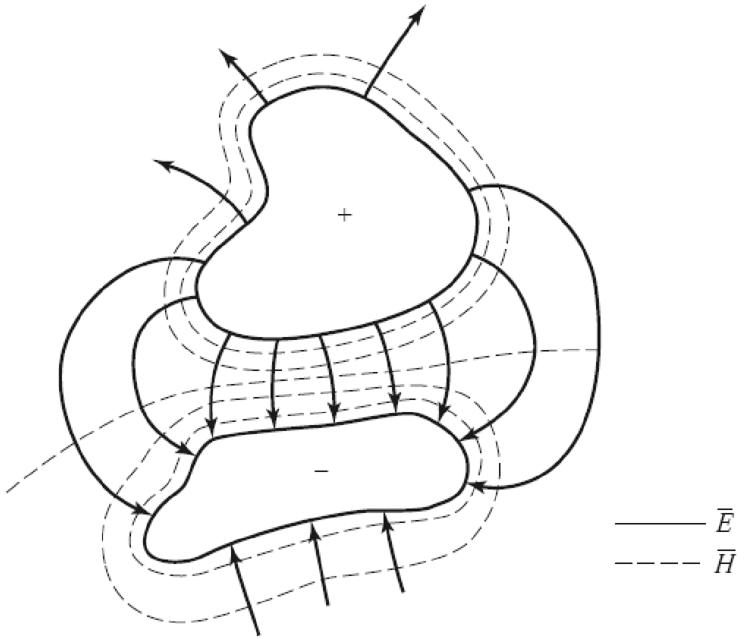
- Microwave analysis and design---- extension to basic circuit and network concepts.
- Field analysis --- solution to Maxwell’s equations gives the electric and magnetic fields at all points in space. However, usually we are only interested in the voltage or current at a set of terminals, the power flow through a device, or some other type of “terminal” quantity.

4.1 IMPEDANCE AND EQUIVALENT VOLTAGES AND CURRENTS

Equivalent Voltages and Currents

● At microwave frequencies the measurement of voltage or current is difficult (or impossible), unless a clearly defined terminal pair is available.

● Terminal pairs exist for TEM-type lines (such as coaxial cable, microstrip line, or stripline), but does not strictly exist for non-TEM lines



$$V = \int_{+}^{-} \bar{E} \cdot d\bar{\ell},$$

where the integration path begins on the + conductor and ends on the − conductor. The is unique and does not depend on the shape of the integration path.

The total current flowing on the + conductor can be determined from an application of Ampere's law as

$$I = \oint_{C+} \bar{H} \cdot d\bar{\ell},$$

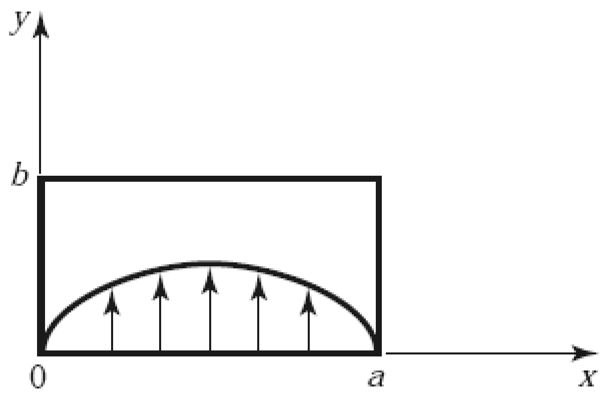
where the integration contour is any closed path enclosing the + conductor (but not the − conductor).

A characteristic impedance Z_0

$$Z_0 = \frac{V}{I}.$$

Transmission line theory can be used for TEM line.

For a rectangular waveguide



Electric field lines for the TE10 mode of a rectangular waveguide.

For the dominant TE10 mode

$$E_y(x, y, z) = \frac{j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = A e_y(x, y) e^{-j\beta z},$$

$$H_x(x, y, z) = \frac{j\beta a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = A h_x(x, y) e^{-j\beta z}.$$

$$V = \frac{-j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} \int_y dy.$$

It is seen that this voltage depends on the position, *x*, as well as the length of the integration contour along the *y* direction.

Integrating from y = 0 to b for x = a/2 gives a voltage that is quite different from that obtained by integrating from y = 0 to b for x = 0. What, then, is the correct voltage?

There are many ways to define equivalent voltage, current, and impedance for waveguides since these quantities are not unique for non-TEM lines, but the following considerations usually lead to the most useful results

- Voltage and current are defined only for a particular waveguide mode
  ---the voltage is proportional to the transverse electric field
  ---the current is proportional to the transverse magnetic field.
- the equivalent voltages and currents should be defined so that their product gives the power flow of the waveguide mode.
- The ratio of the voltage to the current should be equal to the characteristic impedance of the line. This impedance is usually selected as equal to the wave impedance of the line, or else normalized to unity.

For an arbitrary waveguide mode with both positively and negatively traveling waves, the transverse fields can be written as

$$\bar{E}_t(x, y, z) = \bar{e}(x, y)(A^+ e^{-j\beta z} + A^- e^{j\beta z}) = \frac{\bar{e}(x, y)}{C_1}\left(V^+ e^{-j\beta z} + V^- e^{j\beta z}\right),$$

$$\bar{H}_t(x, y, z) = \bar{h}(x, y)\left(A^+ e^{-j\beta z} - A^- e^{j\beta z}\right) = \frac{\bar{h}(x, y)}{C_2}\left(I^+ e^{-j\beta z} - I^- e^{j\beta z}\right),$$

$$\bar{h}(x, y) = \frac{\hat{z} \times \bar{e}(x, y)}{Z_w}.$$

$$C_1 = V^+/A^+ = V^-/A^-$$
$$C_2 = I^+/A^+ = I^-/A^-,$$

define equivalent voltage and current waves as

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z},$$

$$I(z) = I^+ e^{-j\beta z} - I^- e^{j\beta z},$$

with $V^+/I^+ = V^-/I^- = Z_0$.

The complex power flow for the incident wave is

$$P^+ = \frac{1}{2}|A^+|^2 \int_S \bar{e} \times \bar{h}^* \cdot \hat{z} ds = \frac{V^+ I^{+*}}{2C_1 C_2^*} \int_S \bar{e} \times \bar{h}^* \cdot \hat{z} ds.$$

Because we want this power to be equal to $(1/2) V^+ I^{+*}$, we have the result that

$$\boxed{C_1 C_2^* = \int_S \bar{e} \times \bar{h}^* \cdot \hat{z} ds,}$$

The characteristic impedance is

$$Z_0 = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \frac{C_1}{C_2},$$

If it is desired to have $Z_0 = Zw$,

$$\boxed{\frac{C_1}{C_2} = Z_w \, (Z_{\mathrm{TE}} \text{ or } Z_{\mathrm{TM}}).}$$

C1 and C2 can be solved.

Higher order modes can be treated in the same way, so that a general field in a waveguide can be expressed as

$$\bar{E}_t(x, y, z) = \sum_{n=1}^{N} \left(\frac{V_n^+}{C_{1n}} e^{-j\beta_n z} + \frac{V_n^-}{C_{1n}} e^{j\beta_n z} \right) \bar{e}_n(x, y),$$

$$\bar{H}_t(x, y, z) = \sum_{n=1}^{N} \left(\frac{I_n^+}{C_{2n}} e^{-j\beta_n z} - \frac{I_n^-}{C_{2n}} e^{j\beta_n z} \right) \bar{h}_n(x, y),$$

where $V_n^{\pm}$ and $I_n^{\pm}$ are the equivalent voltages and currents for the nth mode, and C_{1n} and C_{2n} are the proportionality constants for each mode.

EXAMPLE 4.1 EQUIVALENT VOLTAGE AND CURRENT FOR A RECTANGULAR WAVEGUIDE

Find the equivalent voltages and currents for a TE10 mode in a rectangular waveguide.

Solution

Waveguide Fields	Transmission Line Model
$E_y = \left(A^+ e^{-j\beta z} + A^- e^{j\beta z}\right) \sin\frac{\pi x}{a}$	$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$
$H_x = \frac{-1}{Z_{\text{TE}}}\left(A^+ e^{-j\beta z} - A^- e^{j\beta z}\right) \sin\frac{\pi x}{a}$	$I(z) = I^+ e^{-j\beta z} - I^- e^{j\beta z}$ $= \frac{1}{Z_0}\left(V^+ e^{-j\beta z} - V^- e^{j\beta z}\right)$
$P^+ = \frac{-1}{2}\int_S E_y H_x^* dx dy = \frac{ab}{4Z_{\text{TE}}}\|A^+\|^2$	$P^+ = \frac{1}{2} V^+ I^{+*}$

Since $C_1 = V^+/A^+ = V^-/A^-$ and $C_2 = I^+/A^+ = I^-/A^-$

$$\frac{ab|A^+|^2}{4Z_{\mathrm{TE}}} = \frac{1}{2}V^+ I^{+*} = \frac{1}{2}|A^+|^2 C_1 C_2^*.$$

If we choose $Z_0 = Z_{TE}$, then

$$\frac{V^+}{I^+} = \frac{C_1}{C_2} = Z_{\mathrm{TE}}.$$

Solving for $C1$, $C2$ gives

$$C_1 = \sqrt{\frac{ab}{2}},$$

$$C_2 = \frac{1}{Z_{\mathrm{TE}}}\sqrt{\frac{ab}{2}},$$

which completes the transmission line equivalence for the TE10 mode.

The Concept of Impedance

- The term impedance was first used by Oliver Heaviside in the nineteenth century to describe the complex ratio *V/I in AC circuits consisting* of resistors, inductors, and capacitors.
- It was then applied to transmission lines, in terms of lumped-element equivalent circuits and the distributed series impedance and shunt admittance of the line.
- In the 1930s, the impedance concept could be extended to electromagnetic fields in a systematic way, and noted that impedance should be regarded as characteristic of the type of field, as well as of the medium.

◆ Microwave concept: intrinsic impedance of the medium, wave impedance
◆ Transmission line concept: characteristic impedance, distributed series impedance
◆ circuit concept: input impedance

- $\eta = \sqrt{\mu/\epsilon} =$ **intrinsic impedance of the medium.**

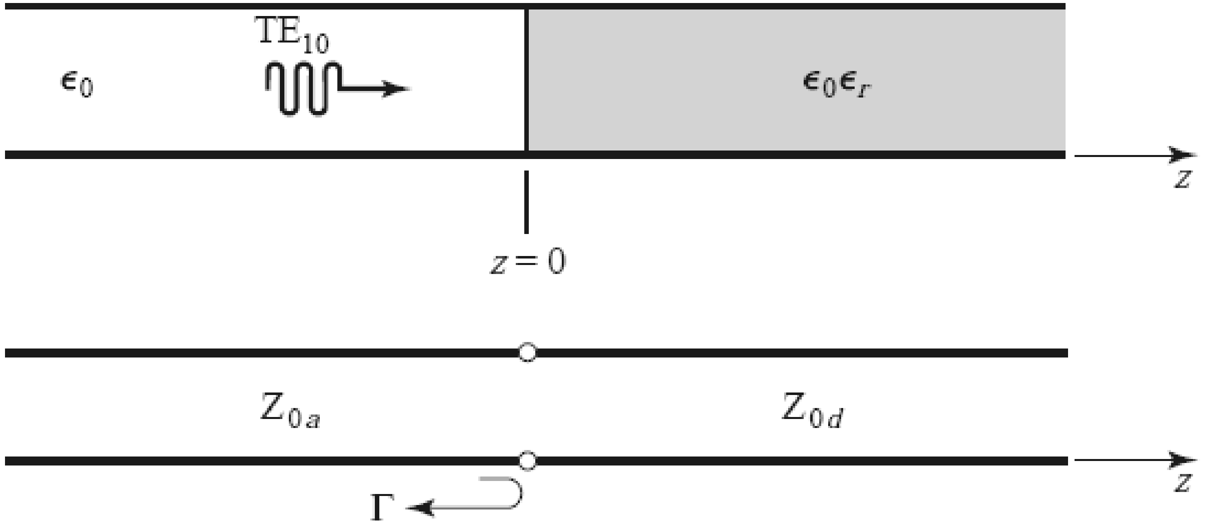
This impedance is dependent only on the material parameters of the medium, and is equal to the wave impedance for plane waves.

- $Z_w = E_t/H_t = 1/Y_w =$ **wave impedance**. This impedance is a characteristic of the particular type of wave. TEM, TM, and TE waves each have different wave impedances (Z_{TEM}, Z_{TM}, Z_{TE}), which may depend on the type of line or guide, the material, and the operating frequency.

- $Z_0 = 1/Y_0 = V^+/I^+ =$ **Characteristic impedance** is the ratio of voltage to current for a traveling wave on a transmission line.

EXAMPLE 4.2 APPLICATION OF WAVEGUIDE IMPEDANCE

Consider a rectangular waveguide with $a = 2.286$ cm and $b = 1.016$ cm (X-band guide), air filled for $z < 0$ and Rexolite filled ($\epsilon_r = 2.54$) for $z > 0$, as shown in Figure 4.3. If the operating frequency is 10 GHz, use an equivalent transmission line model to compute the reflection coefficient of a TE_{10} wave incident on the interface from $z < 0$.



Geometry of a partially filled waveguide and its transmission line equivalent for Example 4.2.

Solution:

Firstly compute the cutoff frequency of the waveguide and determine the propagation mode in the waveguide at 10GHz. (**Do it on class**)

The waveguide propagation constants in the air ($z < 0$) and the dielectric ($z > 0$) regions are

$$\beta_a = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2} = 158.0 \text{ m}^{-1},$$

$$\beta_d = \sqrt{\epsilon_r k_0^2 - \left(\frac{\pi}{a}\right)^2} = 304.1 \text{ m}^{-1},$$

where $k_0 = 209.4 \text{ m}^{-1}$.

We can set up an equivalent transmission line for the TE10 mode in each waveguide, and treat the problem as the reflection of an incident voltage wave at the junction of two infinite transmission lines.

The equivalent characteristic impedances for the two lines are

$$Z_{0_a} = \frac{k_0\eta_0}{\beta_a} = \frac{(209.4)(377)}{158.0} = 500.0\ \Omega,$$

$$Z_{0_d} = \frac{k\eta}{\beta_d} = \frac{k_0\eta_0}{\beta_d} = \frac{(209.4)(377)}{304.1} = 259.6\ \Omega.$$

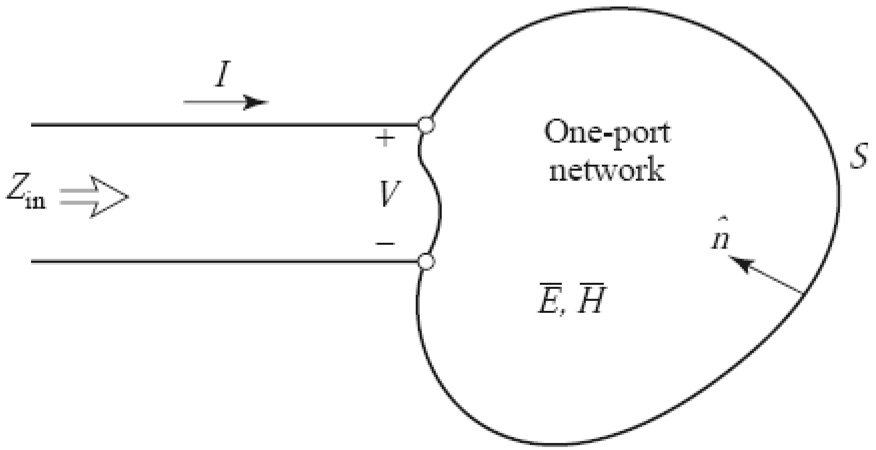
The reflection coefficient seen looking into the dielectric filled region is then

$$\Gamma = \frac{Z_{0_d} - Z_{0_a}}{Z_{0_d} + Z_{0_a}} = -0.316.$$

With this result, expressions for the incident, reflected, and transmitted waves can be written in terms of fields, or in terms of equivalent voltages and currents.

Write the expressions on class.

One port network



We now consider the arbitrary one-port network and derive a general relation between its impedance properties and electromagnetic energy stored in, and the power dissipated by, the network.

The complex power delivered to this network is

$$P = \frac{1}{2}\oint_S \bar{E} \times \bar{H}^* \cdot d\bar{s} = P_\ell + 2j\omega(W_m - W_e),$$

where *Pl* is real and represents the average power dissipated by the network, and *Wm* and *We* represent the stored magnetic and electric energy, respectively.

If we define real transverse modal fields

$$\bar{E}_t(x, y, z) = V(z)\bar{e}(x, y)e^{-j\beta z},$$
$$\bar{H}_t(x, y, z) = I(z)\bar{h}(x, y)e^{-j\beta z},$$

with a normalization such that $\int_S \bar{e} \times \bar{h} \cdot d\bar{s} = 1,$

we can express power in terms of the terminal voltage and current:

$$P = \frac{1}{2}\int_S VI^* \bar{e} \times \bar{h} \cdot d\bar{s} = \frac{1}{2} VI^*.$$

Then the input impedance is

$$Z_{\text{in}} = R + jX = \frac{V}{I} = \frac{VI^*}{|I|^2} = \frac{P}{\frac{1}{2}|I|^2} = \frac{P_\ell + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}.$$

the real part, R, of the input impedance is related to the dissipated power,

$$R = \frac{2P_l}{|I|^2}$$

the imaginary part, *X*, is related to the net energy stored in the network.

$$X = \frac{4\omega(W_m - W_e)}{|I|^2},$$

which is positive for an inductive load (*Wm > We)*, and negative for a capacitive load (*Wm < We).*

Even and Odd Properties of *Z(ω) and* $\Gamma(\omega)$

Consider the driving point impedance, $Z(\omega)$, at the input port of an electrical network. The voltage and current at this port are related as $V(\omega) = Z(\omega)I(\omega)$.

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega.$$

Because $v(t)$ must be real, we have that $v(t) = v*(t)$

$$\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega = \int_{-\infty}^{\infty} V^*(\omega)e^{-j\omega t}d\omega = \int_{-\infty}^{\infty} V^*(-\omega)e^{j\omega t}d\omega,$$

Then $$\boxed{V(-\omega) = V^*(\omega),}$$

which means that Re{V(ω)} is even in ω, while Im{V(ω)} is odd in ω.

$$V^*(-\omega) = Z^*(-\omega)I^*(-\omega) = Z^*(-\omega)I(\omega) = V(\omega) = Z(\omega)I(\omega).$$

Then $\boxed{Z(-\omega) = Z^*(\omega)}$

Since $Z(\omega) = R(\omega) + jX(\omega)$ *R(ω)* is even in *ω and X(ω)* is odd in ω.

Now consider the reflection coefficient at the input port:

$$\Gamma(\omega) = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0} = \frac{R(\omega) - Z_0 + jX(\omega)}{R(\omega) + Z_0 + jX(\omega)}.$$

Then $\boxed{\Gamma(-\omega) = \frac{R(\omega) - Z_0 - jX(\omega)}{R(\omega) + Z_0 - jX(\omega)} = \Gamma^*(\omega),}$

the real and imaginary parts of $\Gamma(\omega)$ are even and odd, respectively.

$$|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega) = \Gamma(\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2,$$

Homework

4.2 Consider a series RLC circuit with a current I. Calculate the power lost and the stored electric and magnetic energies, and show that the input impedance can be expressed as in (4.17).

4.3 Show that the input impedance Z of a parallel RLC circuit satisfies the condition that $Z(-\omega) = Z^*(\omega)$.