Lecture 9

PN junction (I)

Introduction

- **pn junction**
  - **– p-region an n-region in intimate contact**
- **Why is the pn junction worth studying?**

  It is present in virtually every semiconductor device!

Example: CMOS cross-section

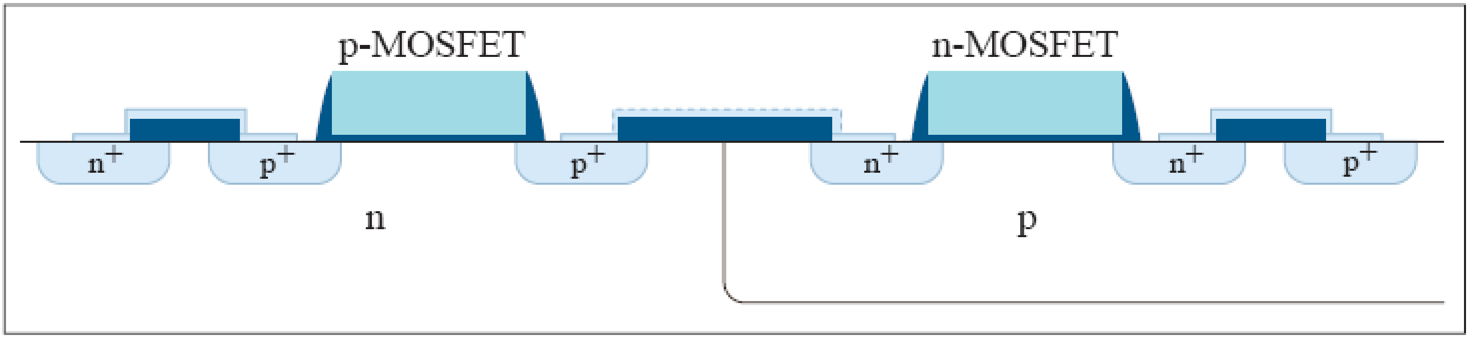

Figure by MIT OpenCourseWare.

Gauss's Law

The relationship of the electric field *E* and charge density.

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$ - The differential form in one dimension

where ε is the electric permittivity (F/cm)

$$E(x) - E(0) = \frac{1}{\varepsilon_s}\int_0^x \rho(x')dx'$$ - The integral form in one dimension

Charge is the source of electric field.

Poisson's equation

- **The electrostatic potential is defined as:**

$$\phi(x) - \phi(0) = -\int_0^x E(x')dx'$$ -potential difference as the integral of electric field.

- **Differentiation of the above**

$$E(x) = -\frac{d\phi(x)}{dx}$$

- **Poisson's equation**

$$\frac{d^2\phi(x)}{dx^2} = -\frac{dE}{dx} = -\frac{\rho(x)}{\varepsilon}$$

Semiconductor Electrostatics in Thermal Equilibrium

Outline

- **Nonuniformly doped semiconductor in thermal equilibrium**
- **Relationships between potential, $\varphi(x)$ and equilibrium carrier concentrations, $p_o(x)$, $n_o(x)$ –Boltzmann relations & "60 *mV* Rule"**
- **Quasi-neutral situation**

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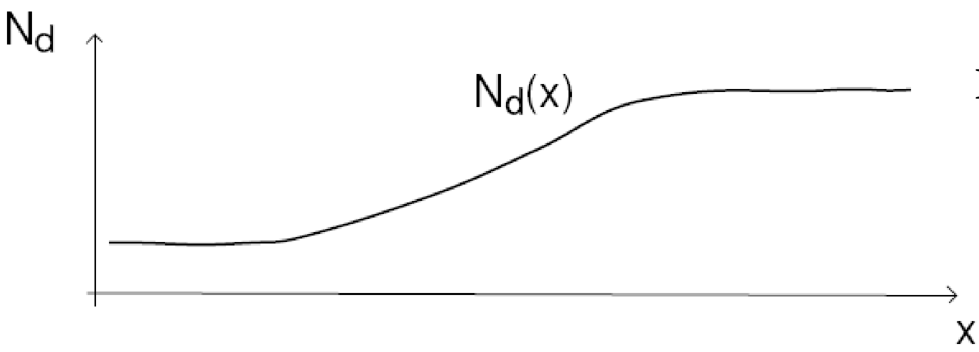
Nonuniformly doped semiconductor in thermal equilibrium

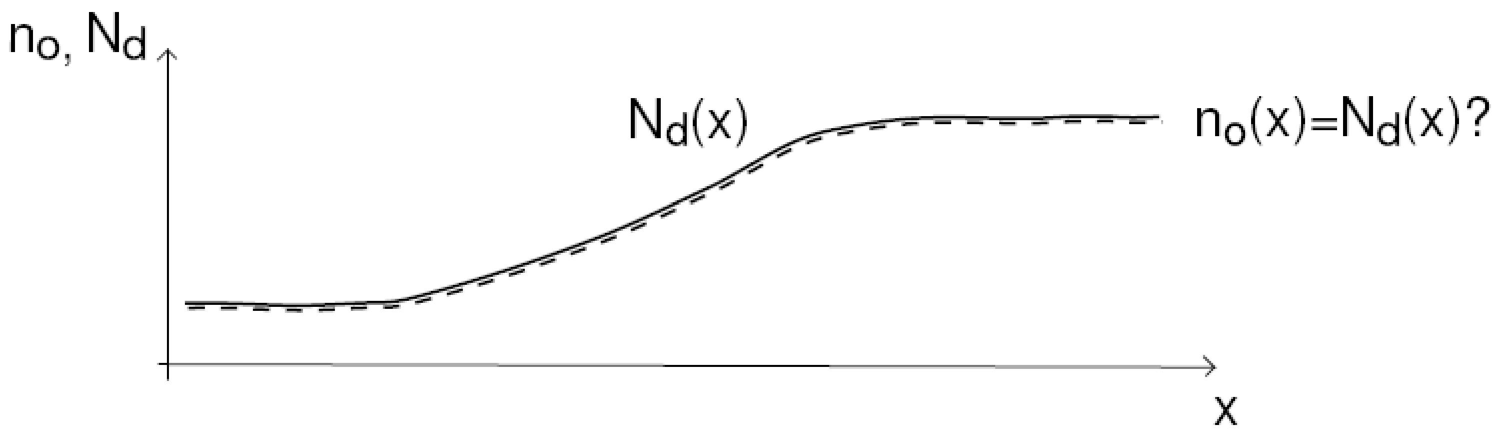
Consider a piece of n-type Si in thermal equilibrium with non-uniform dopant distribution:



n-type
⇒ lots of electrons, few holes
⇒ focus on electrons

What is the resulting electron concentration in thermal equilibrium?

OPTION 1: Electron concentration follows doping concentration EXACTLY ⇒ $n_0(x) = N_d(x)$



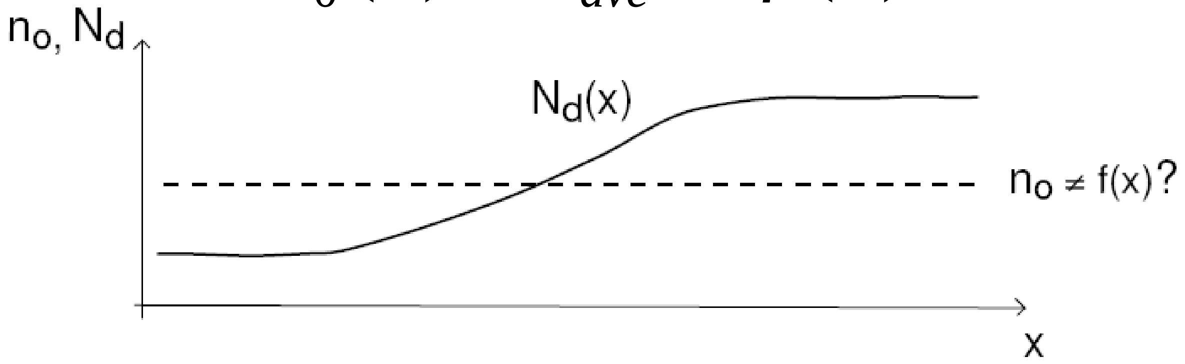
Gradient of electron concentration

⇒ **electron diffusion**

⇒ **not in thermal equilibrium!**

OPTION 2: electron concentration uniform in space

$$n_0(x) = n_{ave} \neq f(x)$$



Think about space charge density: $\rho(x) \approx q[N_d(x) - n_0(x)]$

If $N_d(x) \neq n_0(x)$

$\Rightarrow \rho(x) \neq 0$

$\Rightarrow$electric field

$\Rightarrow$net electron drift

$\Rightarrow$not in thermal equilibrium

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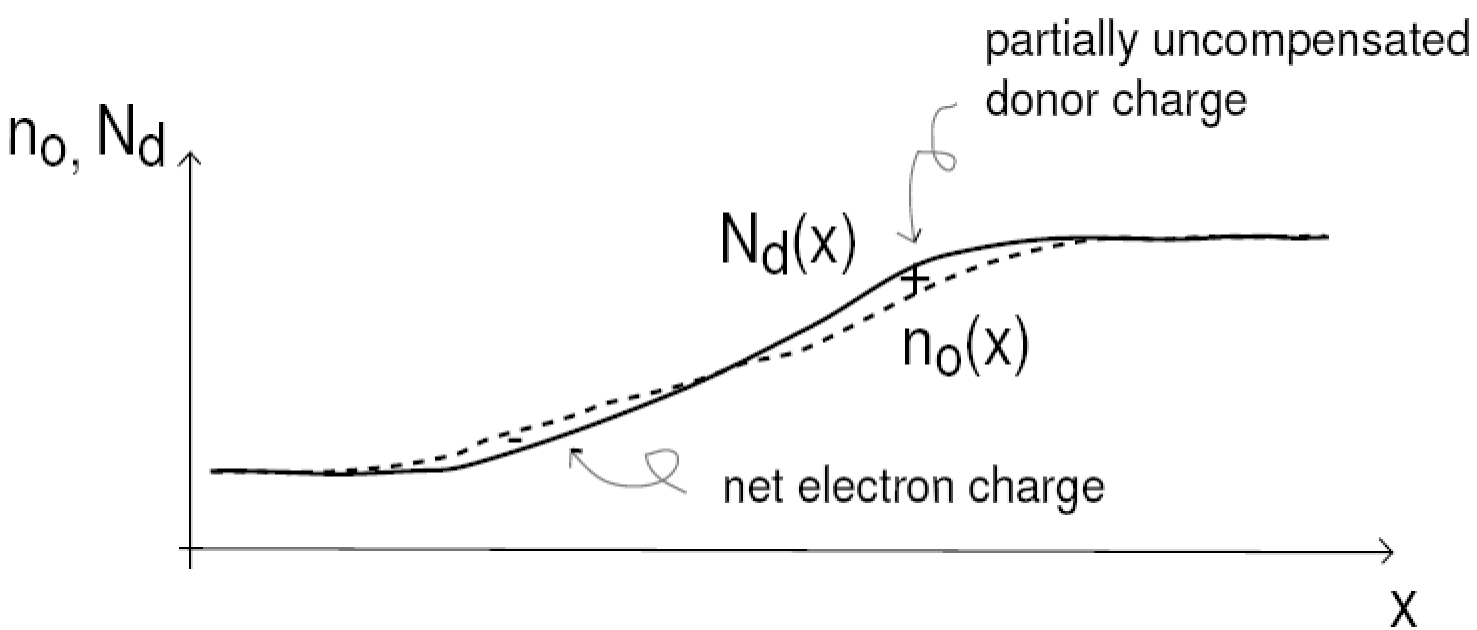
OPTION 3: Demand that $J_n = 0$ in thermal equilibrium at every x ($J_p = 0$ too)

Diffusion precisely balances Drift

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

What is $n_o(x)$ that satisfies this condition?

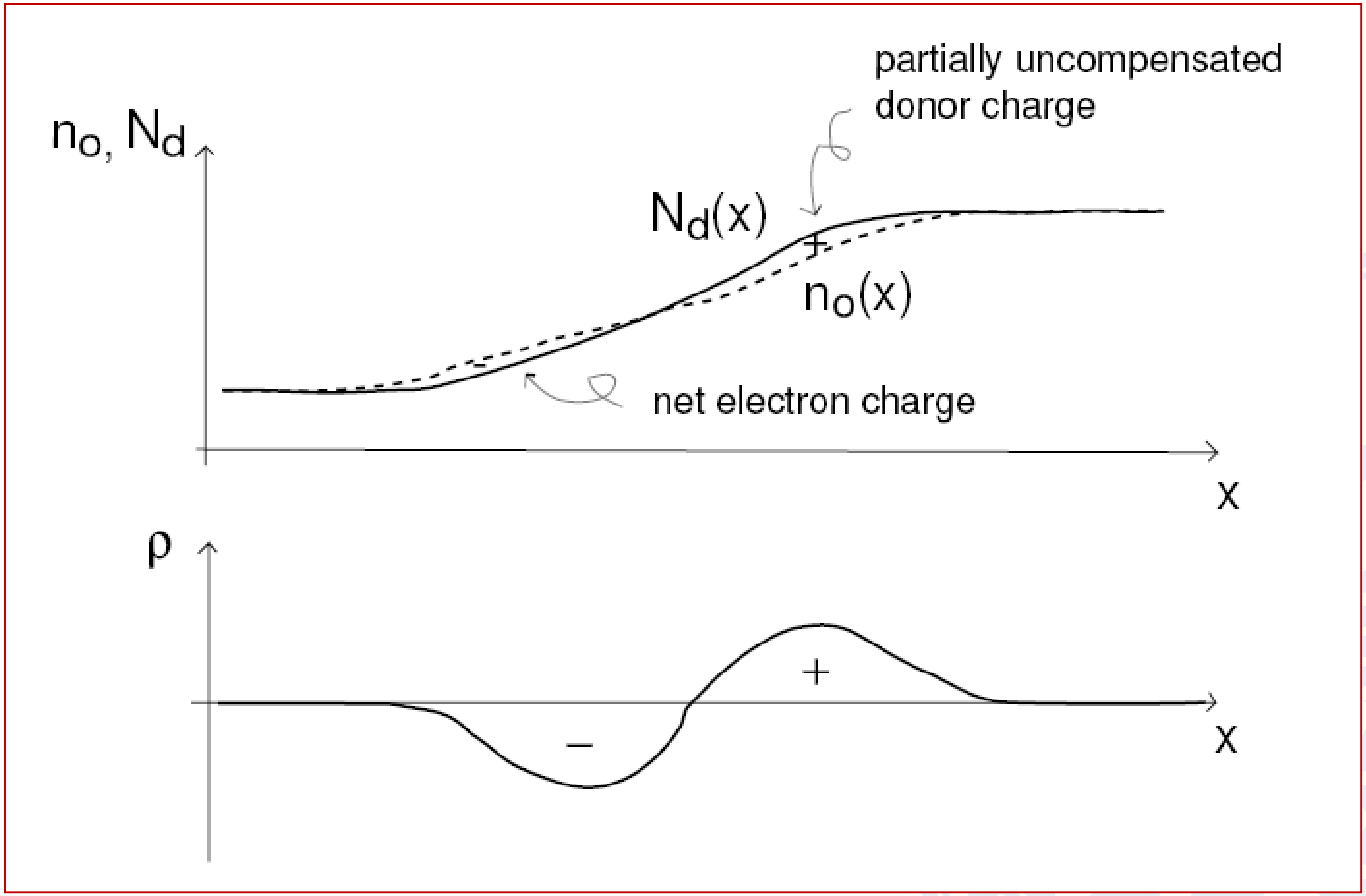


Let us examine the electrostatics implications of

$$n_0(x) \neq N_d(x)$$

Space charge density

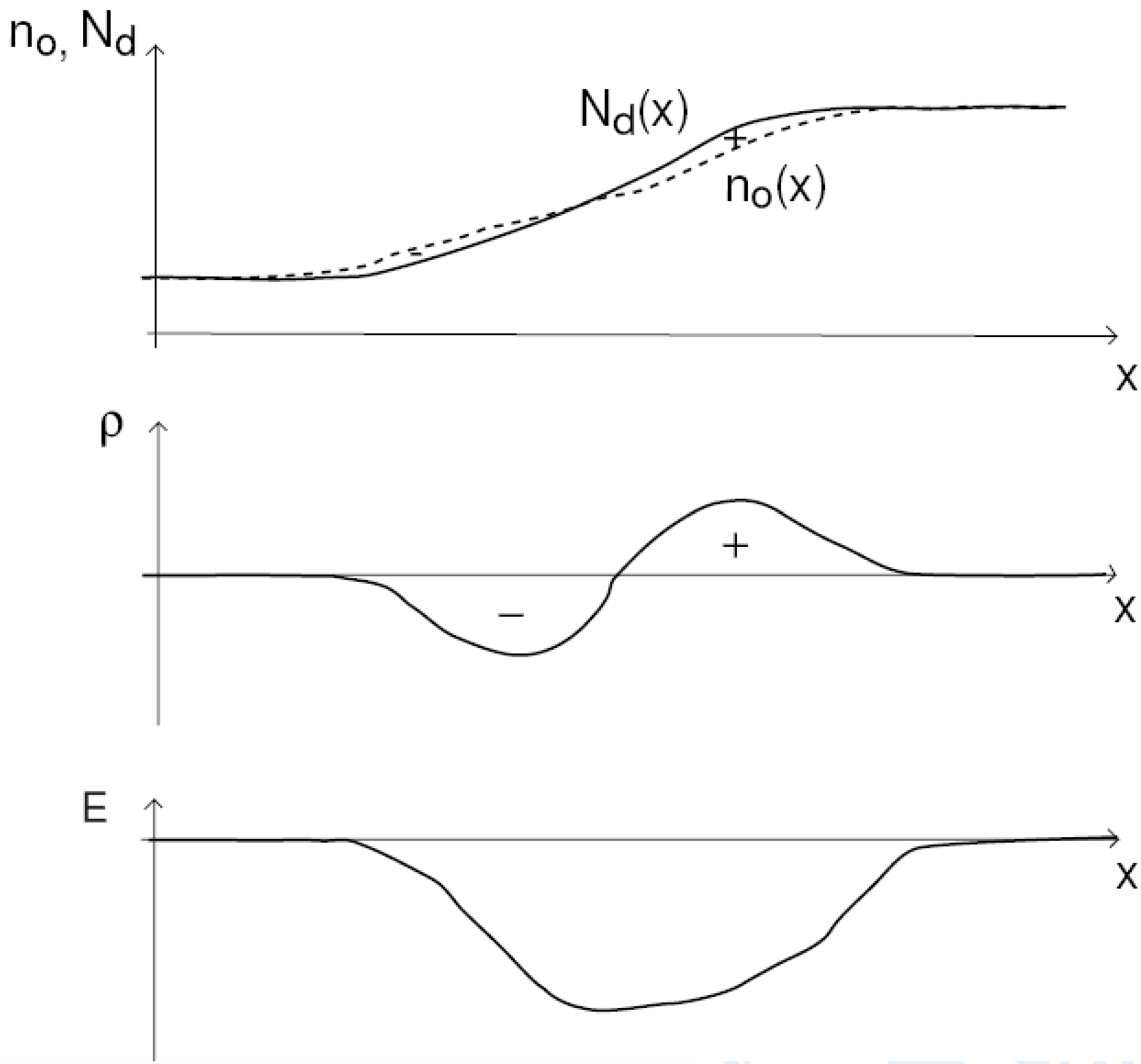
$$\rho(x) = q[N_d(x) - n_0(x)]$$



Electric Field

Gauss's law: $$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_s}$$

Integrate from $x = 0$: $$E(x) - E(0) = \frac{1}{\varepsilon_s}\int_0^x \rho(x')dx'$$


n_o, N_d
N_d(x)
+
n_o(x)
x
ρ
+
−
x
E
x

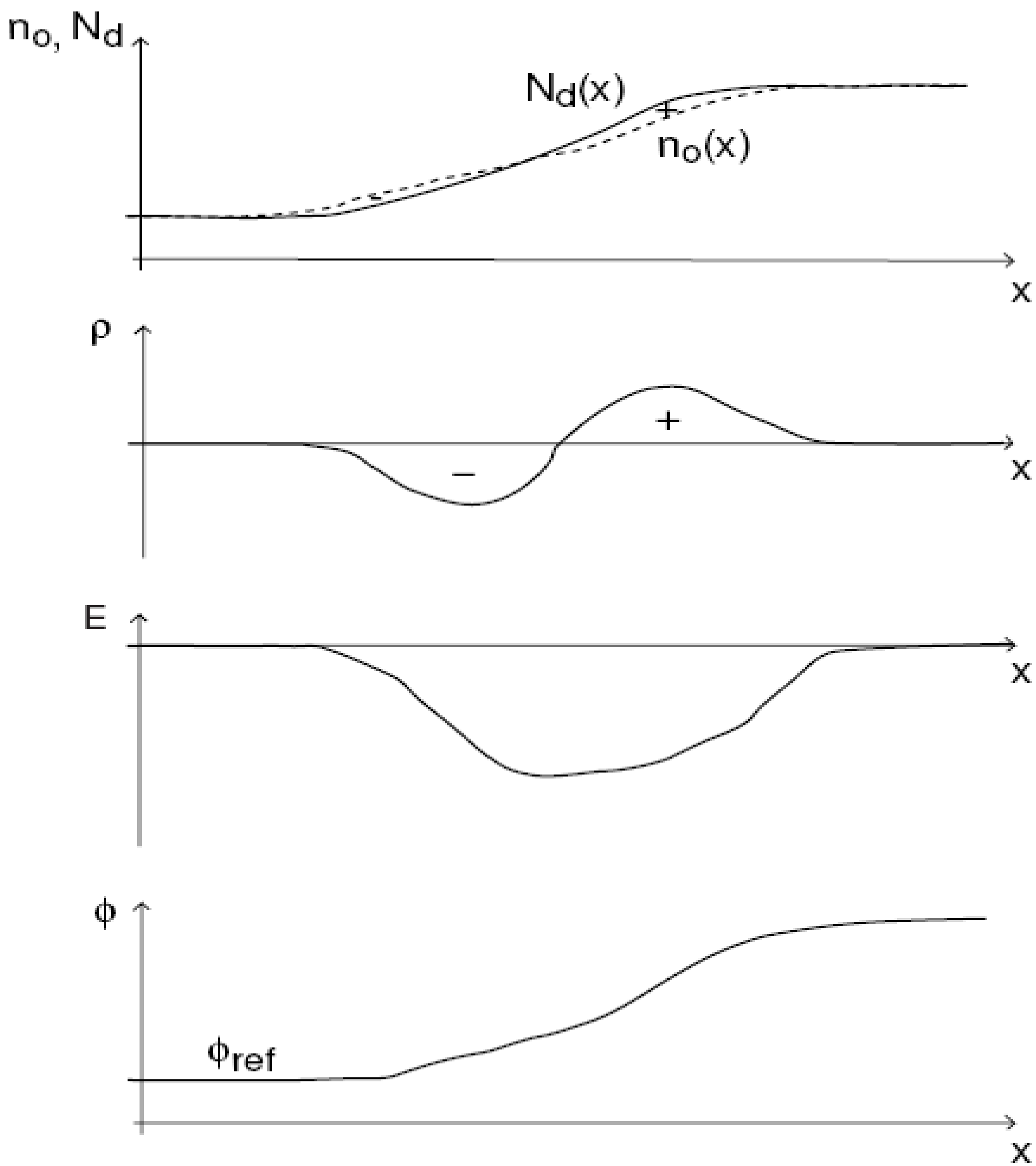
Electrostatic Potential

$$\frac{d\phi}{dx} = -E(x)$$

Integrate from $x=0$:

$$\phi(x) - \phi(0) = -\int_0^x E(x')dx'$$

$$Select \ \ \phi(x=0) = \phi_{ref}$$


n_o, N_d
$N_d(x)$
$n_o(x)$
+
x
ρ
+
−
x
E
x
φ
$φ_{ref}$
x

Boltzmann relations

Relationships between potential, *φ(x)* and equilibrium carrier concentrations, $p_o(x)$, $n_o(x)$

$$J_n = 0 = qn_0\mu_n E + qD_n \frac{dn_0}{dx}$$

$$\frac{\mu_n}{D_n} \bullet \frac{d\phi}{dx} = \frac{1}{n_0} \bullet \frac{dn_0}{dx}$$

Using Einstein relation: $\frac{q}{kT} \bullet \frac{d\phi}{dx} = \frac{d\left(\ln n_0\right)}{dx}$

Integrate:

$$\frac{q}{kT}\left(\phi-\phi_{ref}\right)=\ln n_0-\ln n_{0,ref}=\ln\frac{n_0}{n_{0,ref}}$$

$Then:$

$$n_0=n_{0,ref}\exp\left[\frac{q\left(\phi-\phi_{ref}\right)}{kT}\right]$$

Any reference is good

$$\phi_{ref}=0\,at\;n_{0.ref}=n_i \qquad n_0=n_i e^{q\phi/kT}$$

If we do same with holes (starting with $J_p=0$ in thermal equilibrium, or simply using $n_0p_0=n_i^2$)

$$p_0 = n_i e^{-q\phi/kT}$$

We can rewrite as:

$$\phi = \frac{kT}{q} \bullet \ln \frac{n_0}{n_i}$$

$$\phi = -\frac{kT}{q} \bullet \ln \frac{p_0}{n_i}$$

"60 mV" Rule

At room temperature for Si:

$$\phi = (25mV)\bullet\ln\frac{n_0}{n_i} = (25mV)\bullet\ln(10)\bullet\log\frac{n_0}{n_i}$$

or $$\phi \approx (60mV)\bullet\log\frac{n_0}{n_i}$$

EXAMPLE 1:

$$n_0 = 10^{18}cm^{-3} \Rightarrow \phi = (60mV)\times 8 = 480mV$$

With holes:

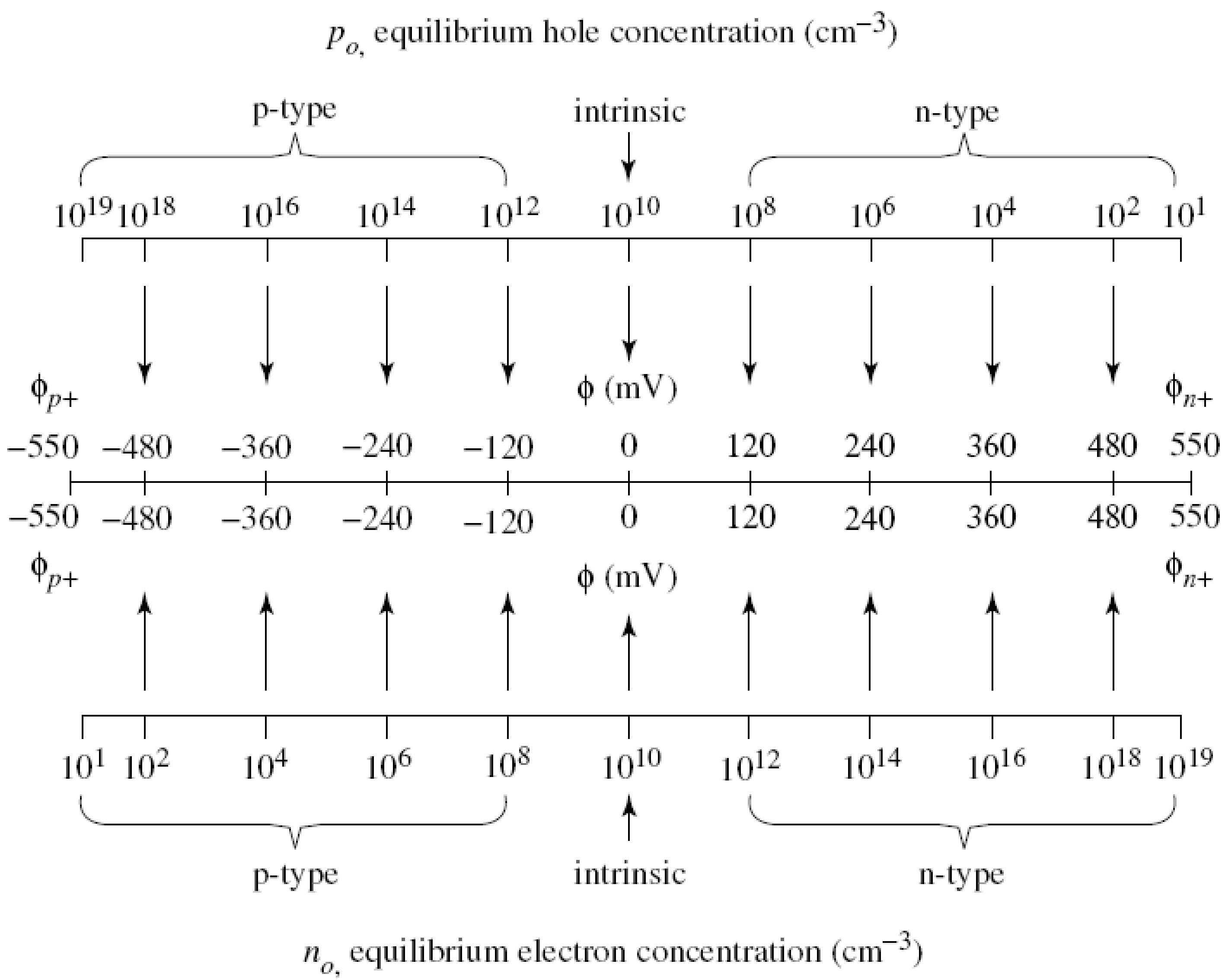
$$\phi = -\frac{kT}{q} \bullet \ln \frac{p_0}{n_i}$$

EXAMPLE 1:

$$n_0 = 10^{18} cm^{-3} \Rightarrow p_0 = 10^{2} cm^{-3}$$

$$\Rightarrow \phi = -(60mV) \times -8 = 480mV$$

Relationship between ϕ, n_0 and p_0 :

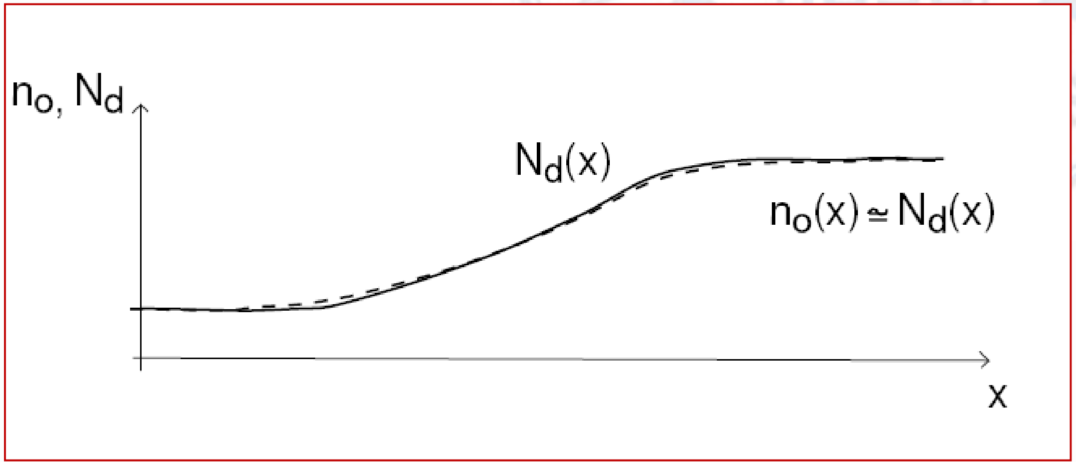


Quasi-neutral situation

If $N_d(x)$ changes slowly with $x \rightarrow n_0(x)$ also changes slowly with x. Why?

- Small dn_o/dx implies a small diffusion current. We do not need a large drift current to balance it.
- Small drift current implies a small electric field and therefore a small space charge

Then $$n_0(x) \approx N_d(x)$$



$n_o(x)$ tracks $N_d(x)$ well
⇒ minimum space charge
⇒ semiconductor is quasineutral

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Summary of Key Concepts

- **It is possible to have an electric field inside a semiconductor in thermal equilibrium**

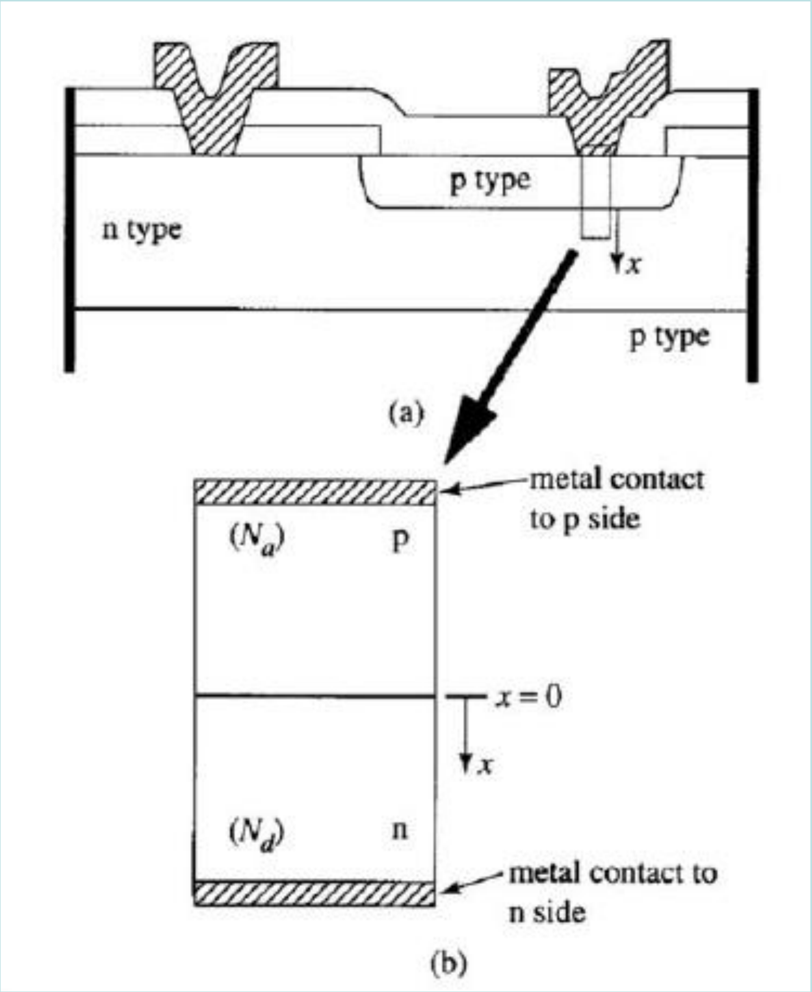
  ⇒ Nonuniform doping distribution.

- **In thermal equilibrium, there is a fundamental relationship between the $\varphi(x)$ and the equilibrium carrier concentrations $n_o(x)$ & $p_o(x)$**

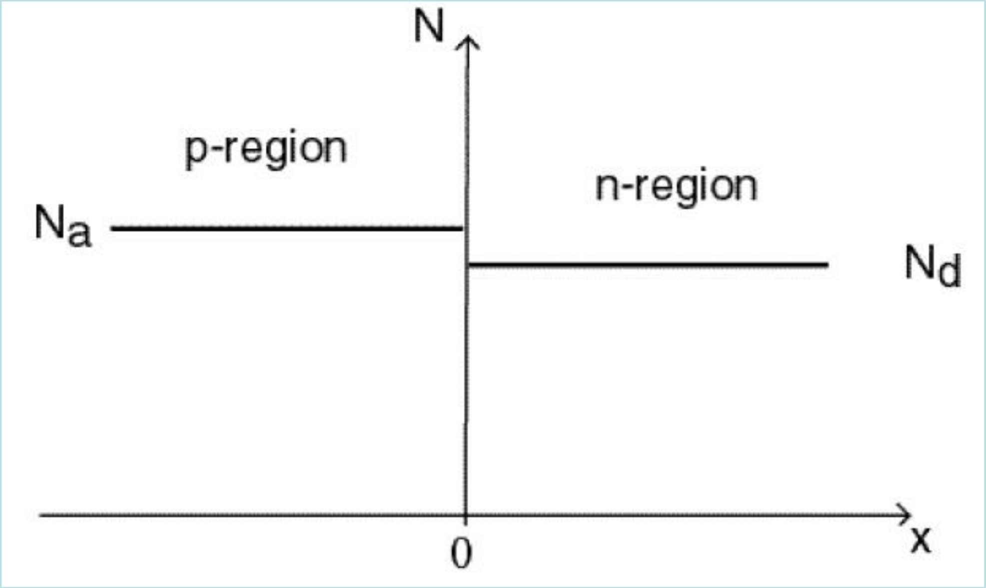
  – Boltzmann relations (or "60 mV Rule").

- **In a slowly varying doping profile, majority carrier concentration tracks well the doping concentration.**

Electrostatics of pn junction in equilibrium

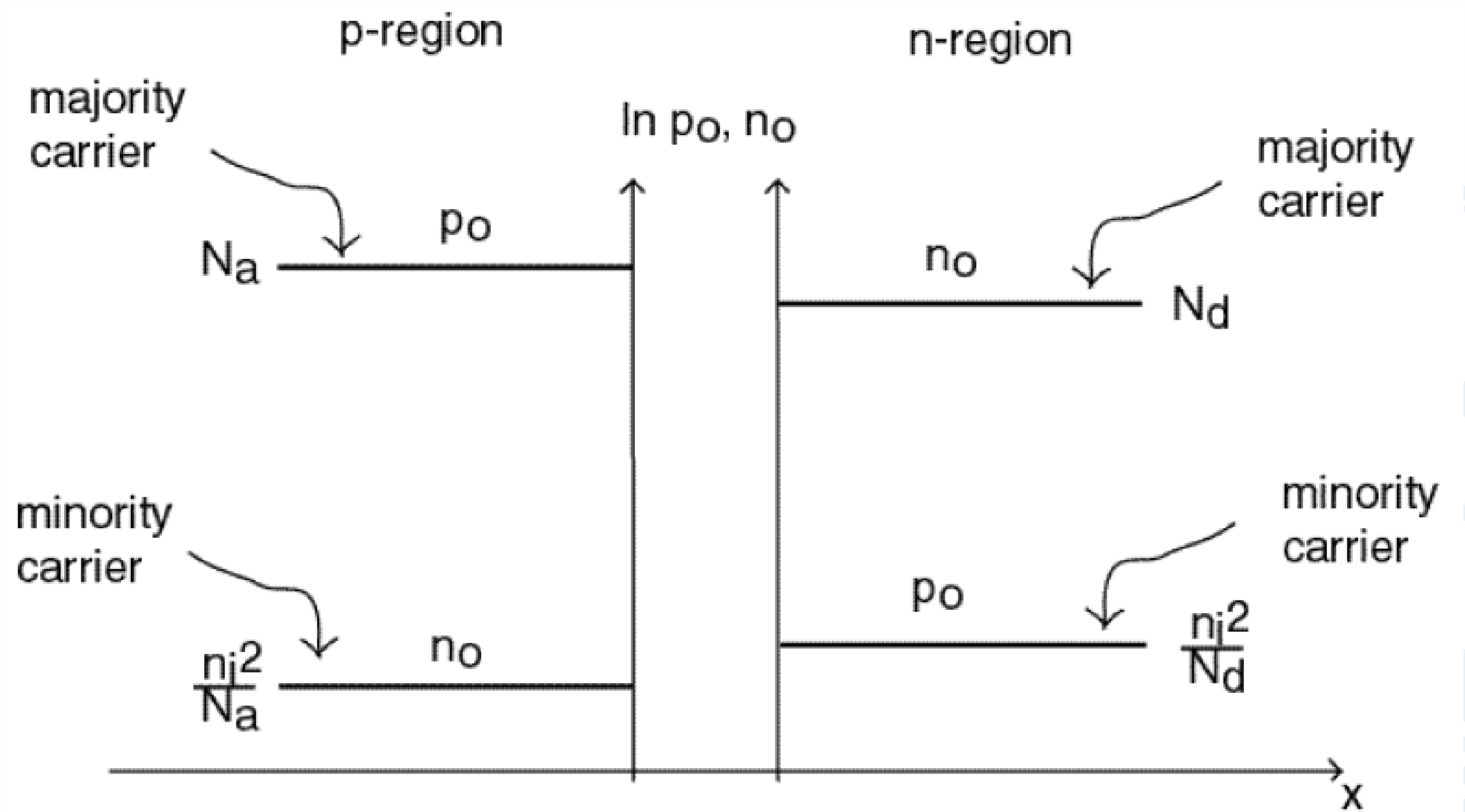


Doping distribution of an **abrupt pn junction (metallurgical junction)**



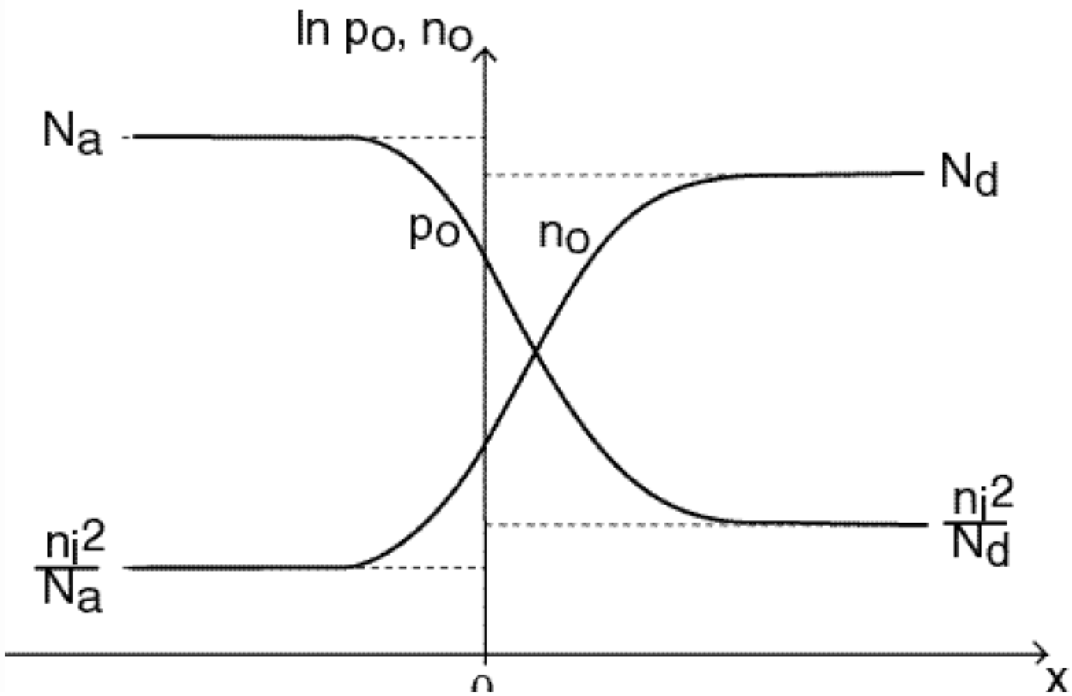
What is the carrier concentration distribution in thermal equilibrium?

First think of the two sides separately:



Now bring the two sides together. What happens?

Resulting carrier concentration profile in thermal equilibrium:



- Far away from the metallurgical junction:
  nothing happens – Two **quasi neutral regions**
- Around the metallurgical junction:
  diffusion of carriers must counterbalance drift
  – **Space charge region (depletion region)**

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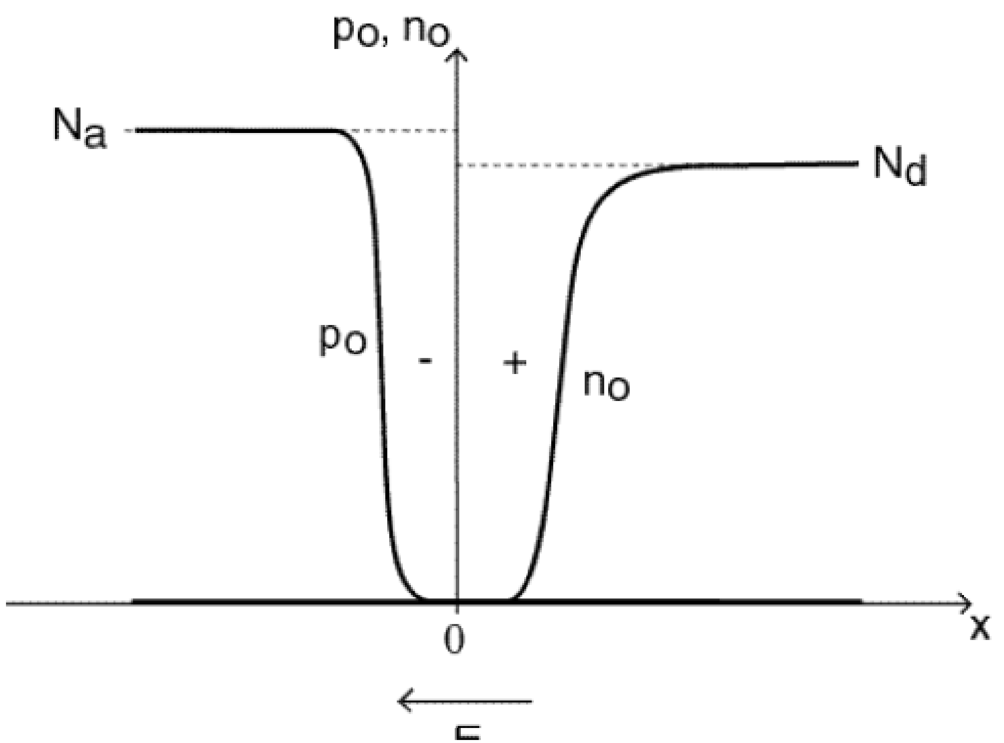
On a linear scale:



Thermal equilibrium: balance between drift and diffusion

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$

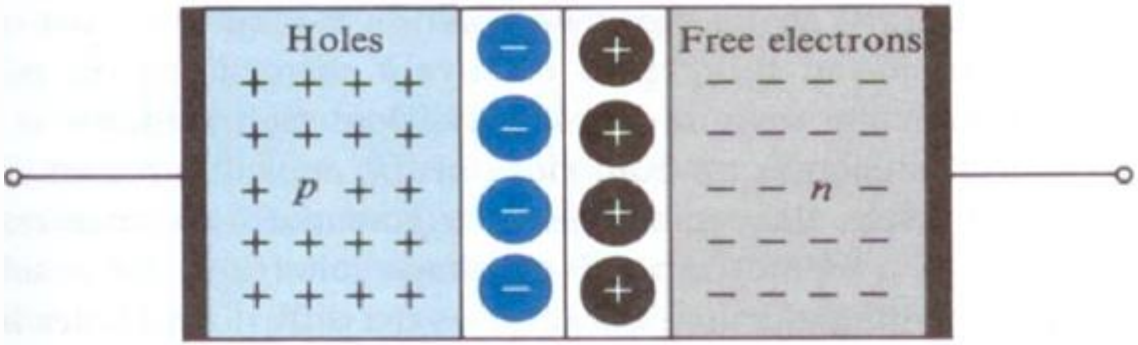
$$J_p(x) = J_p^{drift}(x) + J_p^{diff}(x) = 0$$

Question?

Compare the number of majority electrons in the n-type region (or the number of majority holes in the p-type region) before and after the junction formation.

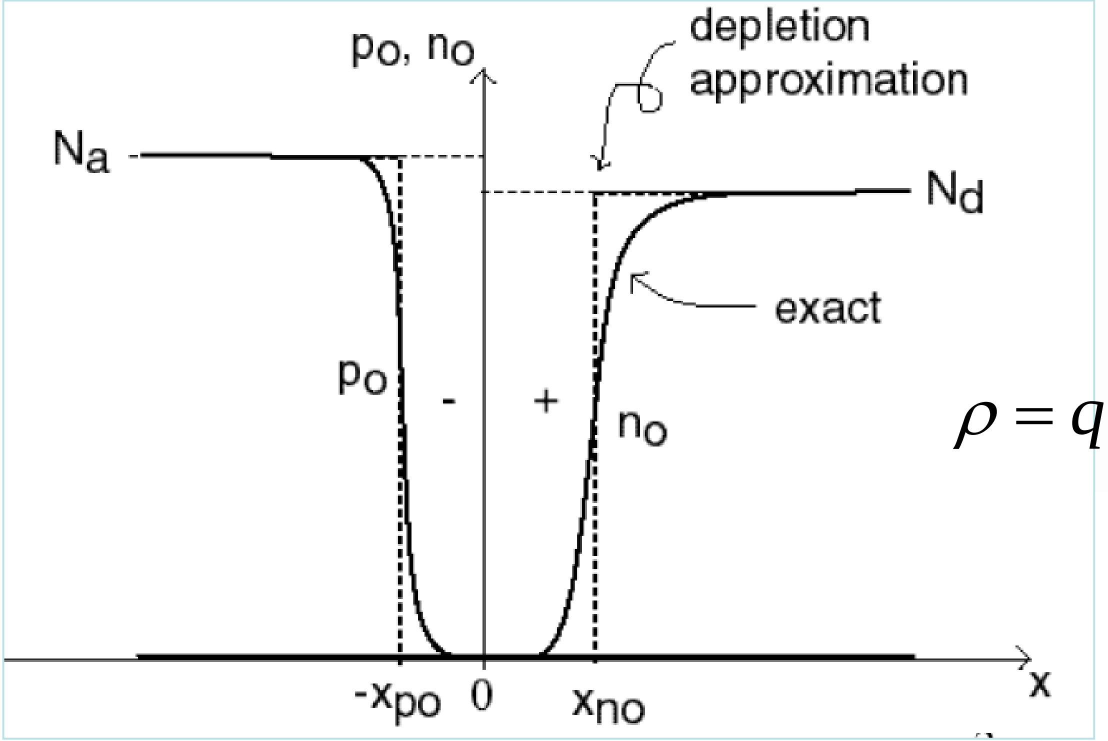
3.The Depletion Approximation

- Assume the QNR's are perfectly **charge neutral**
- Assume the SCR is **depleted** of carriers (complete ionization)
  - *depletion region*
- Transition between SCR and QNR's sharp at
  - $-x_{p0}$ and x_{no} (must calculate where to place these)



Depletion region: dipole layer

Now, we want to know $n_0(x)$, $p_0(x)$, $\rho(x)$, $E(x)$ and $\phi(x)$.
We need to solve Poisson's equation using a simple but powerful approximation



$$\rho = q\left(p_0 - n_0 + N_d - N_a\right)$$

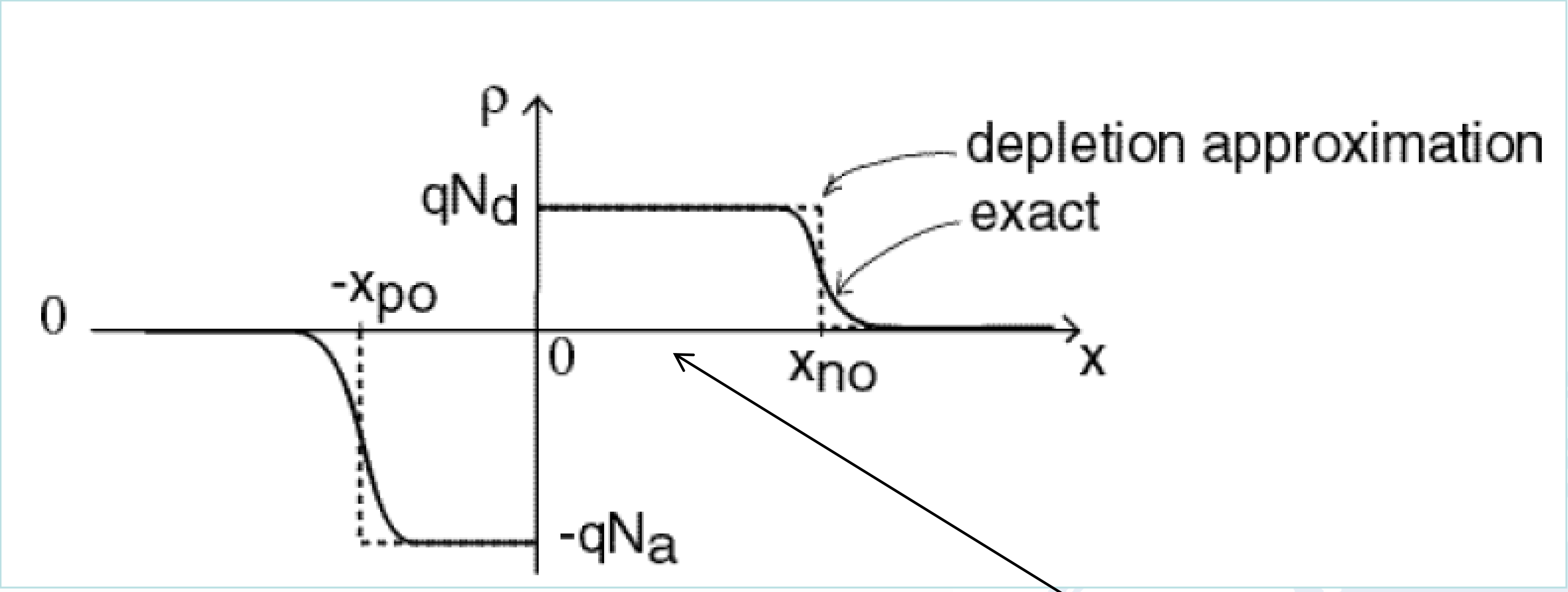
$$x < -x_{po}: \qquad p_o(x) = N_a, \quad n_o(x) = \frac{n_i^2}{N_a}$$

$$-x_{po} < x < 0: \quad p_o(x),\ n_o(x) << N_a$$

$$0 < x < x_{no}: \quad n_o(x),\ p_o(x) << N_d$$

$$x > x_{no}: \qquad n_o(x) = N_d, \quad p_o(x) = \frac{n_i^2}{N_d}$$

Space Charge Density



$$\rho(x) = 0; \qquad x < -x_{po};$$
$$= -qN_a; \quad -x_{po} < x < 0;$$
$$= qN_d; \qquad 0 < x < x_{no};$$
$$= 0; \qquad x > x_{no};$$

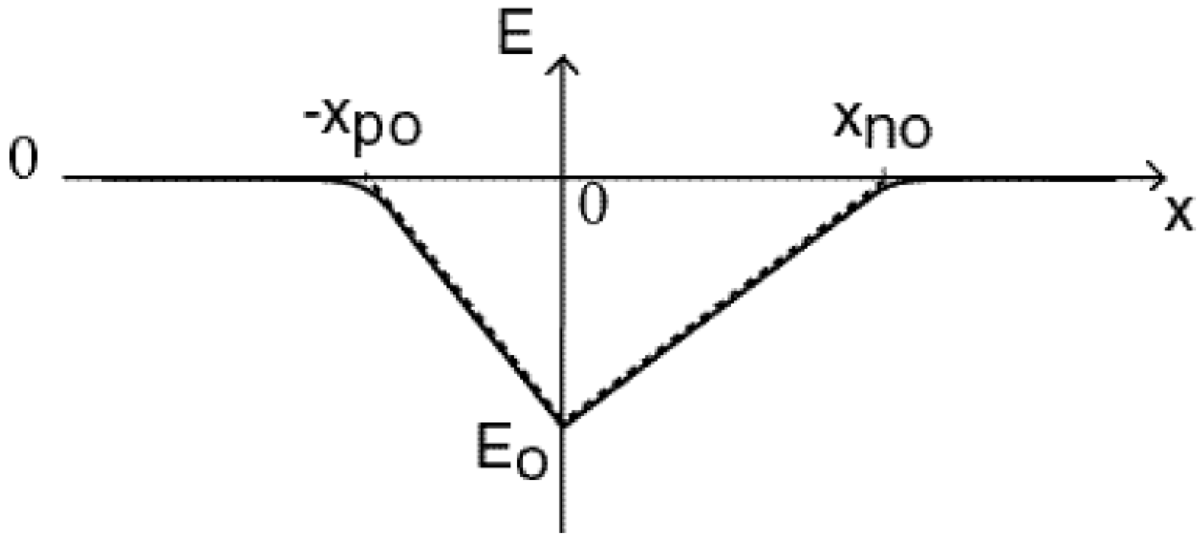
Uncovering of impurity ions near the junction on both sides

Electric Field

Integrate Poisson's equation

$$\nabla \cdot \nabla \varphi = \nabla^2 \varphi = -\frac{\rho}{\varepsilon} \qquad E = -\nabla \varphi$$

$$E(x_2) - E(x_1) = \frac{1}{\varepsilon_s} \int_{x_1}^{x_2} \rho(x) dx$$



Build-in field

$$x < -x_{po}; \qquad E(x) = 0$$

$$-x_{po} < x < 0; \quad E(x) - E(-x_{po}) = \frac{1}{\varepsilon_s} \int_{-x_{po}}^{x} -qN_a dx'$$

$$= \left[-\frac{qN_a}{\varepsilon_s} x \right]_{-x_{po}}^{x} = -\frac{qN_a}{\varepsilon_s} \left(x + x_{po} \right)$$

$$0 < x < x_{no}; \quad E(x) = \frac{qN_d}{\varepsilon_s} (x - x_{no})$$

$$x > x_{no}; \qquad E(x) = 0$$

Electrostatic Potential

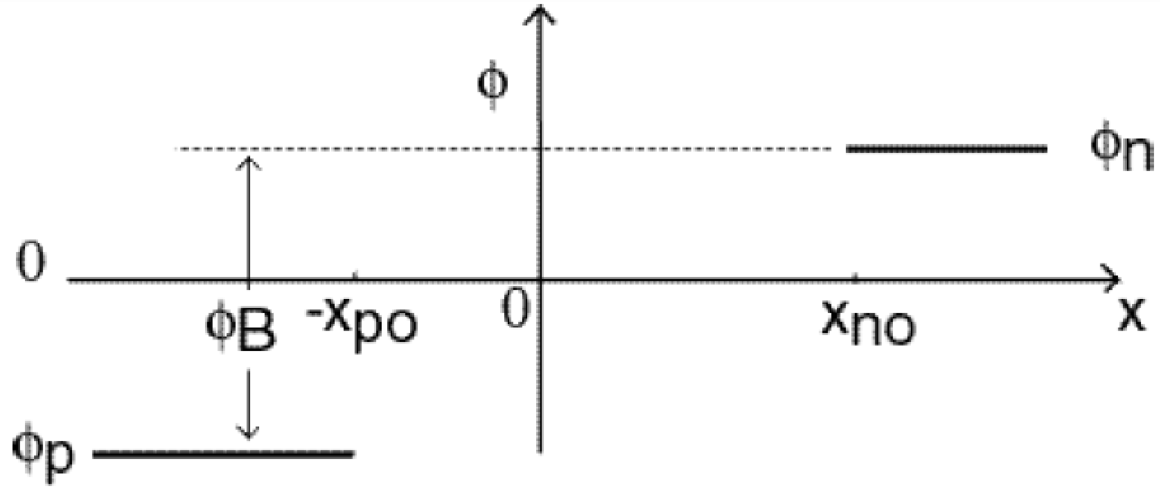
From Boltzman relationship $\phi = \frac{kT}{q} \bullet \ln \frac{n_0}{n_i}$ $\phi = -\frac{kT}{q} \bullet \ln \frac{p_0}{n_i}$

(with $\phi=0$ @ $n_0=p_0=n_i$) — **intrinsic semiconductor**

In QNR's, n_0 and p_0 are known ⇒ can determine ϕ

$$in\ p-QNR:\ p_0 = N_a \Rightarrow \phi_p = -\frac{kT}{q} \bullet \ln \frac{N_a}{n_i}$$

$$in\ n-QNR:\ n_0 = N_d \Rightarrow \phi_n = \frac{kT}{q} \bullet \ln \frac{N_d}{n_i}$$

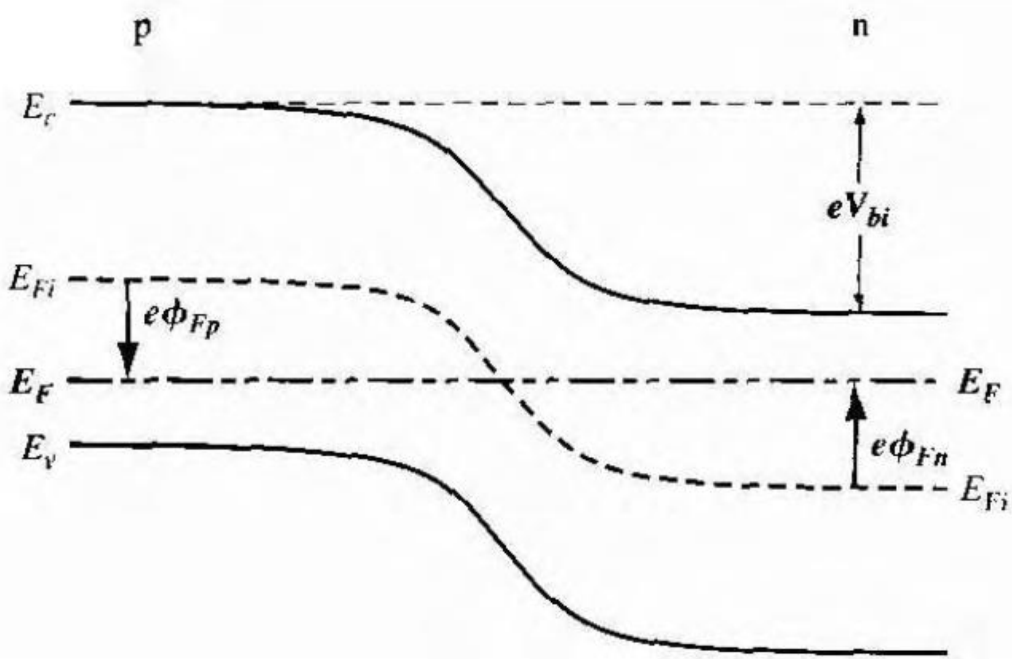


Built-in potential（内建电势差）:

$$\phi_B = \phi_n - \phi_p = \frac{kT}{q} \bullet \ln \frac{N_d N_a}{n_i^2}$$

The built-in potential can be looked as the potential hill or barrier that keeps electrons on the n-type side and keeps holes on the p-type sides.

From energy band diagram



Energy-band diagram of a pn junction in thermal equilibrium

Built-in potential : a potential barrier for electrons in the n region trying to move into the conduction band of the p region.

- In thermal equilibrium, the Fermi energy level is constant throughout the entire system.
- E_{Fi}, E_c, E_v bend with the distribution of carrier concentration.

$$E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

$$E_C - E_F = kT \ln\left(\frac{N_C}{n_0}\right)$$

$$E_F - E_v = kT \ln\left(\frac{N_V}{p_0}\right)$$

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

Built-in potential

$$V_{bi}=|\phi_{Fn}|+|\phi_{Fp}|$$

$$e\phi_{Fn}=E_{Fi}-E_F$$

$$E_{Fi}-E_F=kT\ln\left(\frac{n_i}{n_0}\right)$$

$$\Rightarrow \phi_{Fn}=\frac{-kT}{e}\ln\left(\frac{N_d}{n_i}\right)$$

$$e\phi_{Fp}=E_{Fi}-E_F$$

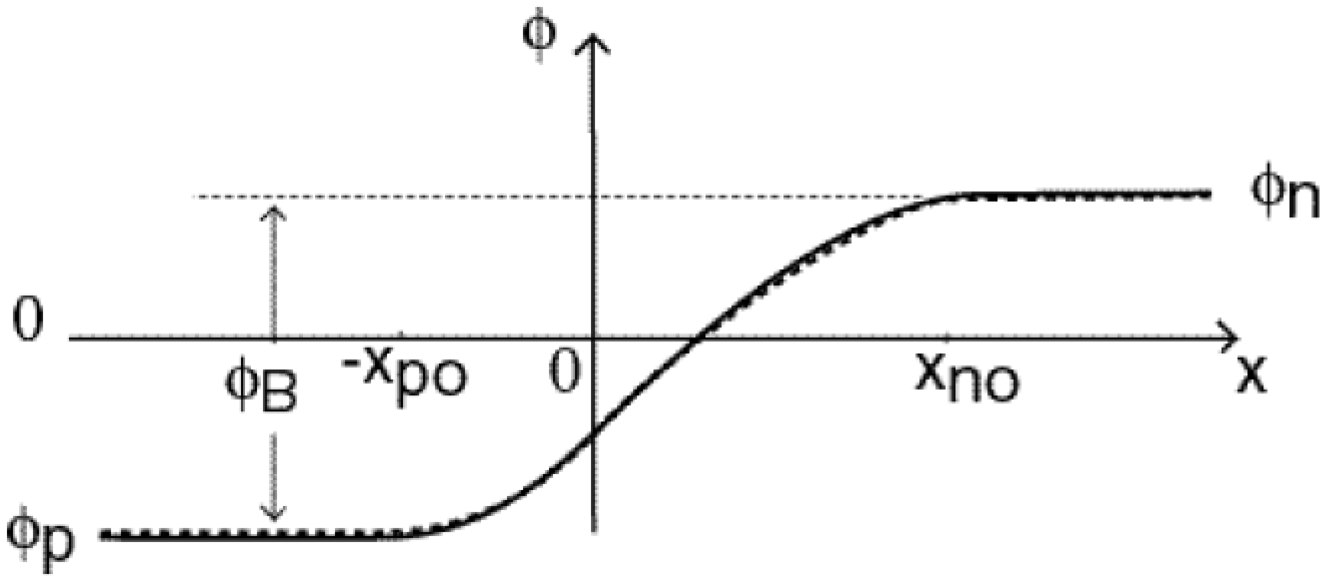
$$E_{Fi}-E_F=kT\ln\left(\frac{p_0}{n_i}\right)$$

$$\Rightarrow \phi_{Fp}=+\frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right)$$

$$\Rightarrow V_{bi}=\frac{kT}{e}\ln\left(\frac{N_aN_d}{n_i^2}\right)=V_T\ln\left(\frac{N_aN_d}{n_i^2}\right)$$

To obtain $\phi(x)$ in between, integrate $E(x)$

$$\phi(x_2) - \phi(x_1) = -\int_{x_1}^{x_2} E(x')dx'$$



$$x < -x_{po}; \qquad \phi(x) = \phi_p$$

$$-x_{po} < x < 0; \quad \phi(x) - \phi(-x_{po}) = -\int_{-x_{po}}^{x} \frac{-qN_a}{\varepsilon_s}(x' + x_{po})dx$$

$$= \frac{qN_a}{2\varepsilon_s}\left(x + x_{po}\right)^2$$

$$\phi(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}\left(x + x_{po}\right)^2$$

$$0 < x < x_{no}; \quad \phi(x) = \phi_n - \frac{qN_a}{2\varepsilon_s}\left(x - x_{no}\right)^2$$

$$x > x_{no}; \qquad \phi(x) = \phi_n$$

Almost done...

Still do not know x_{n0} and x_{p0} ⇒ need two more equations

1. Require overall charge neutrality:

$$qx_{p0}AN_A = qx_{n0}AN_D$$

where x_{p0} and x_{n0} are the width of depletion region in the p side and in the n side, respectively, and A is the cross-sectional area of the junction.

$$\frac{x_{p0}}{x_{n0}} = \frac{N_D}{N_A}$$

The depletion region exists in both the p and n materials and that equal amounts of charge exist on both side. In order to uncover the same amount of charge, the depletion layer will extend deeper into the more lightly doped material.

2. Require *φ(x)* to be continuous at *x=0*;

$$\phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}{}^2 = \phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}{}^2$$

Two equations with two unknowns — obtain solution:

$$x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_B N_a}{q(N_a + N_d) N_d}} \qquad x_{p0} = \sqrt{\frac{2\varepsilon_s \phi_B N_d}{q(N_a + N_d) N_a}}$$

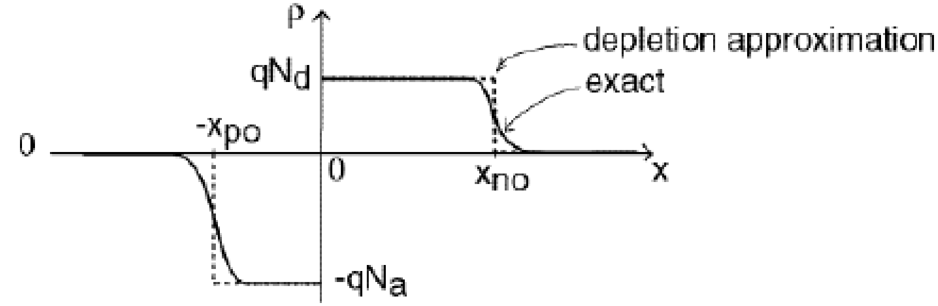
The total width of the depletion region W is

$$W = x_n + x_p$$

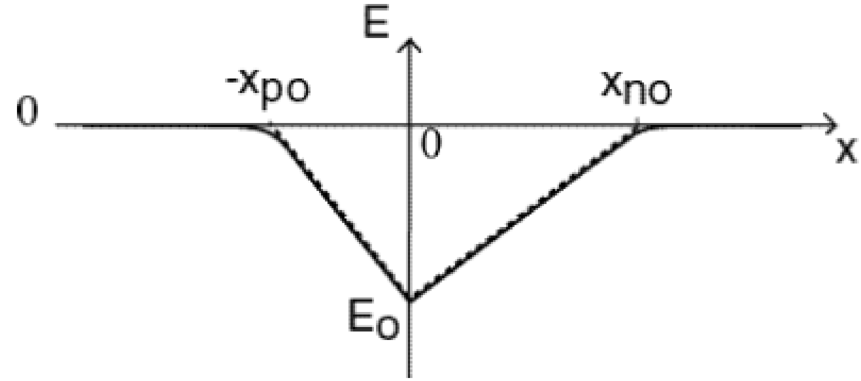
Now problem is completely solved!

Solution Summary

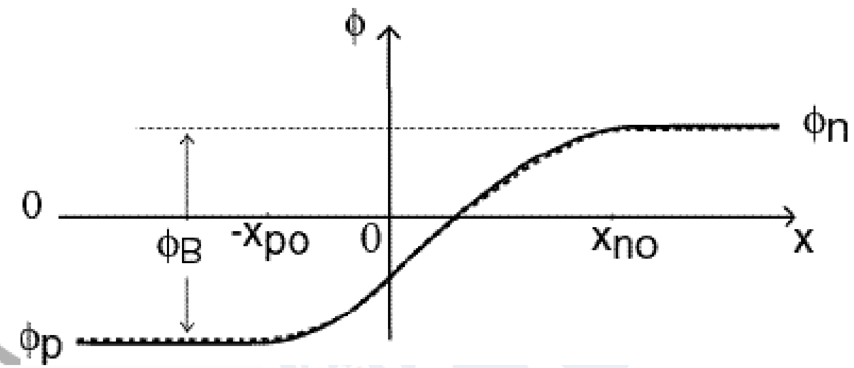
Space Charge Density



Electrostatic Field



Electrostatic Potential



Width of the space charge region:

$$x_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_B \left(N_a + N_d\right)}{qN_a N_d}}$$

Field at the metallurgical junction is maximum:

$$\left|E_0\right| = \sqrt{\frac{2q\phi_B N_a N_d}{\varepsilon_s \left(N_a + N_d\right)}}$$

Three Special Cases

- **Symmetric junction: $N_a = N_d$** $x_{p0} = x_{n0}$

- **Asymmetric junction: $N_a > N_d$** $x_{p0} < x_{n0}$

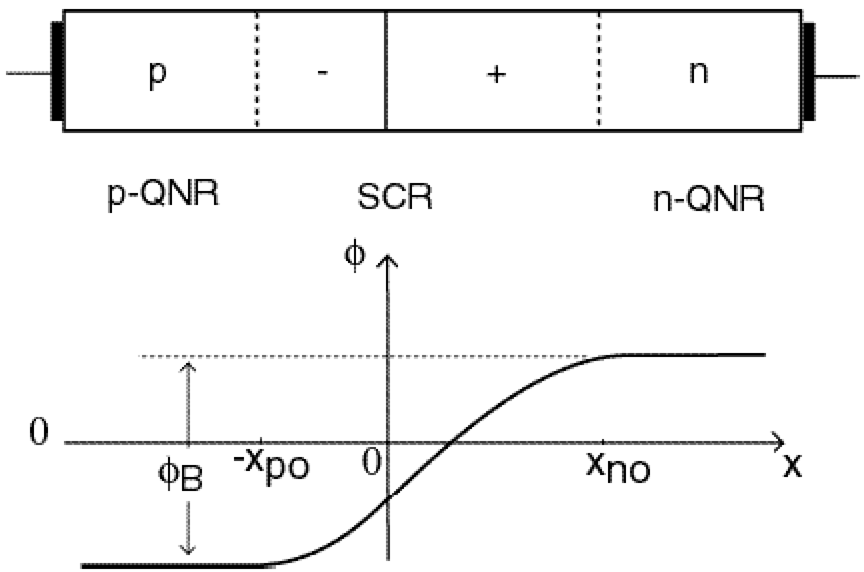
- **Strongly asymmetric junction** p[+]n junction: $N_a >> N_d$

$$x_{p0} \ll x_{n0} \approx \sqrt{\frac{2\varepsilon_s \phi_B}{qN_d}} \qquad |E_0| \approx \sqrt{\frac{2q\phi_B N_d}{\varepsilon_s}}$$

The lightly-doped side controls the electrostatics of the pn junction

Contact Potential

Potential distribution in thermal equilibrium so far:



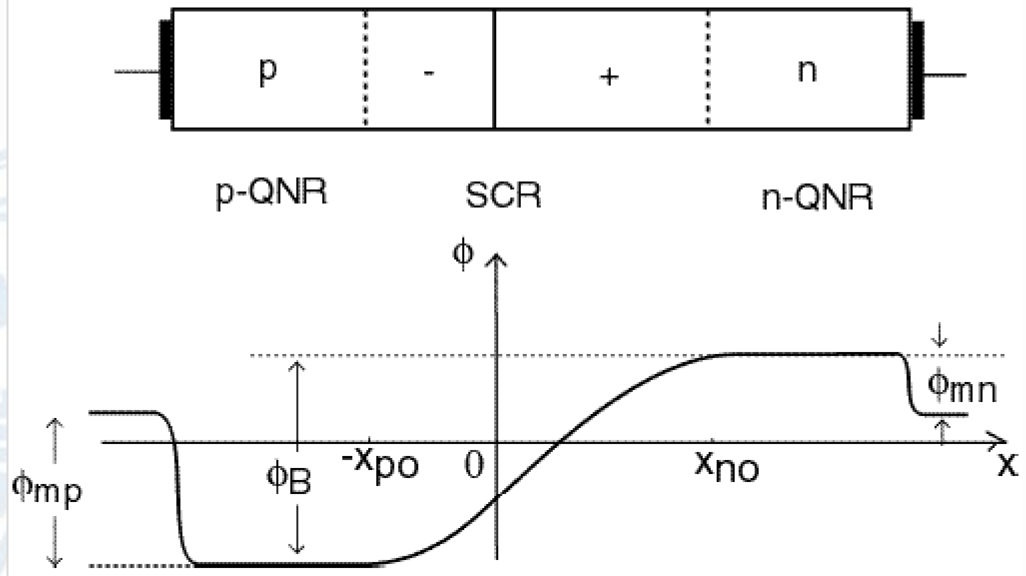
Question 1: If I apply a voltmeter across the pn junction diode, do I measure φ_B?

yes ;no it depends

Question 2: If I short terminals of pn junction diode, does current flow on the outside circuit?

yes ; no ; sometimes

We are missing contact potential at the metal semiconductor contacts:



Net current: macroscopical view
Diffusion current and drift current: microcosmic view.

Metal-semiconductor contacts: junction of dissimilar materials
⇒ built-in potentials at contacts φ_{mn} and φ_{mp} .
Potential difference across structure must be zero
At equilibrium, the diffuse current equals to the drift current, so there is **no net current**.
⇒ Cannot measure φ_B.

$$\phi_B = |\phi_{mn}| + |\phi_{mp}|$$

Summary of Key Concepts

- Electrostatics of pn junction in equilibrium
  - A space charge region surrounded by two quasi-neutral regions formed.
- To first order, carrier concentrations in space charge region are much smaller than the doping level

  ⇒ can use depletion approximation
- From contact to contact, there is no potential buildup across the pn junction diode
  - Contact potential(s).

Homework8

A silicon pn junction in thermal equilibrium at T =300 K is doped such that $E_F - E_{Fi} = 0.365$ eV in the n region and $E_{Fi} - E_F = 0.330$ eV in the p region.

(a) Sketch the energy-band diagram for the pn junction.

(b) Find the impurity doping concentration in each region.

(c) Determine V_{bi}

Homework9

Consider a uniformly doped GaAs pn junction with doping concentrations of $N_a = 2 \times 10^{15}\ \text{cm}^{-3}$ and $N_d = 4 \times 10^{16}\ \text{cm}^{-3}$. Plot the built-in potential barrier V_{bi} versus temperature over the range $200 \leq T \leq 400$ K.