Lec 2电磁场理论基础

Electromagnetic Theory

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

- 电磁场与电路
- 微波与电路理论
- 低频导线与微波传输线

MAXWELL'S EQUATION

Differential form

$$\nabla \times \bar{\mathcal{E}} = \frac{-\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}}, \quad \text{(a)}$$

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}}, \quad \text{(b)}$$

$$\nabla \cdot \bar{\mathcal{D}} = \rho, \quad \text{(c)}$$

$$\nabla \cdot \bar{\mathcal{B}} = 0. \quad \text{(d)}$$

$\bar{\mathcal{E}}$ is the electric field, in volts per meter (V/m).[1]
$\bar{\mathcal{H}}$ is the magnetic field, in amperes per meter (A/m).

$\bar{\mathcal{D}}$ is the electric flux density, in coulombs per meter squared (Coul/m^2).
$\bar{\mathcal{B}}$ is the magnetic flux density, in webers per meter squared (Wb/m^2).
$\bar{\mathcal{M}}$ is the (fictitious) magnetic current density, in volts per meter (V/m^2).
$\bar{\mathcal{J}}$ is the electric current density, in amperes per meter squared (A/m^2).
ρ is the electric charge density, in coulombs per meter cubed (Coul/m^3).

Equations (a)–(d) are linear but are not independent of each other.
Since the divergence of the curl of any vector is zero,
(d) can be derived by (a).
(b) + the continuity equation => (c)

$$\bar{\mathcal{B}} = \mu_0 \bar{\mathcal{H}},$$

$$\bar{\mathcal{D}} = \epsilon_0 \bar{\mathcal{E}},$$

continuity equation $\nabla \cdot \bar{\mathcal{J}} + \frac{\partial \rho}{\partial t} = 0.$

Maxwell's equations → KCL and KVL equations

Integral form

$$\oint_S \bar{\mathcal{D}} \cdot d\bar{s} = \int_V \rho \, dv = Q,$$

$$\oint_S \bar{\mathcal{B}} \cdot d\bar{s} = 0,$$

Faraday's law

$$\oint_C \bar{\mathcal{E}} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int_S \bar{\mathcal{B}} \cdot d\bar{s} - \int_S \bar{\mathcal{M}} \cdot d\bar{s}$$

Ampere's law

$$\oint_C \bar{\mathcal{H}} \cdot d\bar{l} = \frac{\partial}{\partial t} \int_S \bar{\mathcal{D}} \cdot d\bar{s} + \int_S \bar{\mathcal{J}} \cdot d\bar{s} = \frac{\partial}{\partial t} \int_S \bar{\mathcal{D}} \cdot d\bar{s} + \mathcal{I},$$

KVL

At low frequency $\frac{\partial}{\partial t}\bar{B} = 0$

$$\oint_c \bar{E} d\bar{l} = 0 \implies \Sigma V = 0$$

KCL

$$\nabla \cdot \bar{\mathcal{J}} + \frac{\partial \rho}{\partial t} = 0,$$

if $\frac{\partial \rho}{\partial t} = 0 \implies \nabla \cdot \bar{J} = 0$

$$\implies \oint_S \bar{J} \cdot d\bar{S} = 0 \implies \Sigma I = 0$$

微波与电路理论

电路理论与微波理论的区别与联系？

电路：

- 电路理论是电磁场理论（MAXWELL方程）的特定应用，电路模型采用集总参数（lumped circuit elements）。
- 低频，尺寸小于十分之一波长

low frequency (the wavelength λ is much larger than device dimension D) - ($D<\lambda/10$)

微波：

- 电磁场理论（MAXWELL方程），模型采用分布参数（distributed circuit elements）。
- 高频，描述电磁波的特性，尺寸大于十分之一波长。

high frequency (the wavelength is on the order of device dimension) - distributed circuit elements ($D>=\lambda/10$)

低频导线与微波传输线

低频导线与微波传输线的区别？

□低频导线 — 电路理论

- 低频时，电磁波借助有形的导体才能传播。
- 由于电磁变化缓慢，能量几乎无辐射，束缚在导体内。

□微波传输线— 电磁场理论

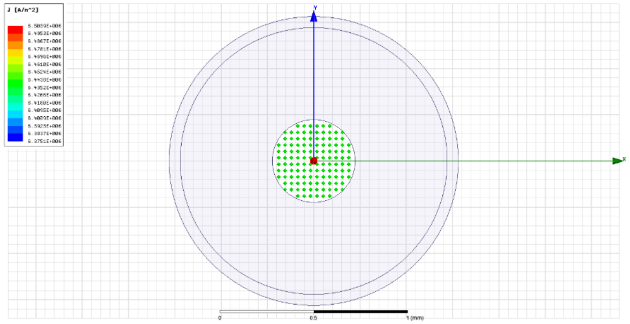
- 高频时，电磁波可以在自由空间（或介质）中传播，也可以束缚在有形导体内传播。
- 微波传输线具有集肤效应：电流不均匀分布，而是集中在导体表面。

$$\bar{J}=\bar{J}_0 e^{-\alpha(r_0-r)}$$

$\bar{J}_0$：$r=r_0$时表面电流密度

- 附录2 低频导线与微波传输线的电磁特性

同轴线电流分布



低频

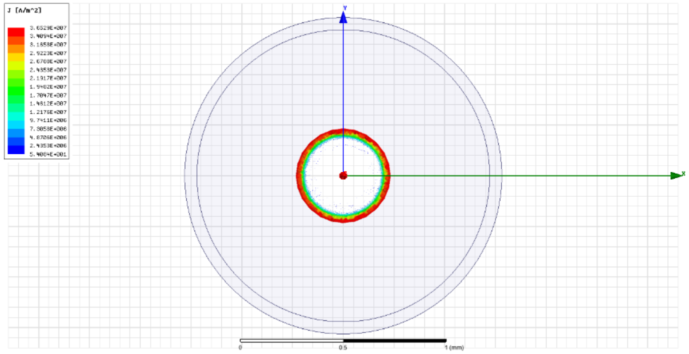
低频（直流）时，位移电流=0，仅有传导电流，为静电场、静磁场，电流密度分布在导体内部

$$\nabla \times \bar{\mathcal{E}} = \frac{-\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}},$$

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}},$$

$$\nabla \cdot \bar{\mathcal{D}} = \rho,$$

$$\nabla \cdot \bar{\mathcal{B}} = 0.$$



高频

高频时，既有位移电流，又有传导电流，导体中位移电流<<传导电流,受趋肤效应影响，电流集中在导体表面。理想导体仅有表面电流。

例： 0.25um集成电路工艺器件工作在30GHz， 应该采用何种电路建模？

解：$\lambda = c/f = 1cm$， $\lambda/10=1mm$

- 有源器件尺寸在um级，采用集总参数电路
- 局部短互连尺寸在um级， 采用集总参数电路
- 全局长互连尺寸在mm，采用分布参数电路（传输线）

FIELDS IN MEDIA AND BOUNDARY CONDITIONS

- **For a dielectric material,** $\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e.$

In a linear medium, the electric polarization (电极化强度) $\bar{P}_e = \epsilon_0 \chi_e \bar{E},$

Xe is the electric susceptibility （电极化率）.

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e = \epsilon_0 (1 + \chi_e) \bar{E} = \epsilon \bar{E},$$

The complex permittivity （复介电常数） of the medium

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0 (1 + \chi_e)$$

$$\begin{aligned} \nabla \times \bar{H} &= j\omega \bar{D} + \bar{J} \\ &= j\omega\epsilon \bar{E} + \sigma \bar{E} \\ &= j\omega\epsilon' \bar{E} + (\omega\epsilon'' + \sigma) \bar{E} \\ &= j\omega\left(\epsilon' - j\epsilon'' - j\frac{\sigma}{\omega}\right)\bar{E}, \end{aligned}$$

the loss tangent（损耗角正切）,

$$\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'},$$

$$\boxed{\epsilon = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta)}$$

上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

□ For magnetic materials.

$$\bar{B} = \mu_0(\bar{H} + \bar{P}_m).$$

In a linear medium, magnetic polarization (or magnetization） $\bar{P}_m = \chi_m \bar{H}$,

$$\bar{B} = \mu_0(1 + \chi_m)\bar{H} = \mu\bar{H},$$

Maxwell's equation in a linear medium,

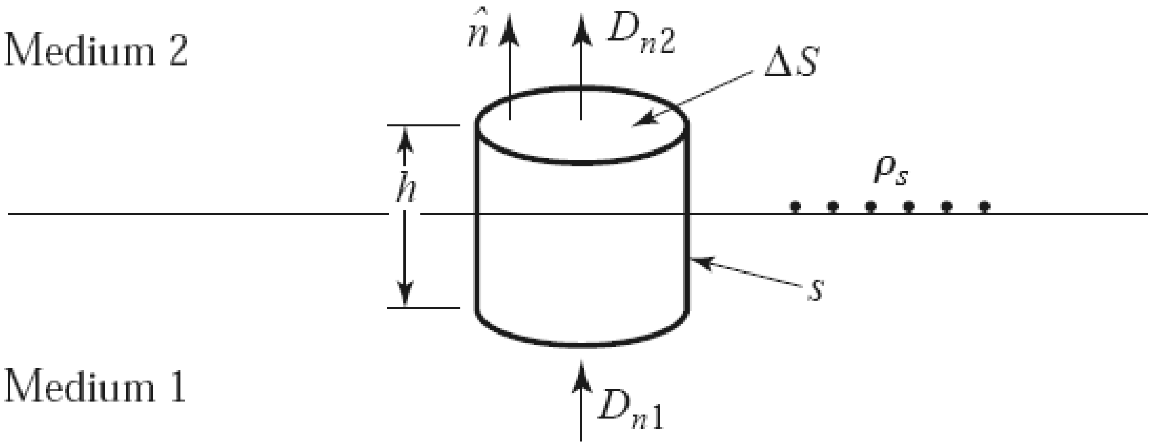
$$\nabla \times \bar{E} = -j\omega\mu\bar{H} - \bar{M},$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} + \bar{J},$$

$$\nabla \cdot \bar{D} = \rho,$$

$$\nabla \cdot \bar{B} = 0.$$

☐ Fields at a General Material Interface

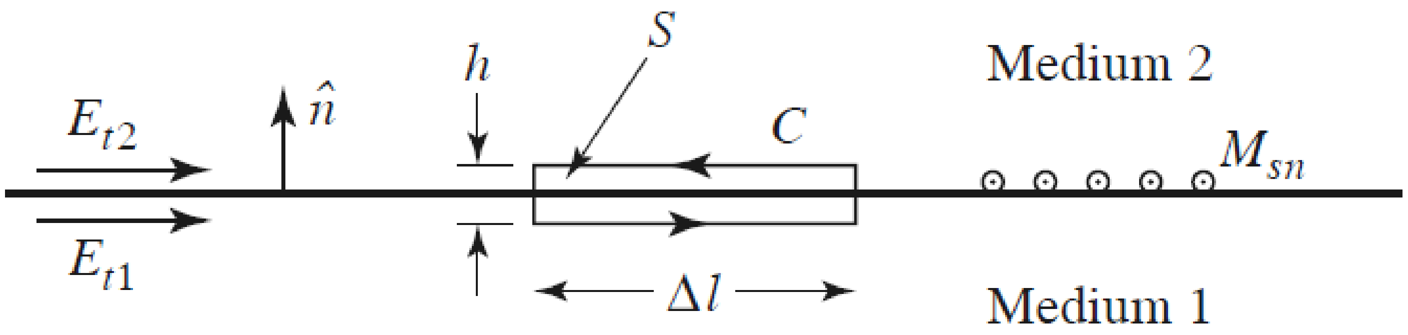


四个边界条件（Four boundary conditions）

$$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho \, dv. \Rightarrow \Delta S D_{2n} - \Delta S D_{1n} = \Delta S \rho_s,$$

$$\Rightarrow D_{2n} - D_{1n} = \rho_s, \Rightarrow \boxed{\hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = \rho_s.}$$

$$\oint_S \bar{B} \cdot d\bar{s} = 0, \Rightarrow \boxed{\hat{n} \cdot \bar{B}_2 = \hat{n} \cdot \bar{B}_1,}$$



$$\oint_C \bar{E} \cdot d\bar{l} = -j\omega \int_S \bar{B} \cdot d\bar{s} - \int_S \bar{M} \cdot d\bar{s},$$

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = \bar{M}_s .$$

$$\oint_C \bar{\mathcal{H}} \cdot d\bar{l} = \frac{\partial}{\partial t} \int_S \bar{\mathcal{D}} \cdot d\bar{s} + \int_S \bar{\mathcal{J}} \cdot d\bar{s}$$

$$\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s ,$$

□ Fields at a Dielectric Interface

At an interface between two lossless dielectric materials, **no charge or surface current densities**

$$\hat{n} \cdot \bar{D}_1 = \hat{n} \cdot \bar{D}_2,$$

$$\hat{n} \cdot \bar{B}_1 = \hat{n} \cdot \bar{B}_2,$$

$$\hat{n} \times \bar{E}_1 = \hat{n} \times \bar{E}_2,$$

$$\hat{n} \times \bar{H}_1 = \hat{n} \times \bar{H}_2.$$

Fields at the Interface with a Perfect Conductor (Electric Wall) $\sigma \to \infty$

All field components must be zero inside the conducting region. The fields in the dielectric

$$\hat{n} \cdot \bar{D} = \rho_s,$$

$$\hat{n} \cdot \bar{B} = 0,$$

$$\hat{n} \times \bar{E} = 0,$$

$$\hat{n} \times \bar{H} = \bar{J}_s,$$

⇨ $E_t=0$

Electrical wall is analogous to an short-circuited transmission line.

The Magnetic Wall Boundary Condition (magnetic wall)

The tangential components of *H* must vanish. The fields in the dielectric

$$\hat{n} \cdot \bar{D} = 0,$$

$$\hat{n} \cdot \bar{B} = 0,$$

$$\hat{n} \times \bar{E} = -\bar{M}_s,$$

$$\hat{n} \times \bar{H} = 0,$$

⇨ $\boldsymbol{H_t=0}$

Magnetic wall is analogous to an open-circuited transmission line.

Space wave (空间波)

- 电磁波的三种形式：

- 空间波- 平面波 （Plane wave）

  无源波，空间传播，电磁波的最大舞台，无线通信，雷达

- 导引波- guided wave

  无源波传输线、波导中传播，波能完全按人类的意愿传播

- 天线波 – antenna wave

  有源波（有J 和 ρ）

Space wave

- 波可以脱离源，数学问题简化

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$
$$\nabla \times \bar{H} = j\omega\epsilon \bar{E}$$
$$\nabla \cdot \bar{D} = \rho$$
$$\nabla \cdot \bar{B} = 0$$

- In a source-free, linear, isotropic(各向同性), homogeneous （均匀） region,

The Helmholtz Equation

$$\nabla \times \bar{E} = -j\omega\mu \bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon \bar{E},$$
$$\nabla \bullet \vec{E} = 0$$
$$\nabla \bullet \vec{H} = 0$$

$\Rightarrow$

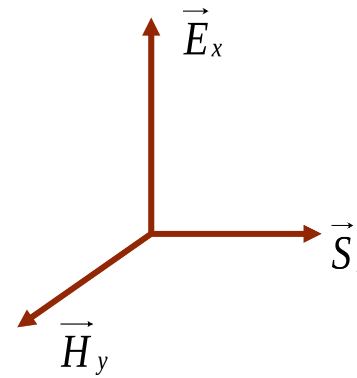
$$\nabla^2 \bar{E} + \omega^2 \mu\epsilon \bar{E} = 0,$$
$$\nabla^2 \bar{H} + \omega^2 \mu\epsilon \bar{H} = 0.$$

$$k = \omega\sqrt{\mu\epsilon}$$

propagation constant (also known as the *phase constant*, or *wave number*), of the medium; its units are 1/m.

□ 一维波动方程

Plane Waves in a Lossless Medium



E_x H_y都是z的函数，z是传播方向

$$\nabla^2 \vec{E} \rightarrow \frac{\partial^2 \vec{E}}{\partial^2 z}$$

$$\frac{\partial^2 E_x}{\partial^2 z} - \frac{1}{c^2}\frac{\partial^2 E_x}{\partial^2 t} = 0 \quad \text{或} \quad \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0.$$

其中相速 $c = \frac{1}{\sqrt{\mu\varepsilon}}$

方程的解：

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz},$$

$$\mathcal{E}_x(z, t) = E^+ \cos(\omega t - kz) + E^- \cos(\omega t + kz),$$

□ 任意波

FFT思想，线性叠加，解为三角函数叠加。

the *phase velocity* is the velocity at which a fixed phase point on the wave travels

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - \text{constant}}{k}\right) = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

In free-space, we have $v_p = c = 2.998 \times 10^8$ m/sec, which is the speed of light.

The *wavelength*, λ, is defined as the distance between two successive maxima (or minima, or any other reference points) on the wave at a fixed instant of time.

$$(\omega t - kz) - [\omega t - k(z + \lambda)] = 2\pi,$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}.$$

Using Maxwell's equations

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{1}{\eta}(E^+ e^{-jkz} - E^- e^{jkz}),$$

$$H_x = H_z = 0,$$

上海交通大學 SHANGHAI JIAO TONG UNIVERSITY

- η is the **intrinsic impedance （本征阻抗） of the medium**.
- The **wave impedance （波阻抗）** is the ratio of the *E* and *H* components。
- For planes waves, the wave impedance is equal to the intrinsic impedance of the media

$$\underline{\eta} = \omega\mu/\underline{k} = \sqrt{\mu/\epsilon}$$

In free-space the intrinsic impedance is $\eta_0 =$ 377 ohm.

The *E* and *H* vectors are orthogonal to each other and orthogonal to the direction of propagation (±^*z*); this is a characteristic of transverse electromagnetic (TEM) waves.

EXAMPLE 1.1 BASIC PLANEWAVE PARAMETERS

A plane wave propagating in a lossless dielectric medium has an electric field given as $E_x = E_0 \cos(\omega t - \beta z)$ with a frequency of 5.0 GHz and a wavelength in the material of 3.0 cm. Determine the propagation constant, the phase velocity, the relative permittivity of the medium, and the wave impedance.

Solution:

The propagation constant $$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.03} = 209.4\ \text{m}^{-1},$$

The phase velocity $$v_p = \frac{\omega}{k} = \frac{2\pi f}{k} = \lambda f = (0.03)\,(5 \times 10^9) = 1.5 \times 10^8\ \text{m/sec}.$$

The relative permittivity of the medium $$\epsilon_r = \left(\frac{c}{v_p}\right)^2 = \left(\frac{3.0 \times 10^8}{1.5 \times 10^8}\right)^2 = 4.0$$

The wave impedance $$\eta = \eta_0/\sqrt{\epsilon_r} = \frac{377}{\sqrt{4.0}} = 188.5\ \Omega$$

Plane Waves in a General Lossy Medium

If the medium is conductive, with a conductivity σ, Maxwell's curl equations

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} + \sigma\bar{E}.$$

$$\nabla^2\bar{E} + \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\bar{E} = 0,$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

$$v_p = \omega/\beta, \qquad \lambda = 2\pi/\beta,$$

$$\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)},$$

If we assume an electric field with only an *x* component and uniform in *x* and *y*,

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0,$$

$$E_x(z) = E^+e^{-\gamma z} + E^-e^{\gamma z}.$$

$$e^{-\gamma z} = e^{-\alpha z}e^{-j\beta z},$$

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{-j\gamma}{\omega\mu}(E^+e^{-\gamma z} - E^-e^{\gamma z}).$$

$$\eta = \frac{j\omega\mu}{\gamma}$$

If the loss is removed, $\sigma = 0$, and we have $\gamma = jk$ and $\boldsymbol{\alpha = 0}$, $\boldsymbol{\beta = k}$.

Plane Waves in a Good Conductor

In a good conductor, the conductive current is much greater than the displacement current, which means that $\sigma \gg \omega\epsilon.$

$$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}.$$

α 衰减常数；

β 相移常数

The skin depth, or characteristic depth of penetration, is $\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}.$

Thus the amplitude of the fields in the conductor will decay by an amount 1/*e*

$$\eta = \frac{j\omega\mu}{\gamma} \simeq (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta_s}.$$

- The phase angle of the impedance is 45◦, **good conductors**.
- The phase angle of the impedance for a **lossless material is 0**◦.
- The phase angle of the impedance of an **arbitrary lossy medium** is between 0◦ and 45◦ .

EXAMPLE 1.2 SKIN DEPTH AT MICROWAVE FREQUENCIES

Compute the skin depth of aluminum, copper, gold, and silver at a frequency of 10 GHz.

Solution

The conductivities for these metals are listed in Appendix F.

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu_0 \sigma}} = 5.03 \times 10^{-3}\sqrt{\frac{1}{\sigma}}.$$

For aluminum: $\delta_s = 5.03 \times 10^{-3}\sqrt{\dfrac{1}{3.816 \times 10^7}} = 8.14 \times 10^{-7}\text{m}.$

For copper: $\delta_s = 5.03 \times 10^{-3}\sqrt{\dfrac{1}{5.813 \times 10^7}} = 6.60 \times 10^{-7}\text{m}.$

For gold: $\delta_s = 5.03 \times 10^{-3}\sqrt{\dfrac{1}{4.098 \times 10^7}} = 7.86 \times 10^{-7}\text{m}.$

For silver: $\delta_s = 5.03 \times 10^{-3}\sqrt{\dfrac{1}{6.173 \times 10^7}} = 6.40 \times 10^{-7}\text{m}.$

These results show that most of the current flow in a good conductor occurs in an extremely thin region near the surface of the conductor.

TABLE Summary of Results for Plane Wave Propagation in Various Media

	Type of Medium		
Quantity	Lossless ($\epsilon'' = \sigma = 0$)	General Lossy	Good Conductor ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
Complex propagation constant	$\gamma = j\omega\sqrt{\mu\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon}$ $= j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\dfrac{\sigma}{\omega\epsilon'}}$	$\gamma = (1+j)\sqrt{\omega\mu\sigma/2}$
Phase constant (wave number)	$\beta = k = \omega\sqrt{\mu\epsilon}$	$\beta = \text{Im}\{\gamma\}$	$\beta = \text{Im}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Attenuation constant	$\alpha = 0$	$\alpha = \text{Re}\{\gamma\}$	$\alpha = \text{Re}\{\gamma\} = \sqrt{\omega\mu\sigma/2}$
Impedance	$\eta = \sqrt{\mu/\epsilon} = \omega\mu/k$	$\eta = j\omega\mu/\gamma$	$\eta = (1+j)\sqrt{\omega\mu/2\sigma}$
Skin depth	$\delta_s = \infty$	$\delta_s = 1/\alpha$	$\delta_s = \sqrt{2/\omega\mu\sigma}$
Wavelength	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$	$\lambda = 2\pi/\beta$
Phase velocity	$v_p = \omega/\beta$	$v_p = \omega/\beta$	$v_p = \omega/\beta$

GENERAL PLANE WAVE SOLUTIONS

In free-space, the Helmholtz equation for E can be written as

$$\nabla^2 \bar{E} + k_0^2 \bar{E} = \frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} + k_0^2 \bar{E} = 0,$$

for each rectangular component $i = x, y, \text{ or } z.$

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} + k_0^2 E_i = 0,$$

solved by the method of separation of variables

$$E_x(x, y, z) = f(x)g(y)h(z).$$

Substituting this form into the partial equation

$$\frac{f''}{f} + \frac{g''}{g} + \frac{h''}{h} + k_0^2 = 0,$$

Each of the terms in the equation must be equal to a constant because they are independent of each other.

$$f''/f = -k_x^2; \quad g''/g = -k_y^2; \quad h''/h = -k_z^2; \quad k_x^2 + k_y^2 + k_z^2 = k_0^2.$$

$$\frac{d^2 f}{dx^2} + k_x^2 f = 0; \quad \frac{d^2 g}{dy^2} + k_y^2 g = 0; \quad \frac{d^2 h}{dz^2} + k_z^2 h = 0.$$

The solution for Ex: $E_x(x, y, z) = Ae^{-j(k_x x + k_y y + k_z z)}$,

The wave number: $\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z} = k_0\hat{n}.$

n is a unit vector in the direction of propagation.

$$E_x(x, y, z) = Ae^{-j\bar{k}\cdot\bar{r}}.$$

The total solution:

$$E_y(x, y, z) = Be^{-j\bar{k}\cdot\bar{r}},$$

$$E_z(x, y, z) = Ce^{-j\bar{k}\cdot\bar{r}}.$$

Since $$\nabla \cdot \bar{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\bar{E}_0 = A\hat{x} + B\hat{y} + C\hat{z}, \qquad \bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}},$$

$$\nabla \cdot \bar{E} = \nabla \cdot (\bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}) = \bar{E}_0 \cdot \nabla e^{-j\bar{k}\cdot\bar{r}} = -j\bar{k} \cdot \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}} = 0,$$

Then $$\bar{k} \cdot \bar{E}_0 = 0,$$

The electric field amplitude vector E_0 must be perpendicular to the direction of propagation *k*.

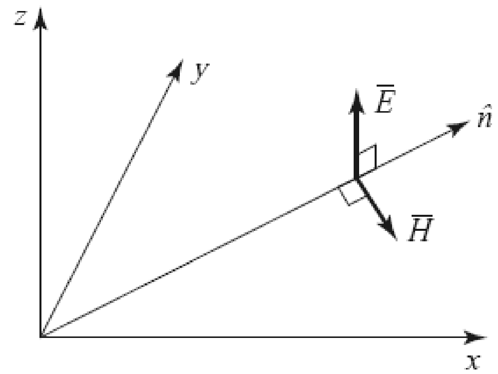
$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H},$$

$$\bar{H} = \frac{j}{\omega\mu_0}\nabla \times \bar{E} = \frac{j}{\omega\mu_0}\nabla \times (\bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}) = \frac{-j}{\omega\mu_0}\bar{E}_0 \times \nabla e^{-j\bar{k}\cdot\bar{r}}$$

$$= \frac{-j}{\omega\mu_0}\bar{E}_0 \times (-j\bar{k})e^{-j\bar{k}\cdot\bar{r}} = \frac{k_0}{\omega\mu_0}\hat{n} \times \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}} = \frac{1}{\eta_0}\hat{n} \times \bar{E},$$

The time domain expression for the electric field

$$\bar{\mathcal{E}}(x, y, z, t) = \mathrm{Re}\{\bar{E}(x, y, z)e^{j\omega t}\}$$
$$= \mathrm{Re}\{\bar{E}_0 e^{-j\bar{k}\cdot\bar{r}} e^{j\omega t}\}$$
$$= \bar{E}_0 \cos(\bar{k}\cdot\bar{r} - \omega t),$$



坐标变换

Orientation of the E, H , and $k = k_0 n$ vectors for a general plane wave.

EXAMPLE 1.3 CURRENT SHEETS AS SOURCES OF PLANEWAVES

An infinite sheet of surface current can be considered as a source for plane waves. If an electric surface current density $\bar{J}_s = J_0\hat{x}$ exists on the $z = 0$ plane in free space, find the resulting fields by assuming plane waves on either side of the current sheet and enforcing boundary conditions.

Solution:
Since the source does not vary with x or y, the fields will not vary with x or y but will propagate away from the source in the $\pm z$ direction. The boundary conditions to be satisfied at $z = 0$ are

$$\hat{n} \times (\bar{E}_2 - \bar{E}_1) = \hat{z} \times (\bar{E}_2 - \bar{E}_1) = 0,$$

$$\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \hat{z} \times (\bar{H}_2 - \bar{H}_1) = J_0\hat{x},$$

→ $\bar{H}$ must have a $\hat{y}$ component.

for $z < 0$, $\bar{E}_1 = \hat{x} A \eta_0 e^{jk_0 z}$, for $z > 0$, $\bar{E}_2 = \hat{x} B \eta_0 e^{-jk_0 z}$,

$\bar{H}_1 = -\hat{y} A e^{jk_0 z}$, $\bar{H}_2 = \hat{y} B e^{-jk_0 z}$,

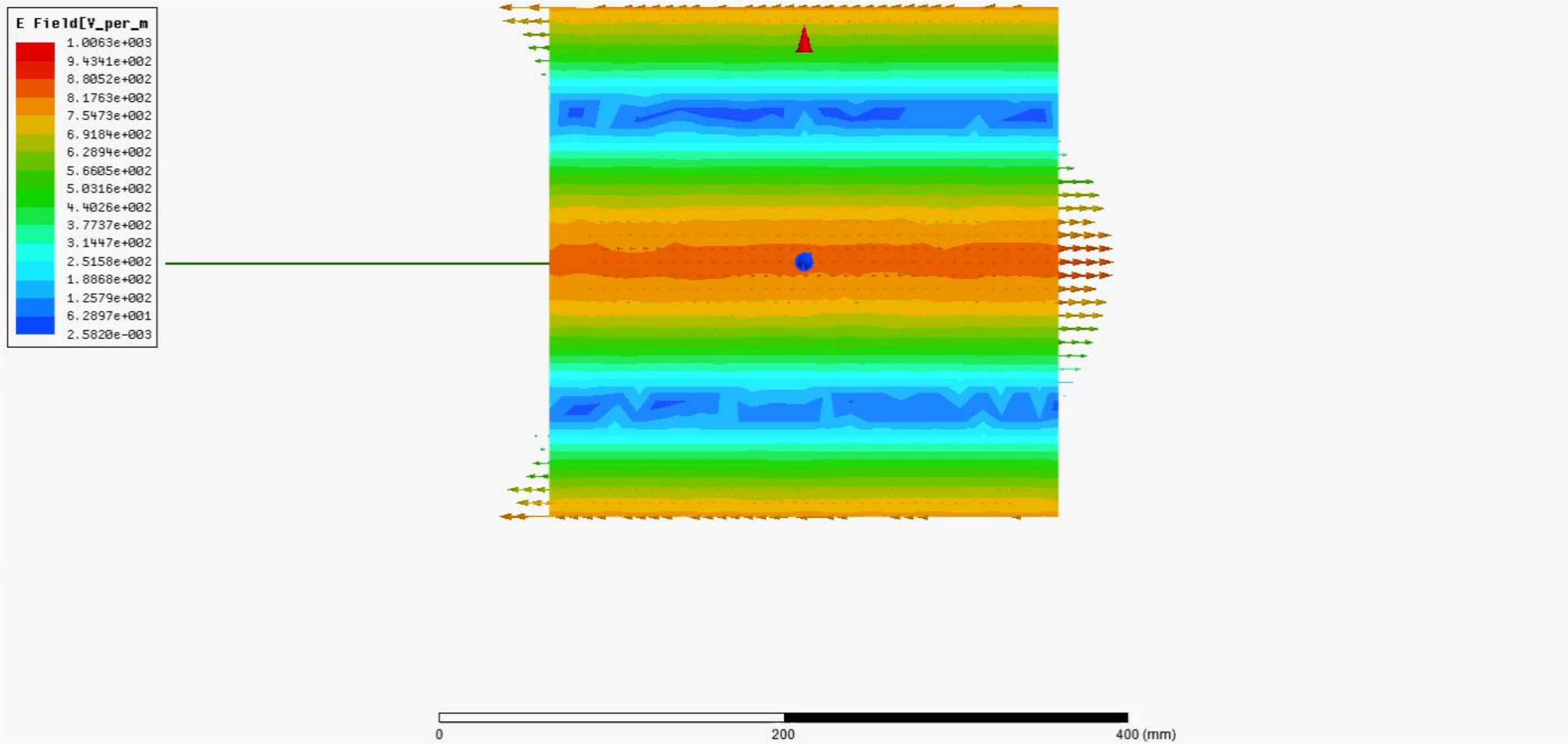
The first boundary condition, that E_x is continuous at $z = 0$, yields $A = B$,

The boundary condition for H yields the equation

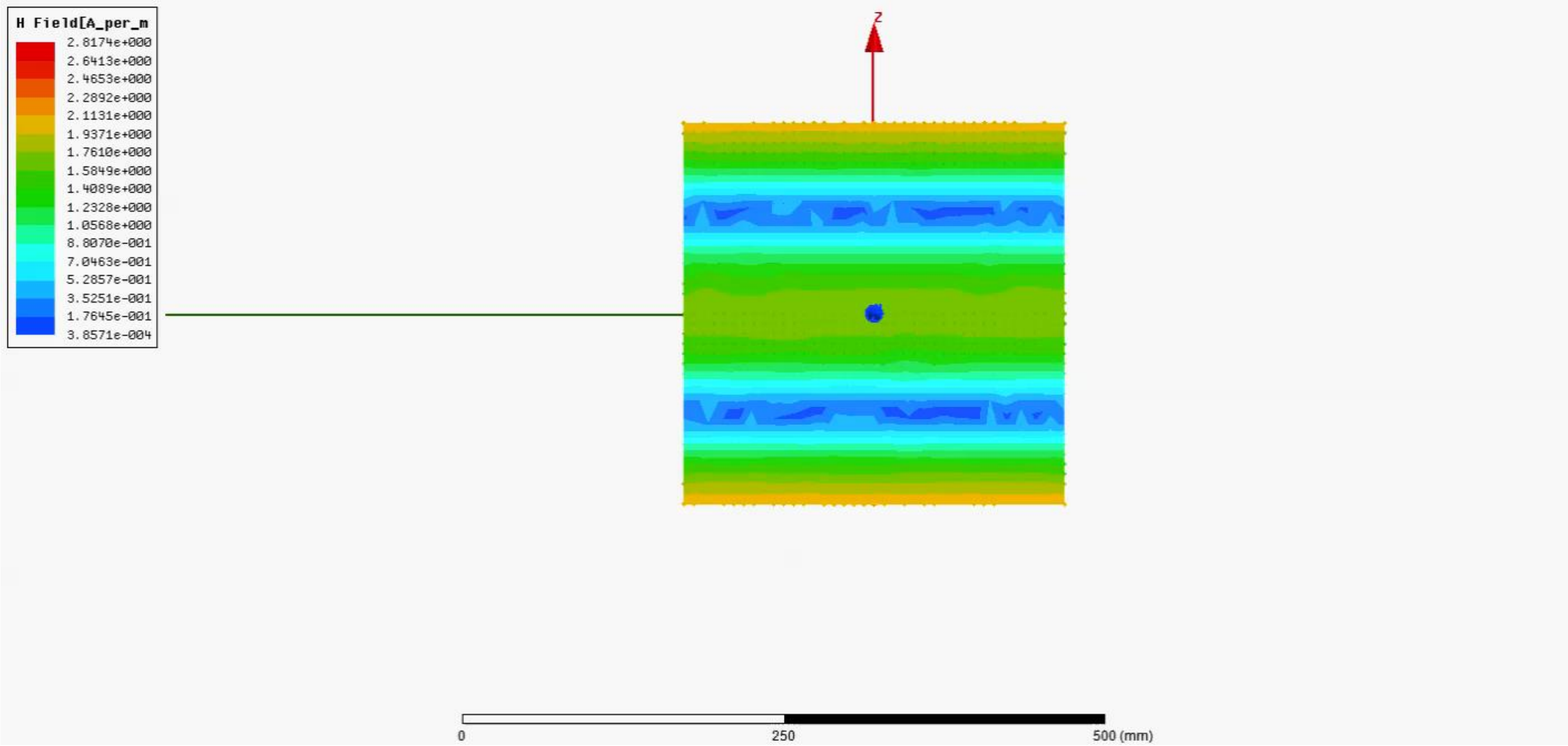
$$-B - A = J_0.$$

Solving for A, B gives

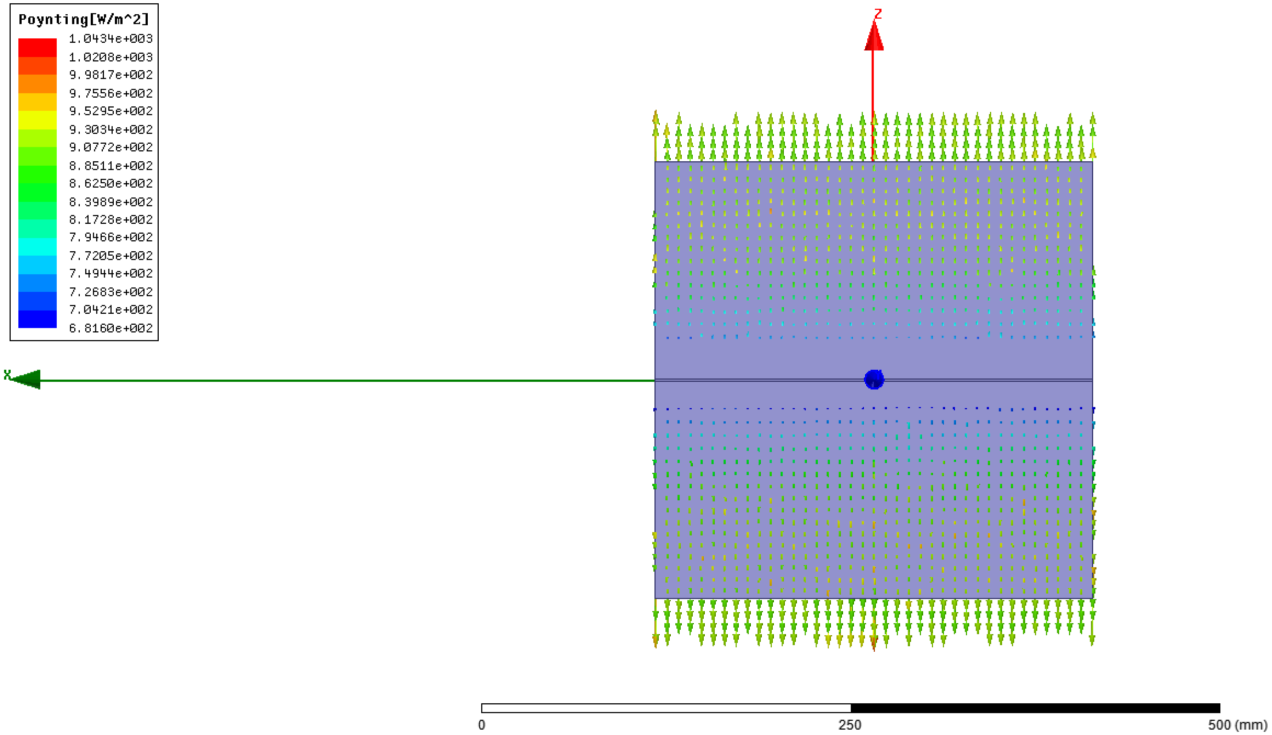
$$A = B = -J_0/2,$$



Magnitude Ex in xoz Plane.
Jx is the source of plane wave.



Magnitude Hy in yoz Plane



Poynting vector

ENERGY AND POWER

- A source of electromagnetic energy sets up fields that store electric and magnetic energy and carry power that may be transmitted or dissipated as loss.

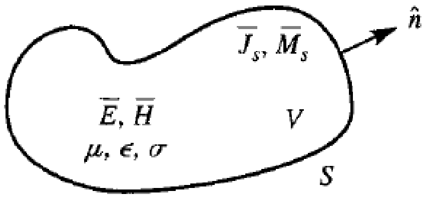
$$W_e = \frac{1}{4}\mathrm{Re}\int_V \bar{E}\cdot\bar{D}^*\,dv,$$

- In the sinusoidal steady-state case, the time-average stored electric energy in a volume *V* is given by

$$W_e = \frac{\epsilon}{4}\int_V \bar{E}\cdot\bar{E}^*\,dv.$$

$$W_m = \frac{1}{4}\mathrm{Re}\int_V \bar{H}\cdot\bar{B}^*\,dv,$$

$$W_m = \frac{\mu}{4}\int_V \bar{H}\cdot\bar{H}^*\,dv,$$



A volume *V*, enclosed by the closed surface *S*, containing fields . *E* , . *H* , and current sources *Js* , *Ms* .

Poynting's theorem leads to energy conservation for electromagnetic fields and sources.

$$-\frac{1}{2}\int_V (\bar{E}\cdot\bar{J}_s^* + \bar{H}^*\cdot\bar{M}_s)\,dv = \frac{1}{2}\oint_S \bar{E}\times\bar{H}^*\cdot d\bar{s} + \frac{\sigma}{2}\int_V |\bar{E}|^2\,dv$$
$$+\frac{\omega}{2}\int_V (\epsilon''|\bar{E}|^2 + \mu''|\bar{H}|^2)\,dv + j\frac{\omega}{2}\int_V (\mu'|\bar{H}|^2 - \epsilon'|\bar{E}|^2)\,dv.$$

- **the complex power *Ps* delivered by the sources**

$$P_s = -\frac{1}{2}\int_V (\bar{E}\cdot\bar{J}_s^* + \bar{H}^*\cdot\bar{M}_s)\,dv.$$

- **The complex power flow out of the closed surface**

$$P_o = \frac{1}{2}\oint_S \bar{E}\times\bar{H}^*\cdot d\bar{s} = \frac{1}{2}\oint_S \bar{S}\cdot d\bar{s}.$$

- The time **average power dissipated** in the volume *V* due to conductivity, dielectric, and magnetic losses.

$$P_\ell = \frac{\sigma}{2}\int_V |\bar{E}|^2\,dv + \frac{\omega}{2}\int_V (\epsilon''|\bar{E}|^2 + \mu''|\bar{H}|^2)\,dv,$$

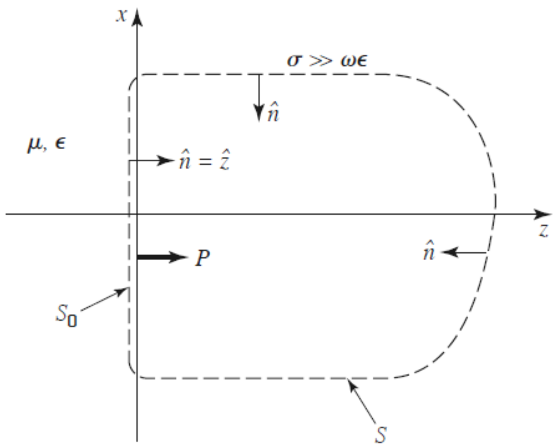
- the **stored electric and magnetic** energies $j\frac{\omega}{2}\int_V (\mu'|\bar{H}|^2 - \epsilon'|\bar{E}|^2)\,dv.$

In summary $$P_s = P_o + P_\ell + 2j\omega(W_m - W_e).$$

Power Absorbed by a Good Conductor

the power dissipated in the conductor can be accomplished using only the fields at the surface of the conductor

$$P_{avg} = \frac{1}{2}\text{Re}\int_{S_0+S} \bar{E} \times \bar{H}^* \cdot \hat{n}\, ds,$$



An interface between a lossless medium and a good conductor with a closed surface $S0 + S$ for computing the power dissipated in the conductor

Since the integral on S surface is zero and

$$\bar{H} = \hat{n} \times \bar{E}/\eta,$$

$$P_{avg} = \frac{R_s}{2}\int_{S_0} |\bar{H}|^2\, ds,$$

the *surface resistance* (表面电阻)of the conductor

$$R_s = \text{Re}\{\eta\} = \text{Re}\left\{(1+j)\sqrt{\frac{\omega\mu}{2\sigma}}\right\} = \sqrt{\frac{\omega\mu}{2\sigma}} = \frac{1}{\sigma\delta_s}$$

the *surface current density*

$$\bar{J}_s = \hat{n} \times \bar{H}$$

Homework

1.2 A plane wave traveling along the x-axis in a polystyrene-filled region with $\epsilon_r = 2.54$ has an electric field given by $E_y = E_0 \cos(\omega t - kx)$. The frequency is 2.4 GHz, and $E_0 = 5.0$ V/m. Find the following: (a) the amplitude and direction of the magnetic field, (b) the phase velocity, (c) the wavelength, and (d) the phase shift between the positions $x_1 = 0.1$ m and $x_2 = 0.15$ m.