Lec3 传输线理论I

Transmission line theory

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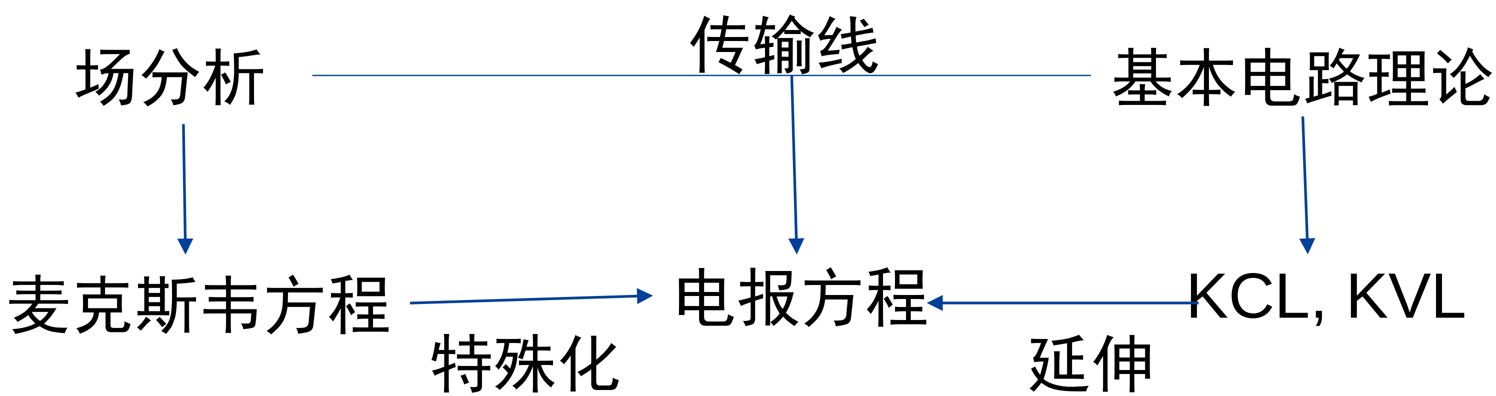
Transmission line theory

- Transmission line theory bridges the gap between field analysis and basic circuit theory.
- an extension of circuit theory
- a specialization of Maxwell's equations

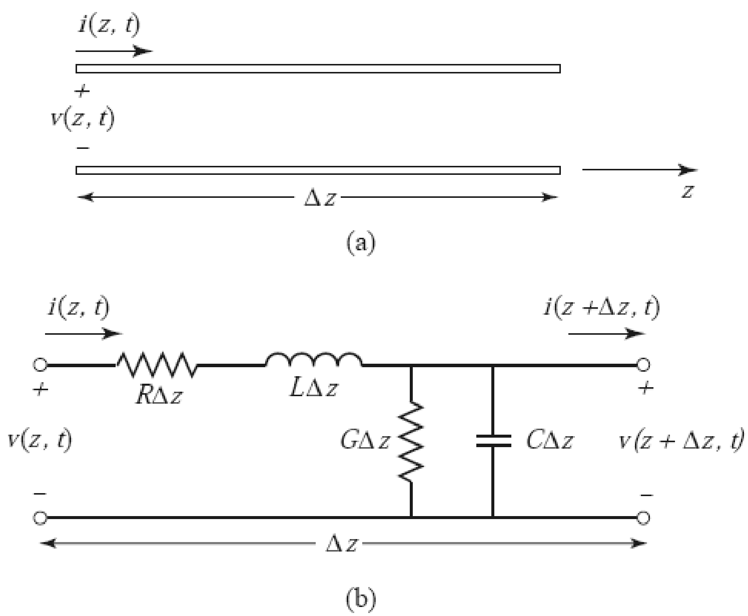


- THE LUMPED-ELEMENT CIRCUIT MODEL FOR A TRANSMISSION LINE
- FIELD ANALYSIS OF TRANSMISSION LINES
- THE TERMINATED LOSSLESS TRANSMISSION LINE

1. THE LUMPED-ELEMENT CIRCUIT MODEL FOR A TRANSMISSION LINE

- **Circuit analysis assumes that the physical dimensions of the network are much smaller than the electrical wavelength. (Less than one tenth of wavelength ）**
- The **transmission lines** may be a considerable fraction of a wavelength, or **many wavelengths**, in size.
- Thus a transmission line is a ***distributed parameter*** **network**, where voltages and currents can vary in magnitude and phase over its length.

A transmission line is often schematically represented as a two-wire line since transmission lines (for transverse electromagnetic [TEM] wave propagation) always have **at least two conductors**.



R = series resistance per unit length, for both conductors, in /m.

L = series inductance per unit length, for both conductors, in H/m.

G = shunt conductance per unit length, in S/m.

C = shunt capacitance per unit length, in F/m.

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z, t) = 0,$$

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0.$$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t},$$
$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}.$$

telegrapher equations.

电报方程？
包含哪些参数，其含义？

For the **sinusoidal steady-state** condition,

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z),$$
$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z).$$

Wave equation

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0,$$
$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0,$$

the complex propagation constant,

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

attenuation constant

phase constant

Traveling wave solutions

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z},$$
$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z},$$

wave propagation in the +z direction

wave propagation in the −z direction

$$I(z) = \frac{\gamma}{R + j\omega L}\left(V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}\right).$$

characteristic impedance $$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}},$$

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}. \qquad I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{\gamma z}.$$

In time domain
$$v(z, t) = |V_o^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_o^-| \cos(\omega t + \beta z + \phi^-) e^{\alpha z},$$

$\phi^\pm$ is the phase angle of the complex voltage $V_o^\pm$.

the wavelength on the line is $$\lambda = \frac{2\pi}{\beta},$$

the phase velocity $$v_p = \frac{\omega}{\beta} = \lambda f.$$

The Lossless Line

In many practical cases, however, the loss of the line is very small and so can be neglected, $R = G = 0$ resulting in **the attenuation constant** α is zero.

$$\beta = \omega\sqrt{LC}, \qquad \gamma = \alpha + j\beta = j\omega\sqrt{LC},$$

$$\alpha = 0.$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z},$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}.$$

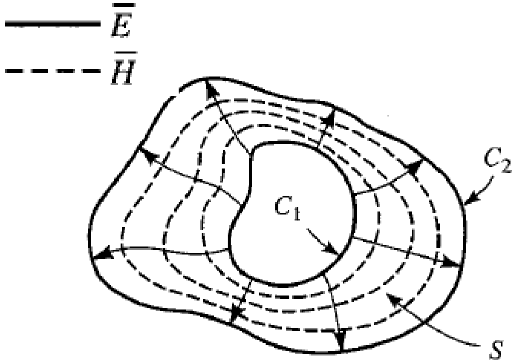
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}, \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}. \qquad Z_0 = \sqrt{\frac{L}{C}},$$

对比： 传输线与平面波的相似性

2. FIELD ANALYSIS OF TRANSMISSION LINES

- Rederive the time-harmonic form of the **telegrapher's equations** starting from Maxwell's equations.
- Derive the transmission line **parameters** *(R, L, G, C)* in terms of the electric and magnetic fields of the transmission line
- Derive the telegrapher equations using these parameters for the specific case of a **coaxial line**.

Transmission Line Parameters



Field lines on an arbitrary TEM transmission line. *S* is the cross sectional surface area of the line

Consider a 1 m length of a uniform transmission line with fields *E* and *H*. Let the voltage between the conductors be $V_o e^{\pm j\beta z}$ and the current be $I_o e^{\pm j\beta z}$.

The time-average stored magnetic energy for this 1m length of line

$$W_m = \frac{\mu}{4}\int_S \bar{H}\cdot\bar{H}^* ds,$$

From circuit theory

$$W_m = L|I_o|^2/4$$

The **self-inductance per unit length**

$$\boxed{L = \frac{\mu}{|I_o|^2}\int_S \bar{H}\cdot\bar{H}^* ds \text{ H/m.}}$$

The time-average stored electric energy per unit length

$$W_e = \frac{\epsilon}{4}\int_S \bar{E}\cdot\bar{E}^* ds,$$

From circuit theory $W_e = C$

The **capacitance per unit length**:

$$\boxed{C = \frac{\epsilon}{|V_o|^2}\int_S \bar{E}\cdot\bar{E}^* ds \text{ F/m.}}$$

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The power loss per unit length due to the finite conductivity of the metallic conductors is

$$P_c = \frac{R_s}{2} \int_{C_1+C_2} \bar{H} \cdot \bar{H}^* d\ell$$

From circuit theory

$$P_c = R|I_o|^2/2.$$

the series resistance R per unit length of line is

$$\boxed{R = \frac{R_s}{|I_o|^2} \int_{C_1+C_2} \bar{H} \cdot \bar{H}^* dl \ \Omega/\text{m}.}$$

the surface resistance of the conductors $R_s = 1/\sigma\delta_s$; $\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$.

the time-average power dissipated per unit length in a lossy dielectric is

$$P_d = \frac{\omega\epsilon''}{2} \int_S \bar{E} \cdot \bar{E}^* ds,$$

From circuit theory

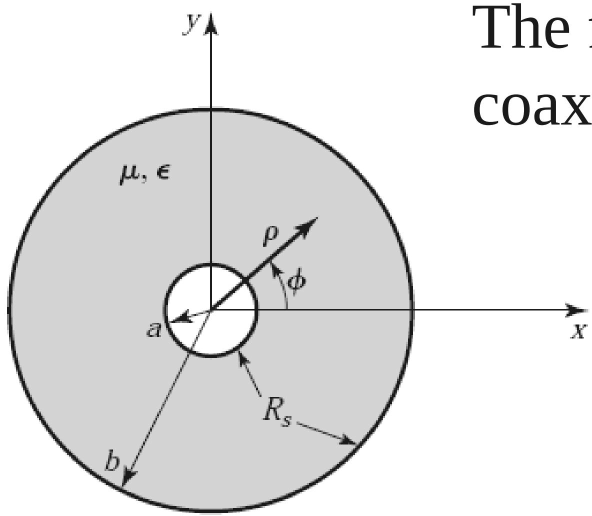
$$P_d = G|V_o|^2/2,$$

the shunt conductance per unit is

$$\boxed{G = \frac{\omega\epsilon''}{|V_o|^2} \int_S \bar{E} \cdot \bar{E}^* ds \ \text{S/m}.}$$

$$\boxed{\epsilon = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta)}$$

EXAMPLE 2.1 TRANSMISSION LINE PARAMETERS OF A COAXIAL LINE



The fields of a traveling TEM wave inside the coaxial line

$$\bar{E} = \frac{V_o \hat{\rho}}{\rho \ln b/a} e^{-\gamma z},$$

$$\bar{H} = \frac{I_o \hat{\phi}}{2\pi\rho} e^{-\gamma z},$$

推导过程
见第三章
同轴波导

where γ is the propagation constant of the line. The conductors are assumed to have a surface resistivity *Rs* , and the material filling the space between the conductors is assumed to have a complex permittivity $\epsilon = \epsilon' - j\epsilon''$ and permeability $\mu = \mu_0 \mu_r$

Determine the transmission line parameters.

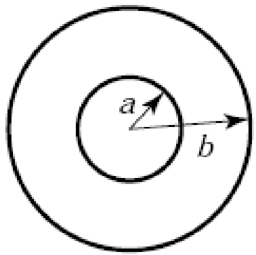
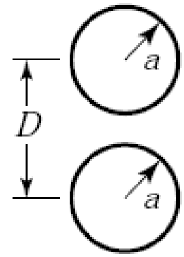
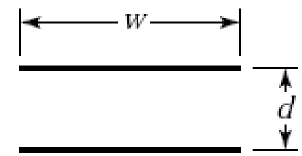
$$L = \frac{\mu}{(2\pi)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho \, d\rho \, d\phi = \frac{\mu}{2\pi} \ln b/a \text{ H/m},$$

$$C = \frac{\epsilon'}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho \, d\rho \, d\phi = \frac{2\pi\epsilon'}{\ln b/a} \text{ F/m},$$

$$R = \frac{R_s}{(2\pi)^2} \left\{ \int_{\phi=0}^{2\pi} \frac{1}{a^2} a \, d\phi + \int_{\phi=0}^{2\pi} \frac{1}{b^2} b \, d\phi \right\} = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right) \Omega/\text{m},$$

$$G = \frac{\omega\epsilon''}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho \, d\rho \, d\phi = \frac{2\pi\omega\epsilon''}{\ln b/a} \text{ S/m}.$$

TABLE Transmission Line Parameters for Some Common Lines

	COAX	TWO-WIRE	PARALLEL PLATE
			
L	$\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$	$\dfrac{\mu}{\pi}\cosh^{-1}\left(\dfrac{D}{2a}\right)$	$\dfrac{\mu d}{w}$
C	$\dfrac{2\pi\epsilon'}{\ln b/a}$	$\dfrac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\dfrac{\epsilon' w}{d}$
R	$\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{R_s}{\pi a}$	$\dfrac{2R_s}{w}$
G	$\dfrac{2\pi\omega\epsilon''}{\ln b/a}$	$\dfrac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\dfrac{\omega\epsilon'' w}{d}$

The Telegrapher Equations Derived from Field Analysis of a Coaxial Line

The fields inside the coaxial line will satisfy Maxwell's curl equations,

$$\nabla \times \bar{E} = -j\omega\mu \bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon \bar{E},$$

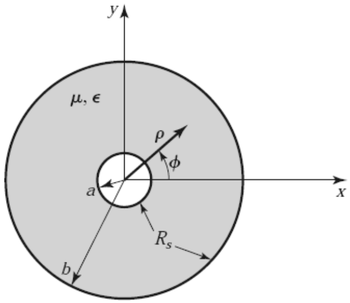
Refer to Appendices

A TEM wave on the coaxial line will be characterized by $E_z = H_z = 0$

Due to symmetry, the fields will have no ϕ variation, so $\partial/\partial\phi = 0$.

where $\epsilon = \epsilon' - j\epsilon''$ may be complex to allow for a lossy dielectric filling. Conductor loss will be ignored here.

$$-\hat{\rho}\frac{\partial E_\phi}{\partial z} + \hat{\phi}\frac{\partial E_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi) = -j\omega\mu(\hat{\rho} H_\rho + \hat{\phi} H_\phi),$$
$$-\hat{\rho}\frac{\partial H_\phi}{\partial z} + \hat{\phi}\frac{\partial H_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi) = j\omega\epsilon(\hat{\rho} E_\rho + \hat{\phi} E_\phi).$$



Since the z components of these two equations must vanish,

$$E_\phi = \frac{f(z)}{\rho},$$

$$H_\phi = \frac{g(z)}{\rho}.$$

To satisfy **the boundary condition** that E_ϕ=0 at ρ= a, b, we must have E_ϕ= 0 everywhere.

From the ρ component $H_\rho = 0.$

$$\frac{\partial E_\rho}{\partial z} = -j\omega\mu H_\phi,$$

$$\frac{\partial H_\phi}{\partial z} = -j\omega\epsilon E_\rho.$$

$$\Rightarrow \quad E_\rho = \frac{h(z)}{\rho}.$$

$$\Rightarrow \quad \frac{\partial h(z)}{\partial z} = -j\omega\mu g(z),$$

$$\frac{\partial g(z)}{\partial z} = -j\omega\epsilon h(z).$$

The voltage between the two conductors

$$V(z) = \int_{\rho=a}^{b} E_\rho(\rho, z) d\rho = h(z) \int_{\rho=a}^{b} \frac{d\rho}{\rho} = h(z) \ln \frac{b}{a},$$

the total current on the inner conductor at $\rho = a$ is

$$I(z) = \int_{\phi=0}^{2\pi} H_\phi(a, z) a d\phi = 2\pi g(z).$$

h(z) and v(z) are in form of waves.

$$h(z) = h(0)e^{-\gamma z} \; V(z) = V_0 e^{-\gamma z}$$

So

$$\bar{E} = \frac{V_o \hat{\rho}}{\rho \ln b/a} e^{-\gamma z},$$

$$\bar{H} = \frac{I_o \hat{\phi}}{2\pi\rho} e^{-\gamma z},$$

$$\frac{\partial V(z)}{\partial z} = -j\frac{\omega\mu \ln b/a}{2\pi} I(z),$$
$$\frac{\partial I(z)}{\partial z} = -j\omega(\epsilon' - j\epsilon'')\frac{2\pi V(z)}{\ln b/a}.$$

L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$

$$\boxed{\begin{aligned}\frac{\partial V(z)}{\partial z} &= -j\omega L I(z),\\ \frac{\partial I(z)}{\partial z} &= -(G + j\omega C)V(z).\end{aligned}}$$

The conductor is perfect so $R=0$.

Propagation Constant, Impedance, and Power Flow for the Lossless Coaxial Line

$$\frac{\partial E_\rho}{\partial z} = -j\omega\mu H_\phi,$$

$$\frac{\partial H_\phi}{\partial z} = -j\omega\epsilon E_\rho.$$

$$\Rightarrow \quad \frac{\partial^2 E_\rho}{\partial z^2} + \omega^2\mu\epsilon E_\rho = 0,$$

Propagation Constant $\gamma^2 = -\omega^2\mu\epsilon$

For lossless media $\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{LC},$

this propagation constant is of the same form as that for plane waves in a lossless dielectric medium.

The wave impedance for the coaxial line is

$$Z_w = \frac{E_\rho}{H_\phi} = \frac{\omega\mu}{\beta} = \sqrt{\mu/\epsilon} = \eta.$$

The characteristic impedance of the coaxial line is

$$Z_0 = \frac{V_o}{I_o} = \frac{E_\rho \ln b/a}{2\pi H_\phi} = \frac{\eta \ln b/a}{2\pi} = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln b/a}{2\pi},$$

The power may be computed From the Poynting vector as

$$P = \frac{1}{2}\int_s \bar{E} \times \bar{H}^* \cdot d\bar{s} = \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b} \frac{V_o I_o^*}{2\pi\rho^2 \ln b/a}\rho\, d\rho\, d\phi = \frac{1}{2} V_o I_o^*,$$

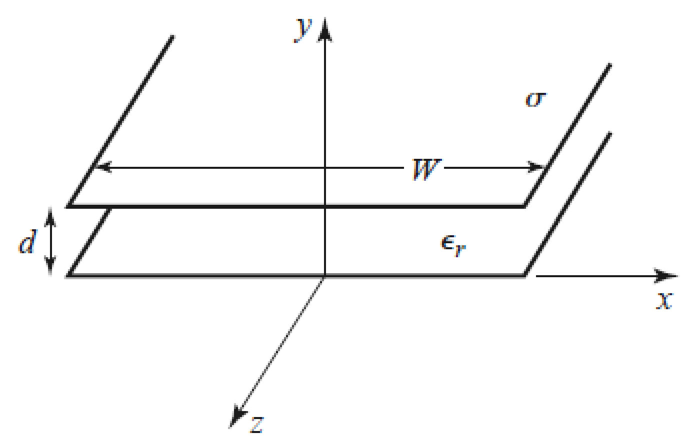
无耗同轴线的波阻抗与特性阻抗？
相速和波长？

Homework

2.1 A 75 Ω coaxial line has a current $i\,(t, z) = 1.8\cos(3.77 \times 10^9\, t - 18.13z)$ mA. Determine (a) the frequency, (b) the phase velocity, (c) the wavelength, (d) the relative permittivity of the line, (e) the phasor form of the current, and (f) the time domain voltage on the line.

2.2 A transmission line has the following per-unit-length parameters: $L = 0.5\ \mu$H/m, $C = 200$ pF/m, $R = 4.0\ \Omega$/m, and $G = 0.02$ S/m. Calculate the propagation constant and characteristic impedance of this line at 800 MHz. If the line is 30 cm long, what is the attenuation in dB? Recalculate these quantities in the absence of loss ($R = G = 0$).

2.5 For the parallel plate line shown in the accompanying figure, derive the R, L, G, and C parameters. Assume $W \gg d$.



Appendice

Cylindrical coordinates:

$$\nabla f = \hat{\rho}\frac{\partial f}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial f}{\partial \phi} + \hat{z}\frac{\partial f}{\partial z}$$

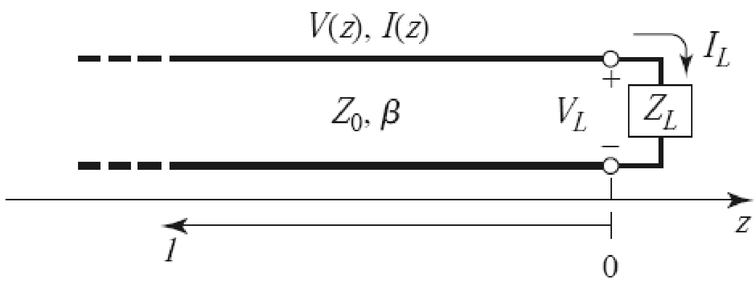
$$\nabla \cdot \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \bar{A} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla \times \nabla \times \bar{A}$$

3. THE TERMINATED LOSSLESS TRANSMISSION LINE



Assume that an incident wave of the form $V_0^+ e^{-j\beta z}$ is generated from a source at $z < 0$. When the line is terminated in an arbitrary load $Z_L \neq Z_0$, the ratio of voltage to current at the load must be Z_L. Thus, a reflected wave must be excited

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}.$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}.$$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_0.$$

$$V_o^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_o^+.$$

the voltage reflection coefficient,

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$V(z) = V_o^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right),$$

$$I(z) = \frac{V_o^+}{Z_0} \left(e^{-j\beta z} - \Gamma e^{j\beta z}\right).$$

the time-average power flow along the line at the point *z*:

$$P_{avg} = \frac{1}{2}\mathrm{Re}\{V(z)I(z)^*\} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\mathrm{Re}\{1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\},$$

$$\boxed{P_{avg} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}(1 - |\Gamma|^2),}$$

负载电阻对负载功率的影响？

If $\Gamma = 0$, maximum power is delivered to the load, $P_{avg} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}$

$|\Gamma| = 1$ no power is delivered $P_{avg} = 0$

When the load is mismatched, not all of the available power from the generator is delivered to the load. This “loss” is called *return loss* (RL),

$$\mathrm{RL} = -20\log|\Gamma|\ \mathrm{dB},$$

A matched load $(\Gamma = 0)$ has a return loss of ∞ dB (no reflected power),

A total reflection $(|\Gamma| = 1)$ has a return loss of 0 dB (all incident power is reflected).

If the load is matched to the line, $\Gamma = 0$ and the magnitude of the voltage on the line is $|V(z)| = |V_o^+|$,

When the load is mismatched, however, the presence of a reflected wave leads to standing waves, and the magnitude of the voltage on the line is not constant but oscillates with position z along the line.

$$|V(z)| = |V_o^+||1 + \Gamma e^{2j\beta z}| = |V_o^+||1 + \Gamma e^{-2j\beta \ell}|$$
$$= |V_o^+||1 + |\Gamma| e^{j(\theta - 2\beta \ell)}|,$$

The maximum value $V_{max} = |V_o^+|(1 + |\Gamma|)$.

The minimum value $V_{min} = |V_o^+|(1 - |\Gamma|)$.

负载与行波，驻波，行驻波关系？

The standing wave ratio (SWR or VSWR) (驻波比), $\text{SWR} = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$.

SWR is a real number such that 1 ⩽ SWR⩽∞,
where SWR = 1 implies a matched load.
As $|\Gamma|$ increases, the ratio of Vmax to Vmin increases

$$|V(z)| = |V_o^+||1 + \Gamma e^{2j\beta z}| = |V_o^+||1 + \Gamma e^{-2j\beta\ell}|$$
$$= |V_o^+||1 + |\Gamma| e^{j(\theta - 2\beta\ell)}|,$$

the distance between two successive voltage maxima (or minima) is

$$l = 2\pi/2\beta = \pi\lambda/2\pi = \lambda/2,$$

the distance between a maximum and a minimum is $l = \pi/2\beta = \lambda/4$,

The reflection coefficient $$\Gamma(\ell) = \frac{V_o^- e^{-j\beta\ell}}{V_o^+ e^{j\beta\ell}} = \Gamma(0) e^{-2j\beta\ell},$$

At a distance $l = -z$ from the load, the input impedance seen looking toward the load is

$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_o^+ \left(e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right)}{V_o^+ \left(e^{j\beta\ell} - \Gamma e^{-j\beta\ell}\right)} Z_0 = \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}} Z_0,$$

在传输线的不同位置上，什么参数不变？
电压电流的周期？输入阻抗的周期？

the transmission line impedance equation

$$V(z) = V_o^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right),$$

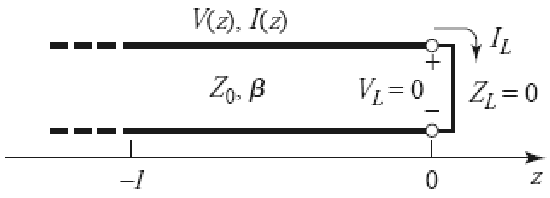
$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right).$$

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$\boxed{\begin{aligned} Z_{\text{in}} &= Z_0 \frac{(Z_L + Z_0)e^{j\beta\ell} + (Z_L - Z_0)e^{-j\beta\ell}}{(Z_L + Z_0)e^{j\beta\ell} - (Z_L - Z_0)e^{-j\beta\ell}} \\ &= Z_0 \frac{Z_L \cos\beta\ell + jZ_0 \sin\beta\ell}{Z_0 \cos\beta\ell + jZ_L \sin\beta\ell} \\ &= Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}. \end{aligned}}$$

输入阻抗的电路含义？

Special Cases of Lossless Terminated Lines



（1）A line is terminated in a short circuit

$$V(z) = V_o^+ \left(e^{-j\beta z} - e^{j\beta z}\right) = -2jV_o^+ \sin\beta z,$$

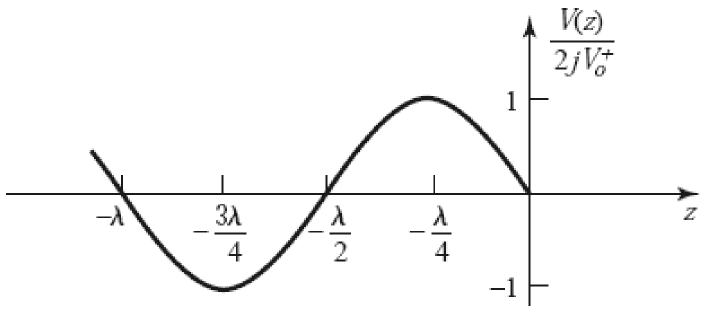
$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} + e^{j\beta z}\right) = \frac{2V_o^+}{Z_0}\cos\beta z,$$

$V = 0$ at the load (for a short circuit), while the current is a maximum there.

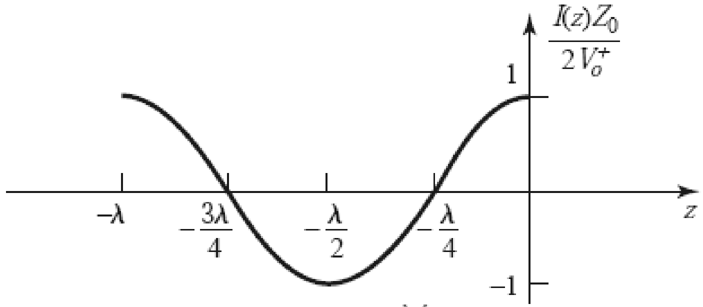
$$Z_{\rm in} = jZ_0 \tan\beta\ell,$$

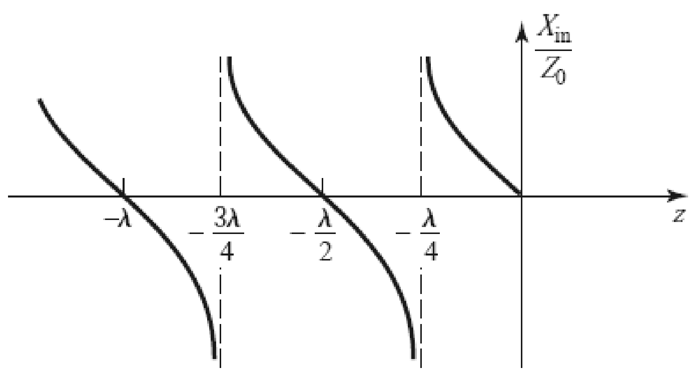
纯驻波时，传输线的输入阻抗的实部为？可看作什么元器件？

- It is purely imaginary for any length and to take on all values between $+j\infty$ and $-j\infty$. When $l = 0$ we have $Z_{in} = 0$, but for $l = \lambda/4$ we have $Z_{in} = \infty$ (open circuit).
- The impedance is periodic in l repeating for multiples of $\lambda/2$.



(a)





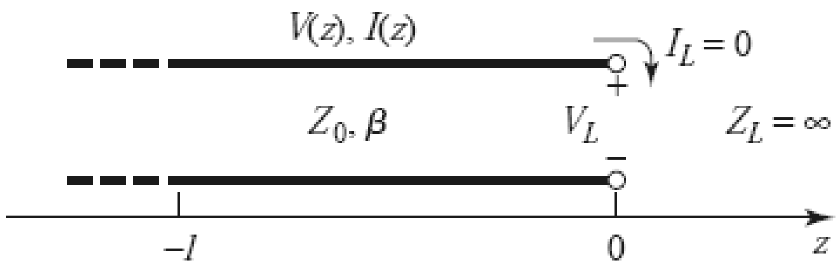
(c)

$$V(z) = V_o^+\left(e^{-j\beta z} - e^{j\beta z}\right) = -2jV_o^+ \sin\beta z,$$

$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} + e^{j\beta z}\right) = \frac{2V_o^+}{Z_0}\cos\beta z,$$

$$Z_{in} = jZ_0 \tan\beta\ell,$$

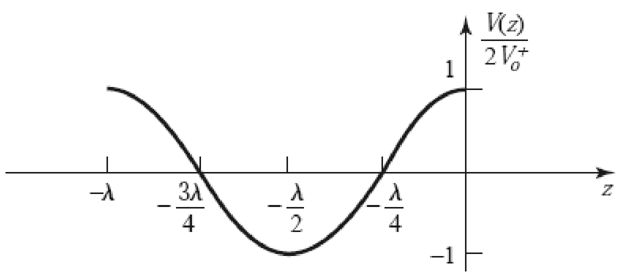
(2) the open-circuited line $Z_L = \infty$. $\Gamma = 1$



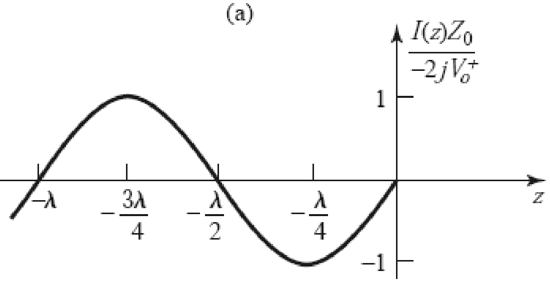
$$V(z) = V_o^+\left(e^{-j\beta z} + e^{j\beta z}\right) = 2V_o^+ \cos\beta z,$$

$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} - e^{j\beta z}\right) = \frac{-2jV_o^+}{Z_0}\sin\beta z,$$

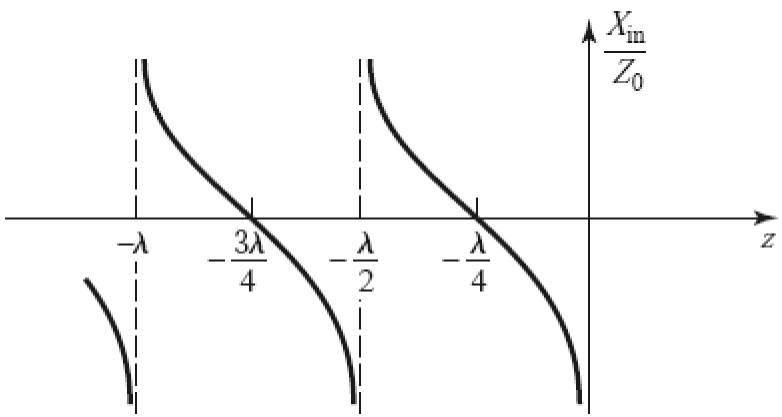
$$Z_{\text{in}} = -jZ_0 \cot\beta\ell,$$



(a)



(b)



(c)

(3) consider terminated transmission lines with some special lengths.

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell}.$$

If $\ell = \lambda/2,\qquad Z_{\text{in}} = Z_L,$

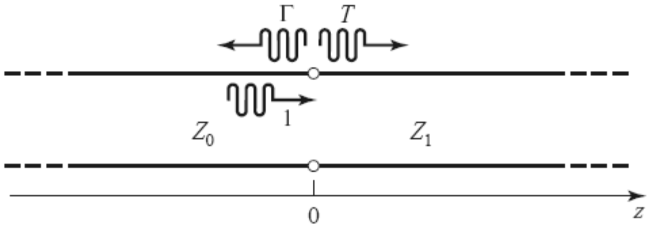
A half-wavelength line (or any multiple of $\lambda/2$) does not alter or transform the load impedance, regardless of its characteristic impedance.

If the line is a quarter-wavelength long $\quad \ell = \lambda/4 + n\lambda/2,$

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L}.$$

A *quarter-wave transformer* because it has the effect of transforming the load impedance in an inverse manner, depending on the characteristic impedance of the line.

Reflection and transmission at the junction of two transmission lines with different characteristic impedances.



Consider a transmission line of characteristic impedance Z_0 feeding a line of different characteristic impedance, Z_1. If the load line is infinitely long, or if it is terminated in its own characteristic impedance, so that there are no reflections from its far end, then the input impedance seen by the feed line is Z_1.

The reflection coefficient is $\Gamma = \dfrac{Z_1 - Z_0}{Z_1 + Z_0}.$

$$V(z) = V_o^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right), \qquad z < 0,$$

$$V(z) = V_o^+ T e^{-j\beta z} \quad \text{for } z > 0.$$

Equating these voltages at z = 0 gives the transmission coefficient,

$$T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}.$$

The transmission coefficient between two points in a circuit is often expressed in dB as the *insertion loss*, IL,

$$\mathrm{IL} = -20 \log |T| \mathrm{dB}.$$

the ratio of two power levels *P*1 and *P*2 in a microwave system is expressed in decibels (dB) as

$$10 \log \frac{P_1}{P_2} \text{ dB}.$$

$$\frac{P_1}{P_2} = 2 \rightarrow \text{3dB} \qquad \frac{P_1}{P_2} = 0.1 \rightarrow \text{-10dB} \qquad \frac{P_1}{P_2} = 0.01 \rightarrow \text{-20dB}$$

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Using power ratios in dB makes it easy to calculate power loss or gain through a series of components since multiplicative loss or gain factors can be accounted for by adding the loss or gain in dB for each stage. For example, a signal passing through a 6 dB attenuator followed by a 23 dB amplifier will have an overall gain of 23 − 6 = 17 dB.

Decibels can be represented by voltage ratio,

$$10\log\frac{V_1^2 R_2}{V_2^2 R_1} = 20\log\frac{V_1}{V_2}\sqrt{\frac{R_2}{R_1}}\ \text{dB},$$

If the load resistances are equal,

$$20\log\frac{V_1}{V_2}\ \text{dB}.$$

The ratio of voltages across equal load resistances can also be expressed in terms of nepers (Np) as

$$\ln\frac{V_1}{V_2}\ \text{Np}.$$

$$1\ \text{Np} = 10\log e^2 = 8.686\ \text{dB}.$$

If we let P_2 =1 mW, then the power P_1 can be expressed in dBm

$$10\log\frac{P_1}{1\ \text{mW}}\ \text{dBm}$$

A power of 1 mW is equivalent to 0 dBm, while a power of 1W is equivalent to 30 dBm.

Homework

2.8 A lossless transmission line of electrical length $= 0.3\lambda$ *is terminated with a complex load impedance* as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.