Lec13 Microwave Network Analysis （V）

4.6 DISCONTINUITIES AND MODAL ANALYSIS （不连续性和模式分析）

不连续性有哪些？

Microwave circuits and networks often consist of transmission lines with various types of discontinuities.

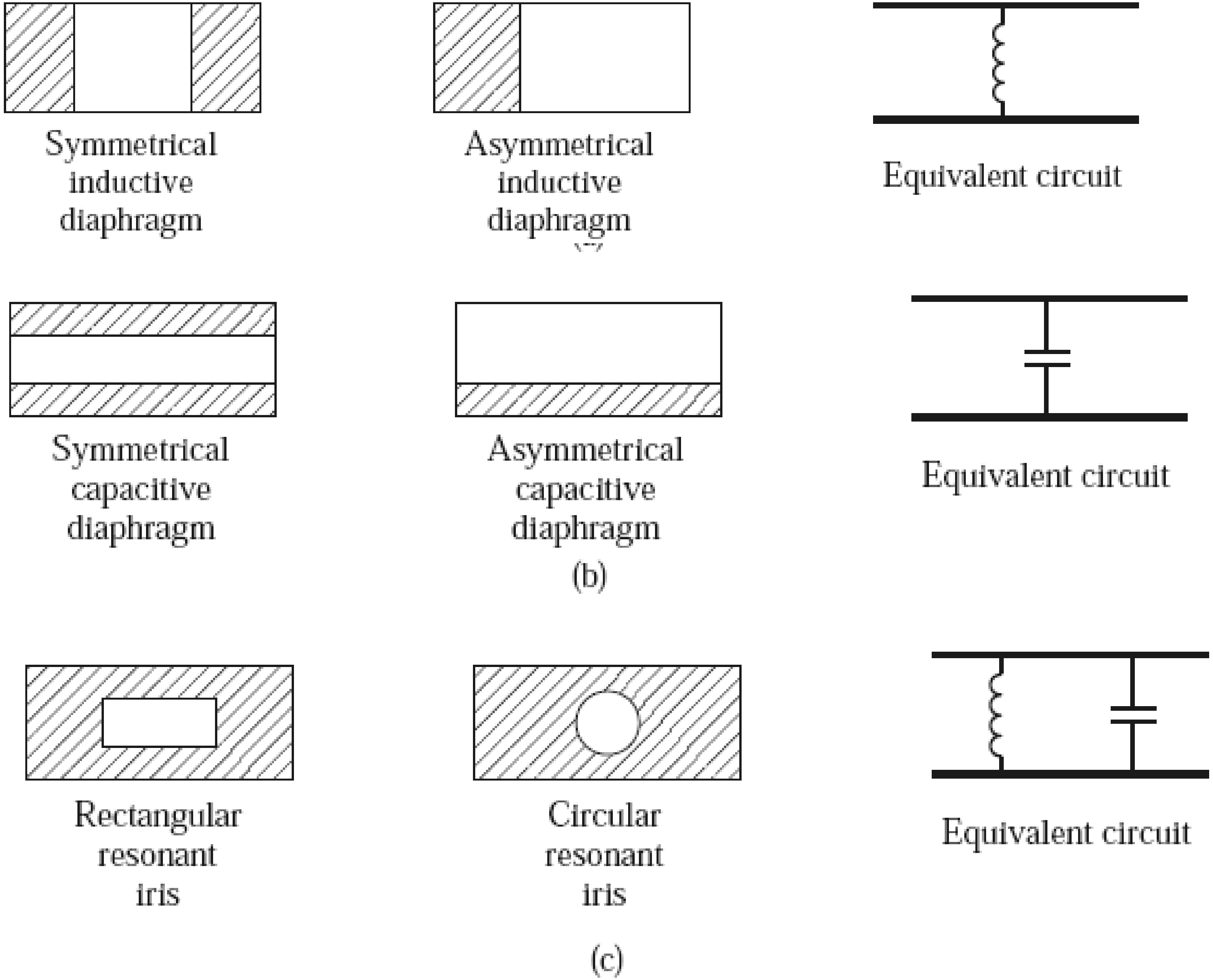
- Some discontinuities are an unavoidable result of mechanical or electrical transitions from one medium to another.
  (e.g. a junction between two waveguides, or a coax-to-microstrip transition)

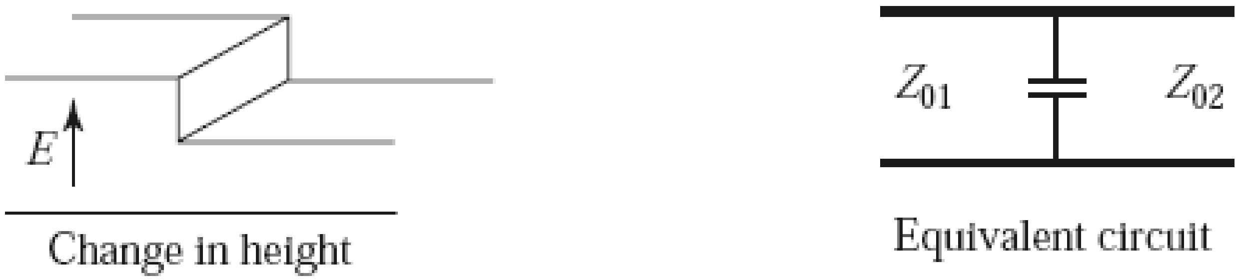
- Some discontinuities may be deliberately introduced into the circuit to perform a certain electrical function.
  (e.g., reactive diaphragms (电抗膜片) in waveguide, or stubs on a microstrip line for matching or filter circuits).

- A transmission line discontinuity can be represented as an equivalent circuit at some point on the transmission line.
- Depending on the type of discontinuity, the equivalent circuit may be a simple shunt or series element across the line or, a T- or π-equivalent circuit .
- Once the equivalent circuit of a given discontinuity is known, its effect can be incorporated into the analysis or design of the network.

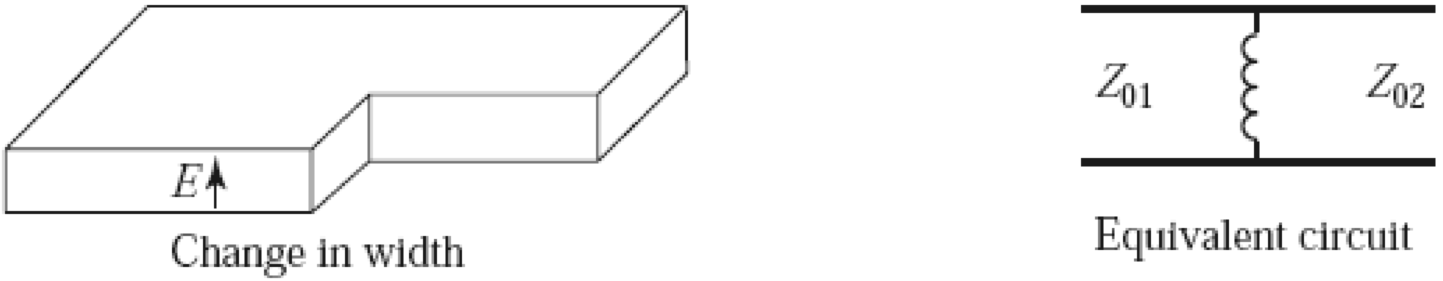
How are equivalent circuits obtained for transmission line discontinuities?

- ➢ One approach is to start with a field theory solution to a discontinuity problem and develop a circuit model with component values, that is, replacing complicated field analyses with circuit concepts.
- ➢ In other cases, it may be easier to measure the network parameters (S parameters) of an isolated discontinuity.


Symmetrical inductive diaphragm
Asymmetrical inductive diaphragm
Equivalent circuit
Symmetrical capacitive diaphragm
Asymmetrical capacitive diaphragm
Equivalent circuit
(b)
Rectangular resonant iris
Circular resonant iris
Equivalent circuit
(c)



(d)



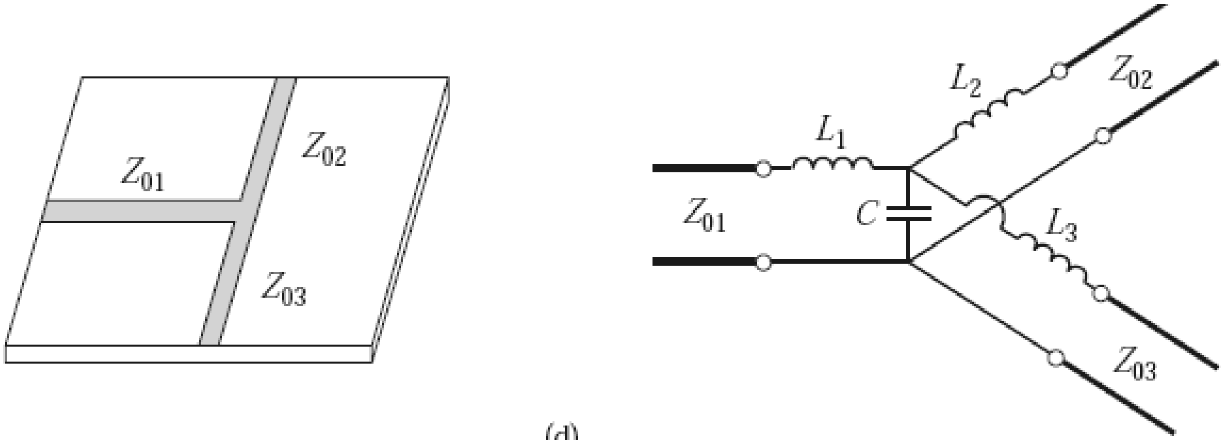
(e)

Rectangular waveguide discontinuities.

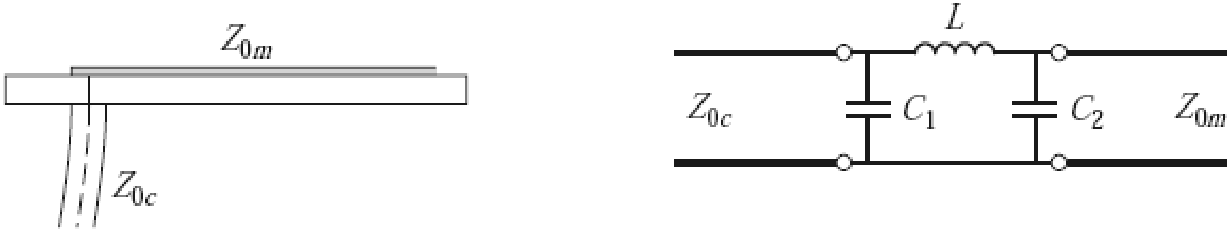
(a)

(b)

(c)



(d)



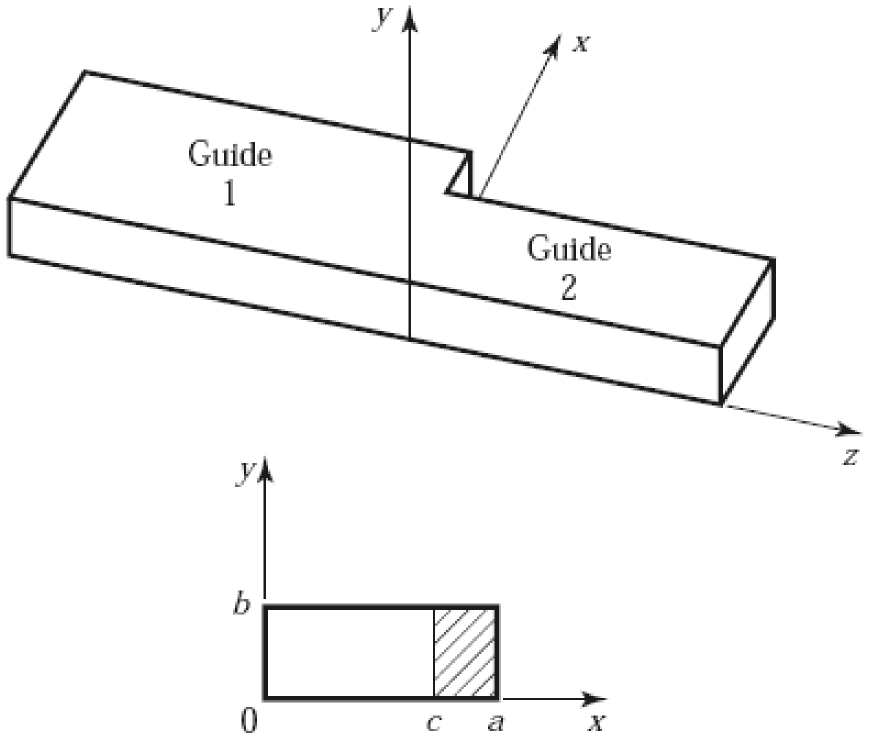
(e)

Some common microstrip discontinuities. (a) Open-ended microstrip. (b) Gap in microstrip. (c) Change in width. (d) T-junction. (e) Coax-to-microstrip junction.

Modal Analysis of an H-Plane Step in Rectangular Waveguide

- The **field analysis** of most transmission line discontinuity problems is difficult.

- The technique of waveguide **modal analysis** is relatively straightforward and similar in principle to the reflection/transmission problems.



It is assumed that only the dominant TE10 mode is propagating in guide 1 *(z < 0)* and is incident on the junction from *z < 0.*

$$E_y^i = \sin\frac{\pi x}{a} e^{-j\beta_1^a z},$$

$$H_x^i = \frac{-1}{Z_1^a}\sin\frac{\pi x}{a} e^{-j\beta_1^a z},$$

where the propagation constant of TEn0 is

$$\beta_n^a = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)^2}$$

Geometry of an H-plane step (change in width) in a rectangular waveguide.

The wave impedance of the TE*n0* mode in guide 1 is

$$Z_n^a = \frac{k_0 \eta_0}{\beta_n^a}$$

- Because of the discontinuity at *z = 0*, there will be reflected and transmitted waves in both guides, consisting of infinite sets of TE*n0 modes* in guides 1 and 2.
- Because there is no y variation introduced by this discontinuity, TEnm modes for m = 0 are not excited, nor are any TM modes.
- Only the TE10 mode will propagate in guide 1, but higher order modes are also important in this problem because they account for stored energy localized near z = 0.

Rectangular waveguide

Quantity	TE_{mn} Mode	TM_{mn} Mode
E_z	0	$B\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
H_z	$A\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	0
E_x	$\frac{j\omega\mu n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\beta m\pi}{k_c^2 a}B\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
E_y	$\frac{-j\omega\mu m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\beta n\pi}{k_c^2 b}B\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
H_x	$\frac{j\beta m\pi}{k_c^2 a}A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{j\omega\epsilon n\pi}{k_c^2 b}B\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$
H_y	$\frac{j\beta n\pi}{k_c^2 b}A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$	$\frac{-j\omega\epsilon m\pi}{k_c^2 a}B\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$
Z	$Z_{TE}=\frac{k\eta}{\beta}$	$Z_{TM}=\frac{\beta\eta}{k}$

The reflected modes in guide 1 may be written, for *z < 0, as*

$$E_y^r = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{a} e^{j\beta_n^a z},$$

$$H_x^r = \sum_{n=1}^{\infty} \frac{A_n}{Z_n^a} \sin\frac{n\pi x}{a} e^{j\beta_n^a z},$$

where An is the unknown amplitude coefficient of the reflected TEn0 mode in guide 1.

The transmitted modes into guide 2 can be written, for *z > 0, as*

$$E_y^t = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{c} e^{-j\beta_n^c z},$$

$$H_x^t = -\sum_{n=1}^{\infty} \frac{B_n}{Z_n^c} \sin\frac{n\pi x}{c} e^{-j\beta_n^c z},$$

where the propagation constant in guide 2 is $\beta_n^c = \sqrt{k_0^2 - \left(\frac{n\pi}{c}\right)^2},$

and the wave impedance in guide 2 is $Z_n^c = \frac{k_0 \eta_0}{\beta_n^c}.$

At $z = 0$, the transverse fields (Ey , Hx) must be continuous for $0 < x < c$;
Ey must be zero for c < x < a because of the step.

$$E_y = \sin\frac{\pi x}{a} + \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{a} = \begin{cases} \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{c} & \text{for } 0 < x < c, \\ 0 & \text{for } c < x < a, \end{cases}$$

$$H_x = \frac{-1}{Z_1^a}\sin\frac{\pi x}{a} + \sum_{n=1}^{\infty}\frac{A_n}{Z_n^a}\sin\frac{n\pi x}{a} = -\sum_{n=1}^{\infty}\frac{B_n}{Z_n^c}\sin\frac{n\pi x}{c} \quad \text{for } 0 < x < c.$$

The above equations constitute a doubly infinite set of linear equations for the modal coefficients *An and Bn*.
We will first eliminate the Bn and then truncate the resulting equation to a finite number of terms and solve for the *An.*

Note：Fields at a General Material Interface

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = \bar{M}_s.$$

$$\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s,$$

The orthogonality (正交) relations:

$$\int_0^a \cos^2 \frac{n\pi x}{a} dx = \int_0^a \sin^2 \frac{n\pi x}{a} dx = \frac{a}{2}, \qquad \text{for } n \geq 1$$

$$\int_0^a \cos \frac{m\pi x}{a} \cos \frac{n\pi x}{a} dx = \int_0^a \sin \frac{m\pi x}{a} \sin \frac{n\pi x}{a} dx = 0, \qquad \text{for } m \neq n$$

$$\int_0^a \cos \frac{m\pi x}{a} \sin \frac{n\pi x}{a} dx = 0$$

Multiplying the equation Ey at z=0 by sin(mπx/a), integrating from x = 0 to a, and using the orthogonality relations yields

$$\frac{a}{2}\delta_{m1} + \frac{a}{2}A_m = \sum_{n=1}^{\infty} B_n I_{mn} = \sum_{k=1}^{\infty} B_k I_{mk},$$

where

$$I_{mn} = \int_{x=0}^{c} \sin \frac{m\pi x}{a} \sin \frac{n\pi x}{c} dx$$

$$\delta_{mn} = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

Similarly, multiplying the equation Hx at z=0 by sin(kπx/c), integrating from x = 0 to c, and using the orthogonality relations yields

$$\frac{-1}{Z_1^a} I_{k1} + \sum_{n=1}^{\infty} \frac{A_n}{Z_n^a} I_{kn} = \frac{-cB_k}{2Z_k^c}.$$

Eliminating *Bk* gives an infinite set of linear equations for the An, where m = 1, 2, . . . ,

$$\frac{a}{2} A_m + \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{2Z_k^c I_{mk} I_{kn} A_n}{cZ_n^a} = \sum_{k=1}^{\infty} \frac{2Z_k^c I_{mk} I_{k1}}{cZ_1^a} - \frac{a}{2} \delta_{m1}.$$

For numerical calculation we can truncate these summations to *N* terms, which will result in *N* linear equations for the first *N* coefficients.

For example, let N = 1.

$$\frac{a}{2}A_1 + \frac{2Z_1^c I_{11}^2}{cZ_1^a}A_1 = \frac{2Z_1^c I_{11}^2}{cZ_1^a} - \frac{a}{2}.$$

Solving for A1 (the reflection coefficient of the incident TE10 mode) gives

$$A_1 = \frac{Z_\ell - Z_1^a}{Z_\ell + Z_1^a} \quad \text{for } N = 1,$$

where $Z_\ell = 4Z_1^c I_{11}^2/ac$, which looks like an effective load impedance to guide 1.

Accuracy is improved by using larger values of *N* and leads to a set of equations that can be written in matrix form as

$$[Q][A] = [P],$$

where [Q] is a square N × N matrix of coefficients,

$$Q_{mn} = \frac{a}{2}\delta_{mn} + \sum_{k=1}^{N} \frac{2Z_k^c I_{mk} I_{kn}}{cZ_n^a},$$

[P] is an N × 1 column vector of coefficients given by

$$P_m = \sum_{k=1}^{N} \frac{2Z_k^c I_{mk} I_{k1}}{cZ_1^a} - \frac{a}{2}\delta_{m1},$$

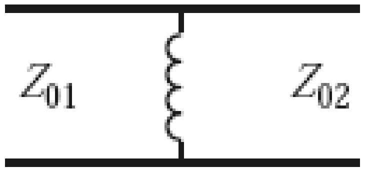
and [A] is an N × 1 column vector of the coefficients An.

If the width *c* of guide 2 is such that all modes are cut off (evanescent), then no real power can be transmitted into guide 2, and all the incident power is reflected back into guide 1.

The evanescent fields on both sides of the discontinuity store reactive power, however, which implies that the step discontinuity and guide 2 beyond the discontinuity look like a reactance (in this case an inductive reactance) to an incident *TE10* mode in guide 1.

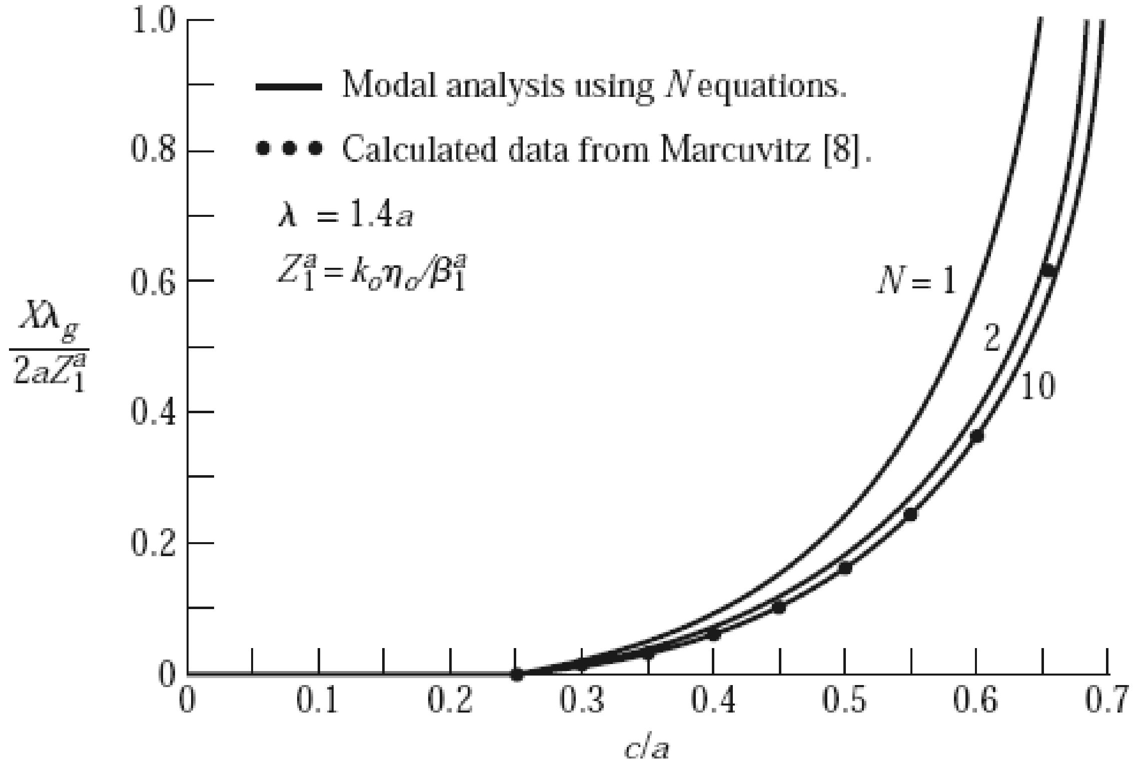
Thus the equivalent circuit of the H-plane step looks like a shunt inductor at the z = 0 plane of guide 1, as shown in Figure 4.22e. The equivalent reactance can be found from the reflection coefficient A1 as

$$X = -jZ_1^a \frac{1 + A_1}{1 - A_1}.$$



Equivalent circuit

Equivalent inductance of an H-plane asymmetric step.



Note that the solution converges very quickly (because of the fast exponential decay of the higher order evanescent modes), and that the result using just two modes is very close to the data of reference

- The fact that the H-plane step appears inductive is a result of the actual value of the reflection coefficient.
- we can verify the inductive nature of the discontinuity by computing the complex power flow into the evanescent modes on either side of the discontinuity.

the complex power flow into guide 2 can be found as

$$P = \int_{x=0}^{c} \int_{y=0}^{b} \bar{E} \times \bar{H}^* \Big|_{z=0^+} \cdot \hat{z}dxdy$$

$$= -b \int_{x=0}^{c} E_y H_x^* dx$$

$$= -b \int_{x=0}^{c} \left[\sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{c} \right] \left[- \sum_{m=1}^{\infty} \frac{B_m^*}{Z_m^{c*}} \sin \frac{m\pi x}{c} \right] dx$$

$$= \frac{bc}{2} \sum_{n=1}^{\infty} \frac{|B_n|^2}{Z_n^{c*}}$$

$$= \frac{jbc}{2k_0\eta_0} \sum_{n=1}^{\infty} |B_n|^2 |\beta_n^c|,$$

the complex power flow into guide 2 is positive imaginary, implying stored magnetic energy and an inductive reactance.

the input impedance is

$$Z_{\text{in}} = R + jX = \frac{V}{I} = \frac{VI^*}{|I|^2} = \frac{P}{\frac{1}{2}|I|^2} = \frac{P_\ell + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}.$$

the real part, R, of the input impedance is related to the dissipated power.

$$R = \frac{P_l}{\frac{1}{2}|I|^2}$$

The reactance X

$$X = \frac{4\omega(W_m - W_e)}{|I|^2},$$

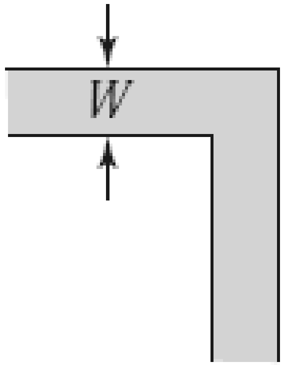
includes the inductance

$$L = \frac{W_m}{\frac{1}{4}|I|^2}$$

and the capacitance

$$C = \frac{W_e}{\frac{1}{4}|I|^2}$$

Microstrip Discontinuity Compensation



Right-angle bend

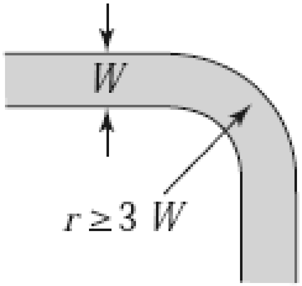
One problem with microstrip circuits (and other planar circuits) is that **the inevitable discontinuities at bends**, step changes in widths, and junctions can **cause degradation in circuit performance**. This is because such discontinuities introduce **parasitic reactances** that can lead to phase and amplitude errors, input and output mismatch, and possibly spurious coupling or radiation.

One approach for eliminating such effects is to construct an equivalent circuit for the discontinuity (perhaps by measurement), including it in the design of the circuit, and compensating for its effect by **adjusting other circuit parameters** (such as line lengths and characteristic impedances, or tuning stubs （可调谐短截线）).

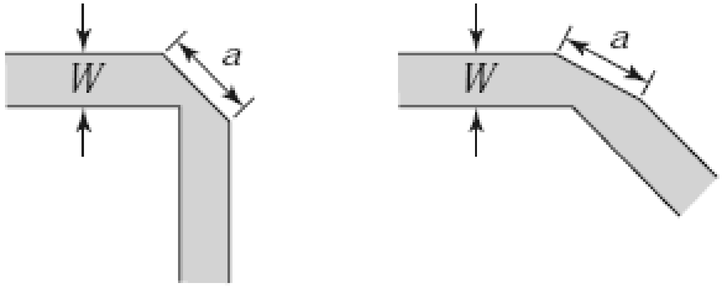
Another approach is to minimize the effect of a discontinuity by compensating the discontinuity directly, often by chamfering (去角) or mitering (斜角) the conductor.

●The straightforward right-angle bend has a parasitic discontinuity capacitance caused by the increased conductor area at the corner of the bend.

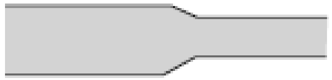
●Swept bend and mitered bend can eliminate this effect.

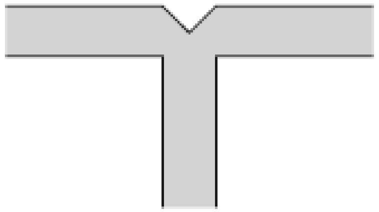


Swept bend



Mitered bends


Mitered step


Mitered *T*-junction

Homework

4.1 Consider the reflection of a TE_{10} mode, incident from $z < 0$, at a step change in the height of a rectangular waveguide, as shown below. Show that if the method of Example 4.2 is used, the result $\Gamma = 0$ is obtained. Do you think this is the correct solution? Why? (This problem shows that the one-mode impedance viewpoint does not always provide a correct analysis.)