Lec16 Microwave Resonators (I)
微波谐振器

Microwave resonators applications:
Filters (滤波器)
Oscillators （振荡器），
frequency meters （频率计）
and tuned amplifiers （可调谐放大器）.

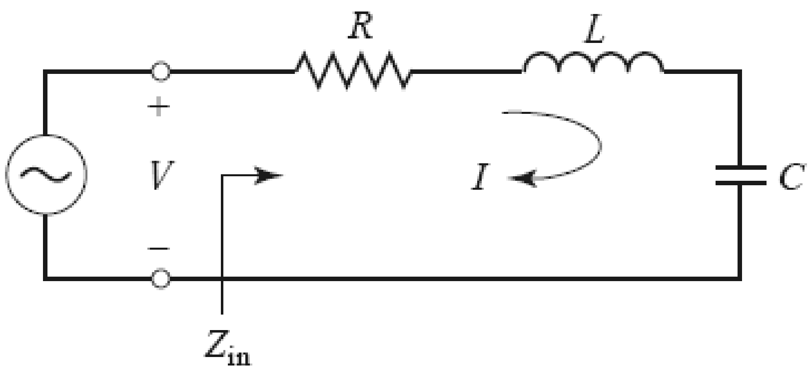
Resonators classification:

1. Lumped-element resonators: series RLC resonant circuits
   and parallel RLC resonant circuits.
2. Microwave resonators **:** transmission lines,
   rectangular waveguides,
   circular waveguides,
   and dielectric cavities.

6.1 SERIES AND PARALLEL RESONANT CIRCUITS

谐振的条件是什么？

Series Resonant Circuit



(a)

The input impedance is

$$Z_{in} = R + j\omega L - j\frac{1}{\omega C},$$

the complex power delivered to the resonator is

$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z_{in}|I|^2 = \frac{1}{2}Z_{in}\left|\frac{V}{Z_{in}}\right|^2$$

$$= \frac{1}{2}|I|^2\left(R + j\omega L - j\frac{1}{\omega C}\right).$$

The power dissipated by the resistor *R is*

$$P_{loss} = \frac{1}{2}|I|^2 R,$$

the average magnetic energy stored in the inductor *L is*

$$W_m = \frac{1}{4}|I|^2 L\,,$$

the average electric energy stored in the capacitor *C is*

$$W_e = \frac{1}{4}|V_c|^2 C = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C},$$

Then the complex power can be rewritten as

$$P_{\text{in}} = P_{\text{loss}} + 2j\omega(W_m - W_e),$$

the input impedance is

$$Z_{\text{in}} = \frac{2P_{\text{in}}}{|I|^2} = \frac{P_{\text{loss}} + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}.$$

Resonance occurs when the average stored magnetic and electric energies are equal, or *Wm* = *We*. Then the input impedance is

$$Z_{\text{in}} = \frac{P_{\text{loss}}}{\frac{1}{2}|I|^2} = R,$$

Wm = *We* implies that the resonant frequency, ω0, can be defined as

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

Quality factor (品质因数) is defined as

$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss/second}}$$

$$= \omega \frac{W_m + W_e}{P_{\text{loss}}}.$$

Q is a measure of the loss of a resonant circuit—lower loss implies a higher Q.

For the series resonant circuit at resonance,

$$Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC},$$

Next, consider the behavior of the input impedance of this resonator near its resonant frequency. Let $\omega = \omega_0 + \omega$, where ω is small. The input impedance can then be rewritten as

$$Z_{\text{in}} = R + j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)$$

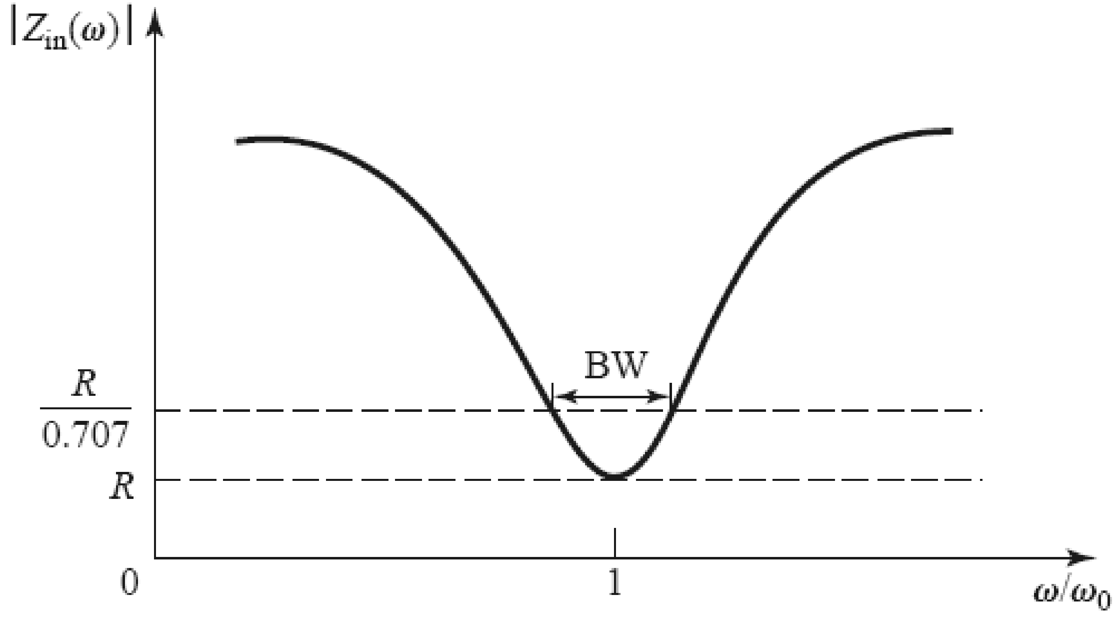
$$= R + j\omega L \left(\frac{\omega^2 - \omega_0^2}{\omega^2}\right),$$

Now $\omega^2 - \omega_0^2 = (\omega - \omega_0)(\omega + \omega_0) = \Delta\omega(2\omega - \Delta\omega) \simeq 2\omega\Delta\omega$

$$Z_{\text{in}} \simeq R + j2L\Delta\omega$$

$$\simeq R + j\frac{2RQ_0\Delta\omega}{\omega_0} = R + j2L\Delta\omega,$$

The half-power fractional bandwidth of the resonator.



(b)

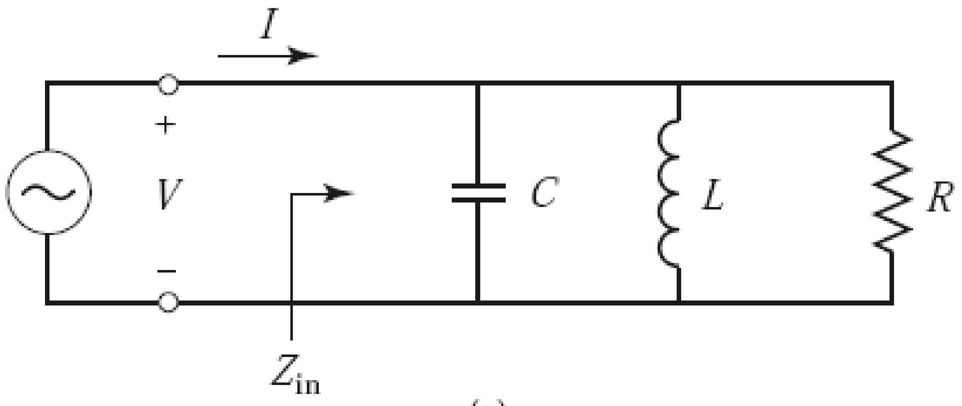
When the frequency is such that $|Zin|^2 = 2R^2$, then the average (real) power delivered to the circuit is one-half that delivered at resonance.

$\Delta\omega/\omega_0 = \mathrm{BW}/2$ at the upper band edge.

$$|R + jRQ_0(\mathrm{BW})|^2 = 2R^2,$$

$$\mathrm{BW} = \frac{1}{Q_0}.$$

Parallel Resonant Circuit



(a)

$$P_{in} = P_{loss} + 2j\omega(W_m - W_e),$$

The input impedance is

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1},$$

the complex power delivered to the resonator is

$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z_{in}|I|^2 = \frac{1}{2}|V|^2\frac{1}{Z_{in}^*}$$
$$= \frac{1}{2}|V|^2\left(\frac{1}{R} + \frac{j}{\omega L} - j\omega C\right).$$

The power dissipated by the resistor, *R, is* $P_{loss} = \frac{1}{2}\frac{|V|^2}{R},$

The average electric energy stored in the capacitor, *C, is* $W_e = \frac{1}{4}|V|^2 C,$

The average magnetic energy stored in the inductor, *L, is*

$$W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}|V|^2\frac{1}{\omega^2 L},$$

As in the series case, resonance occurs when *Wm* = *We*. Then the input impedance is

$$Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R,$$

Wm = We implies that the resonant frequency, ω0, can be defined as

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

the unloaded Q of the parallel resonant circuit can be expressed as

$$Q_0 = \omega_0 \frac{2W_m}{P_{loss}} = \frac{R}{\omega_0 L} = \omega_0 RC,$$

Again letting ω = ω0 + ω, where ω is small,

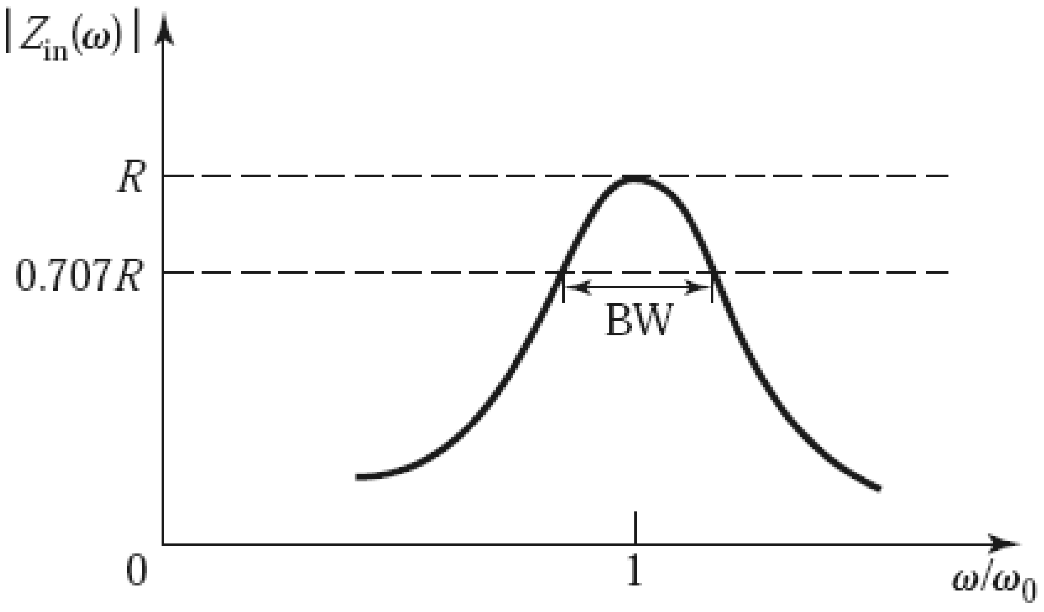
$$Z_{in} \simeq \frac{R}{1 + 2j\Delta\omega RC} = \frac{R}{1 + 2jQ_0\Delta\omega/\omega_0},$$

The half-power bandwidth edges occur at frequencies

$$|Z_{in}|^2 = \frac{R^2}{2},$$

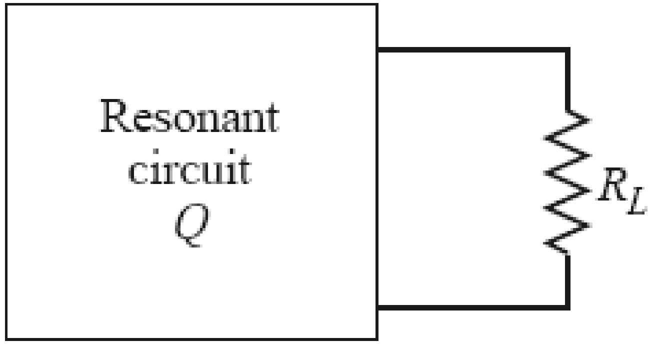
The bandwidth is

$$BW = \frac{1}{Q_0},$$



(b)

Loaded and Unloaded Q



A resonant circuit connected to an external load, *RL*

The unloaded Q, Q_0 is a characteristic of the resonator itself in the absence of any loading.

The loaded Q, Q_L is a characteristic of the resonator with the external load resistor, RL.

- If the resonator is **a series RLC circuit**, the load resistor **R_L adds in series with R**, so the effective resistance is **$R + R_L$** .
- If the resonator is **a parallel RLC circuit**, the load resistor **R_L combines in parallel with R**, so the effective resistance in is **$RR_L/(R + R_L)$.**

If we define an external Q, Qe, as

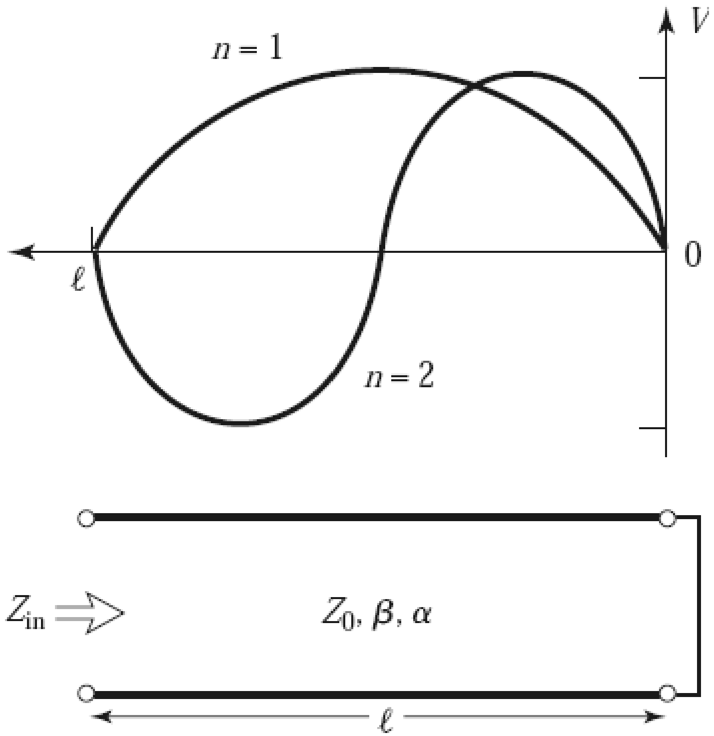
$$Q_e = \begin{cases} \dfrac{\omega_0 L}{R_L} & \text{for series circuits} \\ \dfrac{R_L}{\omega_0 L} & \text{for parallel circuits,} \end{cases}$$

then the loaded Q can be expressed as

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_0}.$$

6.2 TRANSMISSION LINE RESONATORS (传输线谐振器)

Short-Circuited *λ/2* Line



The input impedance *Zin* at a distance from the load is

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = Z_0 \frac{Z_L + Z_0 \tanh \gamma \ell}{Z_0 + Z_L \tanh \gamma \ell}.$$

Then the input impedance of a short-circuited line is

$$Z_{in} = Z_0 \tanh(\alpha + j\beta)\ell.$$

$$Z_{\text{in}} = Z_0 \frac{\tanh \alpha\ell + j \tan \beta\ell}{1 + j \tan \beta\ell \tanh \alpha\ell}.$$

Observe that $Z_{\text{in}} = jZ_0 \tan \beta\ell$ if $\alpha = 0$ (a lossless line).

In practice it is usually desirable to use a low-loss transmission line, so we assume that $\alpha\ell \ll 1$, and then $\tanh \alpha\ell \simeq \alpha\ell$. Again let $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega$ is small. Then, assuming a TEM line, we have

$$\beta\ell = \frac{\omega\ell}{v_p} = \frac{\omega_0\ell}{v_p} + \frac{\Delta\omega\ell}{v_p},$$

Because $\ell = \lambda/2 = \pi v_p/\omega_0$

$$\beta\ell = \pi + \frac{\Delta\omega\pi}{\omega_0},$$

$$\tan \beta\ell = \tan\left(\pi + \frac{\Delta\omega\pi}{\omega_0}\right) = \tan \frac{\Delta\omega\pi}{\omega_0} \simeq \frac{\Delta\omega\pi}{\omega_0}.$$

$$Z_{\text{in}} \simeq Z_0 \frac{\alpha\ell + j(\Delta\omega\pi/\omega_0)}{1 + j(\Delta\omega\pi/\omega_0)\alpha\ell} \simeq Z_0\left(\alpha\ell + j\frac{\Delta\omega\pi}{\omega_0}\right),$$

It is of the form

$$Z_{\text{in}} = R + 2jL\Delta\omega,$$

which is the input impedance of a series RLC resonant circuit with

the equivalent resistance $R = Z_0\alpha\ell,$

the equivalent inductance $L = \frac{Z_0\pi}{2\omega_0}.$

the equivalent capacitance $C = \frac{1}{\omega_0^2 L}.$

Resonance also occurs for *l= nλ/2, n = 1, 2, 3, ...*

The unloaded *Q* of this resonator is

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\pi}{2\alpha\ell} = \frac{\beta}{2\alpha},$$

since $\beta = \pi$ at the first resonance. This result shows that the Q decreases as the attenuation of the line increases.

EXAMPLE 6.1 Q OF HALF-WAVE COAXIAL LINE RESONATORS

A $\lambda/2$ resonator is made from a piece of copper coaxial line having an inner conductor radius of 1 mm and an outer conductor radius of 4 mm. If the resonant frequency is 5 GHz, compare the unloaded *Q* of an air-filled coaxial line resonator to that of a Teflon-filled coaxial line resonator.

Solution

We first compute the attenuation of the coaxial line, using the results of Examples 2.6 or 2.7. From Appendix F, the conductivity of copper is $\sigma = 5.813 \times 107$ *S/m*. The surface resistivity at 5 GHz is

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 1.84 \times 10^{-2}\ \Omega.$$

The attenuation due to conductor loss for the air-filled line is

$$\alpha_c = \frac{R_s}{2\eta \ln b/a}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$= \frac{1.84\times 10^{-2}}{2(377)\ln(0.004/0.001)}\left(\frac{1}{0.001}+\frac{1}{0.004}\right) = 0.022 \text{ Np/m.}$$

For Teflon, *r = 2.08 and tan δ = 0.0004*, so the attenuation due to conductor loss for the Teflon-filled line is

$$\alpha_c = \frac{1.84\times 10^{-2}\sqrt{2.08}}{2(377)\ln(0.004/0.001)}\left(\frac{1}{0.001}+\frac{1}{0.004}\right) = 0.032 \text{ Np/m.}$$

The dielectric loss of the air-filled line is zero, but the dielectric loss of the Teflonfilled line is

$$\alpha_d = k_0\frac{\sqrt{\epsilon_r}}{2}\tan\delta$$

$$= \frac{(104.7)\sqrt{2.08}(0.0004)}{2} = 0.030 \text{ Np/m.}$$

Thus

$$Q_{\text{air}} = \frac{\beta}{2\alpha} = \frac{104.7}{2(0.022)} = 2380,$$

$$Q_{\text{Teflon}} = \frac{\beta}{2\alpha} = \frac{104.7\sqrt{2.08}}{2(0.032+0.030)} = 1218.$$

Short-Circuited *λ/4 Line*

The input impedance of a shorted line of length *is*

$$Z_{\text{in}} = Z_0 \tanh(\alpha + j\beta)\ell$$
$$= Z_0 \frac{\tanh \alpha\ell + j \tan \beta\ell}{1 + j \tan \beta\ell \tanh \alpha\ell}$$
$$= Z_0 \frac{1 - j \tanh \alpha\ell \cot \beta\ell}{\tanh \alpha\ell - j \cot \beta\ell},$$

Now assume that *l= λ/4 at ω = ω0, and let ω = ω0 + ω.* Then, for a TEM line

$$\beta\ell = \frac{\omega_0 \ell}{v_p} + \frac{\Delta\omega \ell}{v_p} = \frac{\pi}{2} + \frac{\pi\,\Delta\omega}{2\omega_0},$$

$$\cot \beta\ell = \cot\left(\frac{\pi}{2} + \frac{\pi\,\Delta\omega}{2\omega_0}\right) = -\tan \frac{\pi\,\Delta\omega}{2\omega_0} \simeq \frac{-\pi\,\Delta\omega}{2\omega_0}.$$

$$Z_{\text{in}} = Z_0 \frac{1 + j\alpha\ell\pi\,\Delta\omega/2\omega_0}{\alpha\ell + j\pi\,\Delta\omega/2\omega_0} \simeq \frac{Z_0}{\alpha\ell + j\pi\,\Delta\omega/2\omega_0},$$

This result is of the same form as the impedance of a parallel *RLC* circuit,

$$Z_{in} = \frac{1}{(1/R) + 2j\Delta\omega C}.$$

the resistance of the equivalent circuit as $R = \frac{Z_0}{\alpha\ell}$

the capacitance of the equivalent circuit as $C = \frac{\pi}{4\omega_0 Z_0}.$

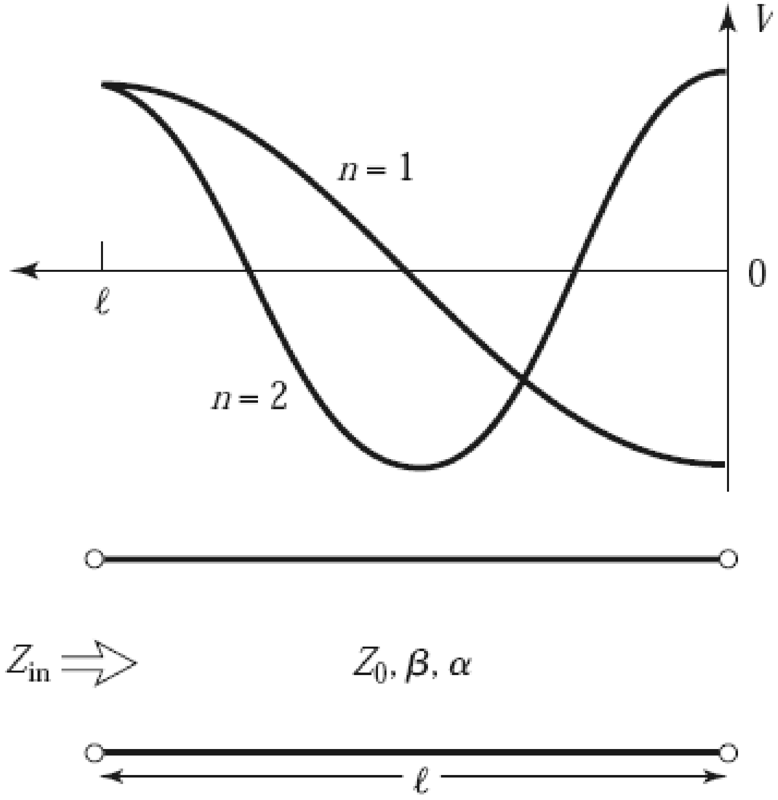
the capacitance of the equivalent circuit as $L = \frac{1}{\omega_0^2 C}.$

has a parallel-type resonance for *l= λ/4, with an* input impedance at resonance of

$$Z_{in} = R = Z_0/\alpha\ell.$$

$$Q_0 = \omega_0 RC = \frac{\pi}{4\alpha\ell} = \frac{\beta}{2\alpha},$$

Open-Circuited *λ/2 Line*



This resonator will behave as a parallel resonant circuit when the length is *λ/2, or multiples of λ/2.*

The input impedance of an open-circuited lossy transmission line of length *is*

$$Z_{\text{in}} = Z_0 \coth(\alpha + j\beta)\ell = Z_0 \frac{1 + j \tan \beta\ell \tanh \alpha\ell}{\tanh \alpha\ell + j \tan \beta\ell}.$$

As before, assume that *= λ/2 at ω = ω0, and let ω = ω0 + ω. Then,*

$$\beta\ell = \pi + \frac{\pi \Delta\omega}{\omega_0},$$

$$\tan \beta\ell = \tan \frac{\Delta\omega\pi}{\omega} \simeq \frac{\Delta\omega\pi}{\omega_0},$$

$$Z_{in} = \frac{Z_0}{\alpha\ell + j(\Delta\omega\pi/\omega_0)}.$$

Comparison with the input impedance of a parallel resonant circuit,

$$Z_{in} = \frac{1}{(1/R) + 2j\Delta\omega C}.$$

the resistance of the equivalent circuit as $R = \frac{Z_0}{\alpha\ell}$,

the capacitance of the equivalent circuit as $C = \frac{\pi}{2\omega_0 Z_0}$.

the capacitance of the equivalent circuit as $L = \frac{1}{\omega_0^2 C}$.

the unloaded Q is $Q_0 = \omega_0 RC = \frac{\pi}{2\alpha\ell} = \frac{\beta}{2\alpha}$,

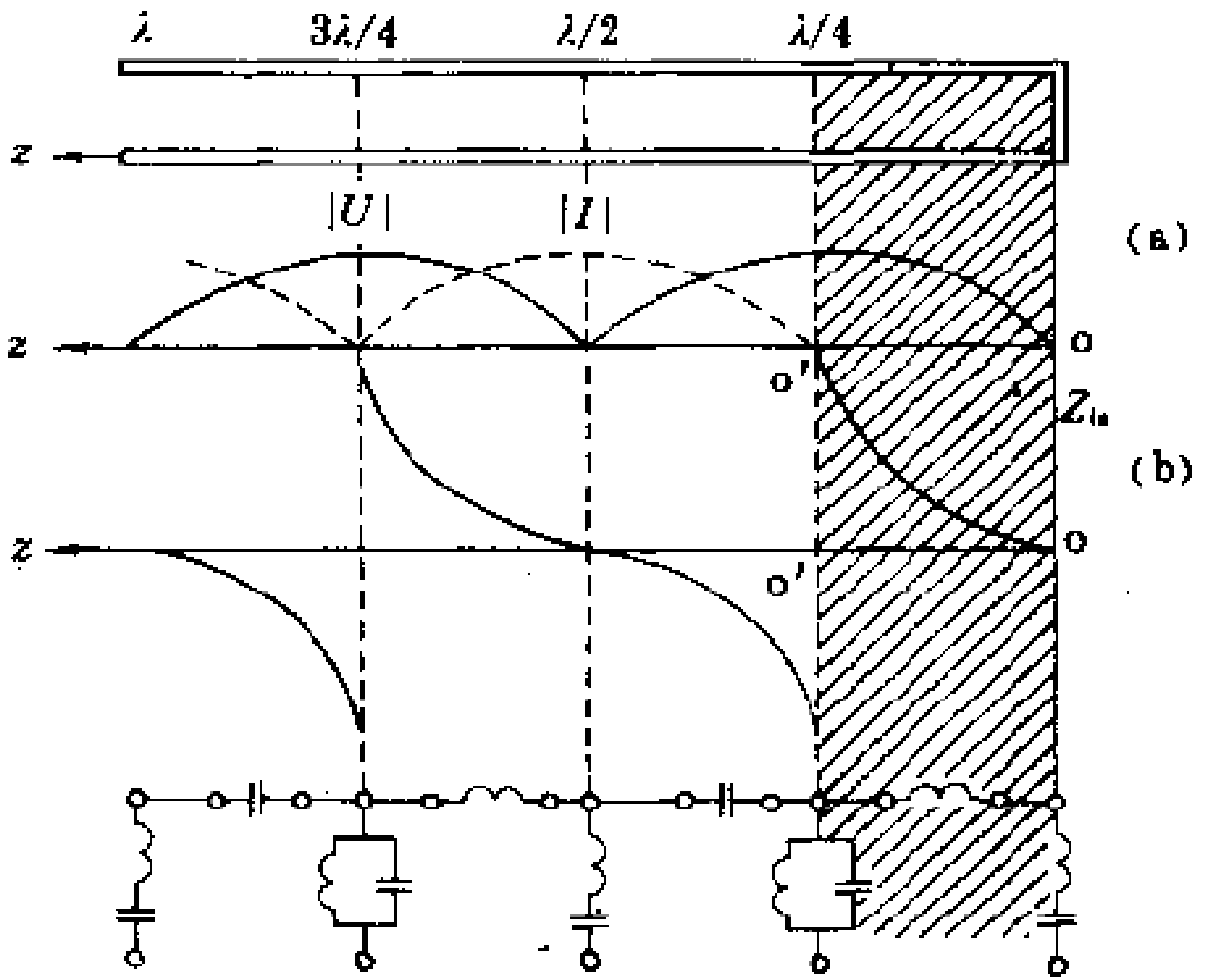


图 2.5 无耗开路线的驻波特性

(a)电压、电流振幅分布图;(b)阻抗特性曲线

开路λ/4传输线是何种谐振电路?

EXAMPLE 6.2 A HALF-WAVE MICROSTRIP RESONATOR

Consider a microstrip resonator constructed from a *λ/2 length of 50* open circuited microstrip line. The substrate is Teflon (e*r = 2.08, tan δ = 0.0004), with*
a thickness of 0.159 cm, and the conductors are copper. Compute the required length of the line for resonance at 5 GHz, and the unloaded *Q of the resonator.*
Ignore fringing fields at the end of the line.

Solution:
From

$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A}-2} & \text{for } W/d < 2 \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B-1) + \dfrac{\epsilon_r - 1}{2\epsilon_r}\left\{\ln(B-1) + 0.39 - \dfrac{0.61}{\epsilon_r}\right\}\right] & \text{for } W/d > 2, \end{cases} \quad (3.197)$$

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}.$$

the width of a 50ohm microstrip line on this substrate is found to be *W = 0.508 cm*, and the effective permittivity is e = 1.80. The resonant length can then be calculated as

$$\ell = \frac{\lambda}{2} = \frac{v_p}{2f} = \frac{c}{2f\sqrt{\epsilon_e}} = \frac{3 \times 10^8}{2(5 \times 10^9)\sqrt{1.80}} = 2.24 \text{ cm.}$$

The propagation constant is

$$\beta = \frac{2\pi f}{v_p} = \frac{2\pi f\sqrt{\epsilon_e}}{c} = \frac{2\pi(5 \times 10^9)\sqrt{1.80}}{3 \times 10^8} = 151.0 \text{ rad/m.}$$

the attenuation due to conductor loss is

$$\alpha_c = \frac{R_s}{Z_0 W} = \frac{1.84 \times 10^{-2}}{50(0.00508)} = 0.0724 \text{ Np/m,}$$

the attenuation due to dielectric loss is

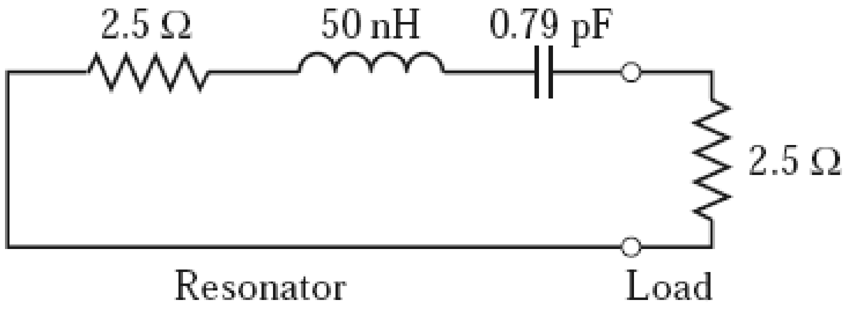
$$\alpha_d = \frac{k_0\epsilon_r(\epsilon_e - 1)\tan\delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} = \frac{(104.7)(2.08)(0.80)(0.0004)}{2\sqrt{1.80}(1.08)} = 0.024 \text{ Np/m.}$$

the unloaded *Q is*

$$Q_0 = \frac{\beta}{2\alpha} = \frac{151.0}{2(0.0724 + 0.024)} = 783.$$

Homework

6.1 A series RLC resonator with an external load is shown below. Find the resonant frequency, the unloaded Q, and the loaded Q.



6.3 A transmission line resonator is fabricated from a $\lambda/4$ length of open-circuited line. Find the unloaded Q of this resonator if the complex propagation constant of the line is $\alpha + j\beta$.

6.5 A resonator is constructed from a 3.0 cm length of 100 air-filled coaxial line, shorted at one end and terminated with a capacitor at the other end, as shown below. (a) Determine the capacitor value to achieve the lowest order resonance at 6.0 GHz. (b) Now assume that loss is introduced by placing a 10,000 resistor in parallel with the capacitor. Calculate the unloaded Q.