Lec4 Transmission line theory (III)

4 THE QUARTER-WAVE TRANSFORMER

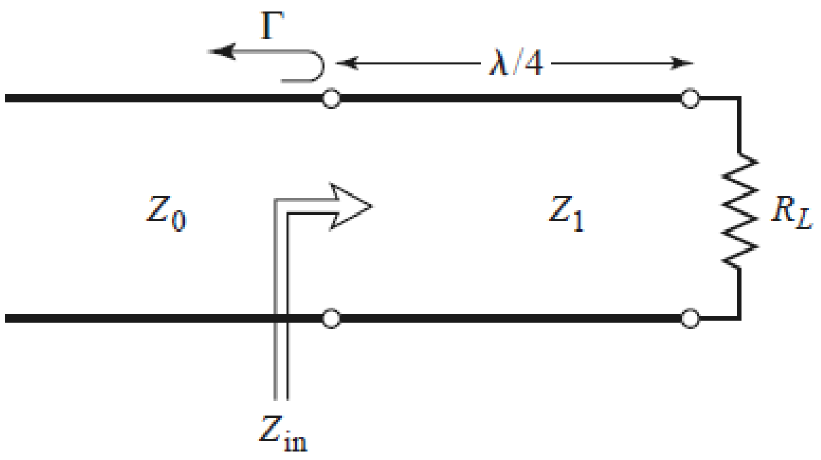
- The quarter-wave transformer is a useful and practical circuit for impedance matching.
- A lossless piece of transmission line of (unknown) characteristic impedance Z_1 and length $\lambda/4$.
- to match the load to the Z_0 line by using the $\lambda/4$ section of line and so make $\Gamma=0$ looking into the $\lambda/4$ matching section.

$$Z_{\text{in}} = Z_1 \frac{R_L + jZ_1 \tan \beta\ell}{Z_1 + jR_L \tan \beta\ell}.$$

$$\beta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2, \qquad Z_{\text{in}} = \frac{Z_1^2}{R_L}.$$

In order for $\Gamma = 0$, we must have $Z_{\text{in}} = Z_0$

$$Z_1 = \sqrt{Z_0 R_L},$$



The quarter-wave matching transformer.

阻抗变换器实现阻抗匹配，受频率影响吗？

There will be no standing waves on the feedline（SWR = 1）, although there will be standing waves on the $\lambda/4$ matching section.

the above condition applies only when the length of the matching section is $\lambda/4$ or an odd multiple of $\lambda/4$, long, so that a perfect match may be achieved at one frequency, but impedance mismatch will occur at other frequencies.

EXAMPLE 2.5 FREQUENCY RESPONSE OF A QUARTER-WAVE TRANSFORMER

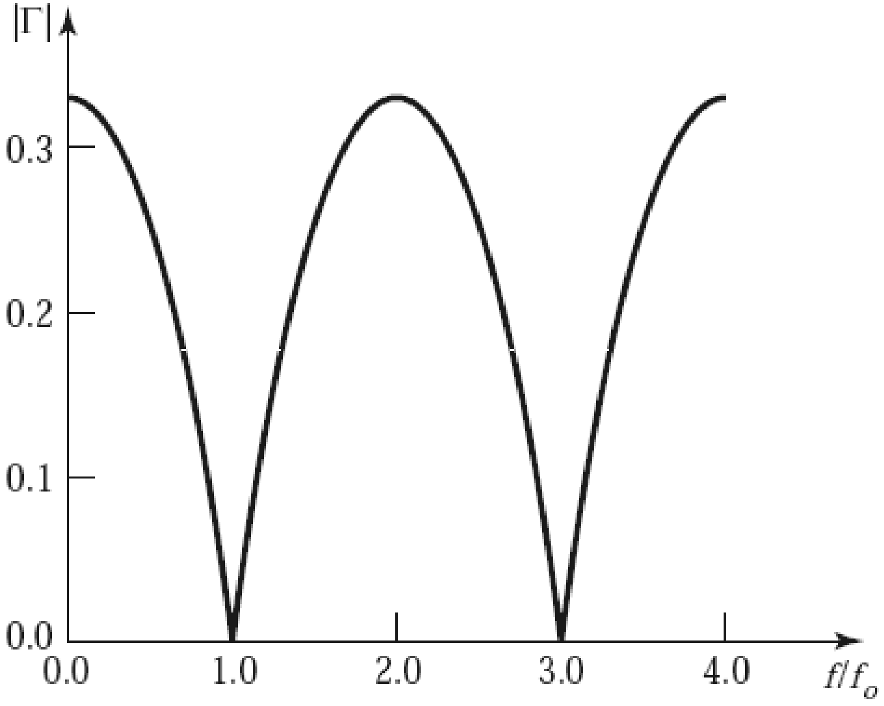
Consider a load resistance $R_L =$ 100 to be matched to a 50 line with a quarter-wave transformer. Find the characteristic impedance of the matching section and plot the magnitude of the reflection coefficient versus normalized frequency, f/fo, where fo is the frequency at which the line is $\lambda/4$ long.

Solution

The necessary characteristic impedance is $Z_1 = \sqrt{(50)(100)} = 70.71\ \Omega.$

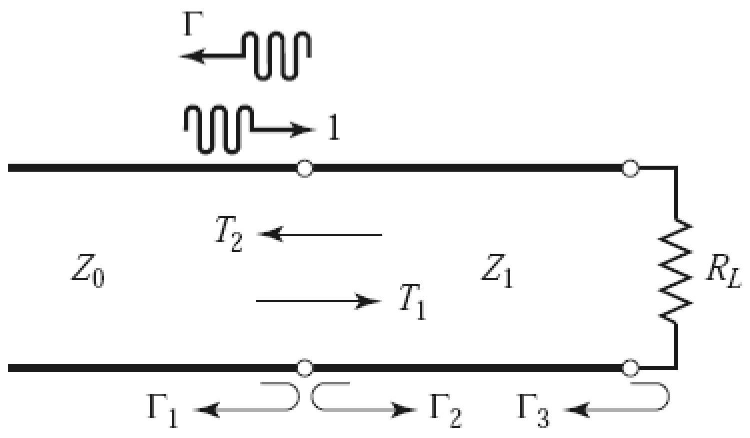
$$|\Gamma| = \left|\frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}\right|, \qquad Z_{\text{in}} = Z_1 \frac{R_L + jZ_1 \tan\beta\ell}{Z_1 + jR_L \tan\beta\ell}.$$

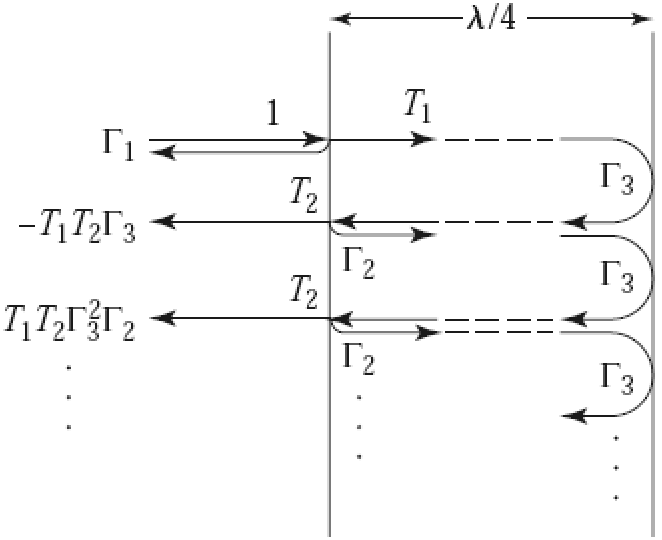
$$\beta\ell = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4f_o}\right) = \frac{\pi f}{2f_o}, \qquad \beta\ell = \pi/2 \text{ for } f = f_o,$$



Reflection coefficient versus normalized frequency for the quarter-wave transformer

5. The Multiple-Reflection Viewpoint





The partial reflection coefficient of a wave incident on a load *Z*1, from the *Z*0 line.

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

partial reflection coefficient of a wave incident on a load *Z*0, from the *Z*1 line.

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

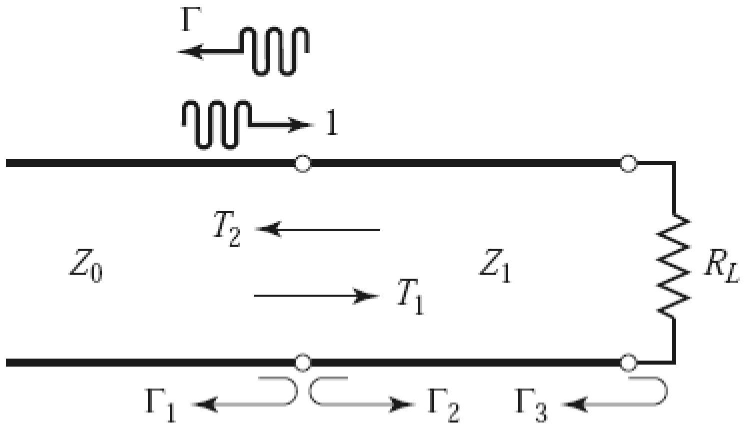
partial reflection coefficient of a wave incident on a load *RL*, from the *Z*1 line.

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

partial transmission coefficient of a wave from the $Z0$ line into the $Z1$ line.

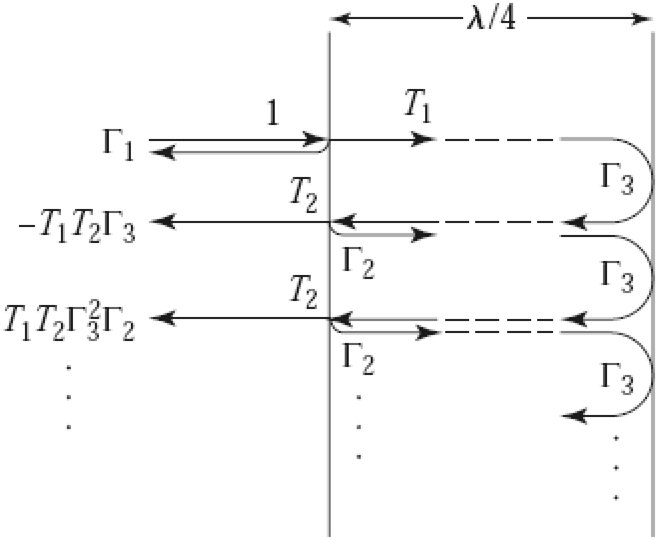
$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

$$T = 1 + \Gamma$$



partial transmission coefficient of a wave from the Z_1 line into the Z_0 line.

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$



total, reflection coefficient of a wave incident on the $\lambda/4$ transformer

$$\Gamma = \Gamma_1 - T_1T_2\Gamma_3 + T_1T_2\Gamma_2\Gamma_3^2 - T_1T_2\Gamma_2^2\Gamma_3^3 + \cdots$$

$$= \Gamma_1 - T_1T_2\Gamma_3\sum_{n=0}^{\infty}(-\Gamma_2\Gamma_3)^n.$$

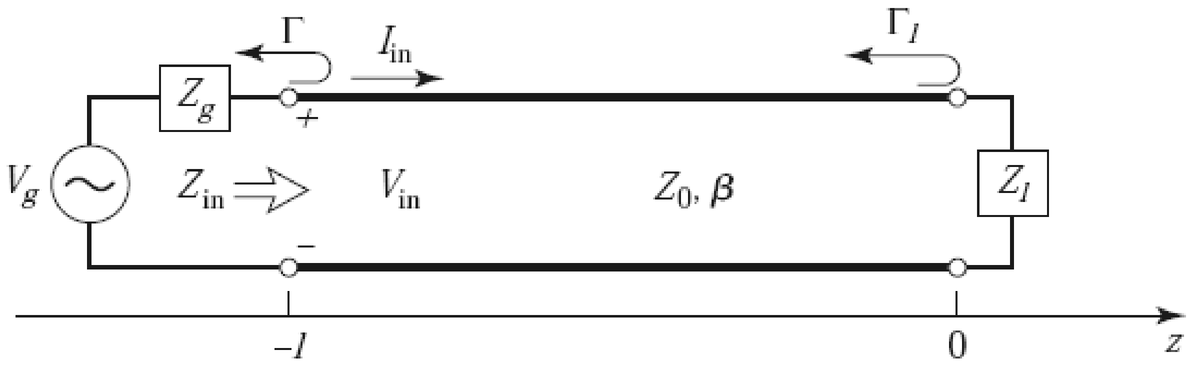
$$= \frac{2(Z_1^2 - Z_0R_L)}{(Z_1 + Z_0)(R_L + Z_1)},$$

if we choose $Z_1 = \sqrt{Z_0R_L}$. $\Gamma = 0$

Under steady-state conditions, an infinite sum of waves traveling in the same direction with the same phase velocity can be combined into a single traveling wave.

6. GENERATOR AND LOAD MISMATCHES

Both generator and load may present mismatched impedances to the transmission line.



The input impedance looking into the terminated transmission line from the generator end is,

$$Z_{in} = Z_0 \frac{1 + \Gamma_\ell e^{-2j\beta\ell}}{1 - \Gamma_\ell e^{-2j\beta\ell}} = Z_0 \frac{Z_\ell + jZ_0 \tan\beta\ell}{Z_0 + jZ_\ell \tan\beta\ell},$$

The reflection coefficient of the load:

$$\Gamma_\ell = \frac{Z_\ell - Z_0}{Z_\ell + Z_0}.$$

The voltage on the line

$$V(z) = V_o^+ \left(e^{-j\beta z} + \Gamma_\ell e^{j\beta z}\right),$$

we can find V_0^+ from the voltage at the generator end of the line,

$$V(-\ell) = V_g \frac{Z_{in}}{Z_{in} + Z_g} = V_o^+ \left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right),$$

$$V_o^+ = V_g \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g} \frac{1}{\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)} \cdot = V_g \frac{Z_0}{Z_0 + Z_g} \frac{e^{-j\beta\ell}}{\left(1 - \Gamma_\ell \Gamma_g e^{-2j\beta\ell}\right)},$$

the reflection coefficient seen looking into the generator:

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}.$$

The standing wave ratio on the line is then $\text{SWR} = \dfrac{1 + |\Gamma_\ell|}{1 - |\Gamma_\ell|}.$

The power delivered to the load is

$$P = \frac{1}{2}\text{Re}\{V_{\text{in}} I_{\text{in}}^*\} = \frac{1}{2}|V_{\text{in}}|^2 \text{Re}\left\{\frac{1}{Z_{\text{in}}}\right\} = \frac{1}{2}|V_g|^2 \left|\frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g}\right|^2 \text{Re}\left\{\frac{1}{Z_{\text{in}}}\right\}.$$

let $Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$ and $Z_g = R_g + jX_g$; then

$$P = \frac{1}{2}|V_g|^2 \frac{R_{\text{in}}}{(R_{\text{in}} + R_g)^2 + (X_{\text{in}} + X_g)^2}.$$

$$P = \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2+(X_{in}+X_g)^2}.$$

传输线获得最大功率的条件是什么？

assume that the generator impedance, *Zg*, is fixed, and consider three cases of load impedance.

Case1：Load Matched to Line

$$Z_l = Z_0, \text{ so } \Gamma_\ell = 0, \text{ and SWR} = 1, \quad Z_{in} = Z_0, \qquad P = \frac{1}{2}|V_g|^2 \frac{Z_0}{(Z_0+R_g)^2+X_g^2}.$$

Case2：Generator Matched to Loaded Line

$$Z_{in} = Z_g, \quad \Gamma = \frac{Z_{in}-Z_g}{Z_{in}+Z_g} = 0. \qquad P = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g^2+X_g^2)}.$$

Case3：Conjugate Matching

Assuming that the generator series impedance *Zg* is fixed, we may vary the input impedance *Zin* until we achieve the maximum power delivered to the load.

$$P = \frac{1}{2}|V_g|^2 \frac{R_{\text{in}}}{(R_{\text{in}} + R_g)^2 + (X_{\text{in}} + X_g)^2}.$$

$$\frac{\partial P}{\partial R_{\text{in}}} = 0 \rightarrow \frac{1}{(R_{\text{in}} + R_g)^2 + (X_{\text{in}} + X_g)^2} + \frac{-2R_{\text{in}}(R_{\text{in}} + R_g)}{[(R_{\text{in}} + R_g)^2 + (X_{\text{in}} + X_g)^2]^2} = 0,$$

$$R_g^2 - R_{\text{in}}^2 + (X_{\text{in}} + X_g)^2 = 0,$$

$$\frac{\partial P}{\partial X_{\text{in}}} = 0 \rightarrow \frac{-2R_{\text{in}}(X_{\text{in}} + X_g)}{[(R_{\text{in}} + R_g)^2 + (X_{\text{in}} + X_g)^2]^2} = 0,$$

$$X_{\text{in}}(X_{\text{in}} + X_g) = 0.$$

$$\rightarrow R_{\text{in}} = R_g, \quad X_{\text{in}} = -X_g,$$

$$Z_{\text{in}} = Z_g^*.$$

$$P = \frac{1}{2}|V_g|^2 \frac{1}{4R_g},$$

The maximum available power from the generator.

Note that the reflection coefficients *may be nonzero*. Physically, this means that in some cases the power in the multiple reflections on a mismatched line may add in phase to deliver more power to the load than would be delivered if the line were flat (no reflections).

If the generator impedance is real *(Xg = 0), then the last two cases reduce to the same result, which is that maximum power* is delivered to the load when the loaded line is matched to the generator (*Rin = Rg, with Xin = Xg = 0).*

4. LOSSY TRANSMISSION LINES

与无耗传输线相比，低耗传输线的传播常数？特性阻抗？

The Low-Loss Line

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)},$$

$$= \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}.$$

$R \ll \omega L$ and $G \ll \omega C$.

$$\gamma \simeq j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}.$$

we will instead use the first two terms of the Taylor series expansion for

$$\sqrt{1 + x} \simeq 1 + x/2 + \cdots$$

$$\gamma \simeq j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right],$$

$$\alpha \simeq \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right),$$

$$\beta \simeq \omega\sqrt{LC},$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \simeq \sqrt{\frac{L}{C}}.$$

the propagation constant and characteristic impedance for a low-loss line can be closely approximated by considering the line as lossless.

EXAMPLE 2.6 ATTENUATION CONSTANT OF THE COAXIAL LINE

In Example 2.1 the L, C, R, and G parameters were derived for a lossy coaxial line. Assuming the loss is small, derive the attenuation constant from (2.85a) with the results from Example 2.1.

Solution

From (2.85a),

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right).$$

Using the results for R and G derived in Example 2.1 gives

$$\alpha = \frac{1}{2}\left[\frac{R_s}{\eta \ln b/a}\left(\frac{1}{a} + \frac{1}{b}\right) + \omega\epsilon''\eta\right],$$

where $\eta = \sqrt{\mu/\epsilon'}$ is the intrinsic impedance of the dielectric material filling the coaxial line. In addition, $\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon'}$ and $Z_0 = \sqrt{L/C} = (\eta/2\pi) \ln b/a$. ■

The Distortionless Line

（什么叫无色散传输线？有耗线都色散吗？）

for the propagation constant of a lossy line, the phase term β is generally a complicated function of frequency ω when loss is present.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}, \quad \gamma = \alpha + j\beta$$

If β is not a linear function of frequency (of the form $\beta = a\omega$), then the phase velocity $v_p = \omega/\beta$ will vary with frequency.

If β is not a linear function of frequency (of the form β = aω), then the phase velocity vp = ω/β will vary with frequency. the various frequency components of a wideband signal will travel with different phase velocities and so arrive at the receiver end of the transmission line at slightly different times. This will lead to *dispersion, a distortion of the signal, and is generally an* undesirable effect.

There is a special case, however, of a lossy line that has a linear phase factor as a function of frequency.

Such a line is called a *distortionless line*, satisfying

$$\frac{R}{L} = \frac{G}{C}.$$

$$\gamma = \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}.$$

$$= j\omega\sqrt{LC}\sqrt{1 - 2j\frac{R}{\omega L} - \frac{R^2}{\omega^2 L^2}}$$

$$= j\omega\sqrt{LC}\left(1 - j\frac{R}{\omega L}\right)$$

$$= R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} = \alpha + j\beta,$$

$$\alpha = R\sqrt{C/L}$$

$$\beta = \omega\sqrt{LC}$$

the distortionless line is not loss free but is capable of passing a pulse or modulation envelope without distortion.

The Terminated Lossy Line

$$V(z) = V_o^+\left(e^{-\gamma z} + \Gamma e^{\gamma z}\right),$$

$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-\gamma z} - \Gamma e^{\gamma z}\right),$$

Lossless

$$\Gamma(\ell) = \frac{V_o^- e^{-j\beta\ell}}{V_o^+ e^{j\beta\ell}} = \Gamma(0)e^{-2j\beta\ell},$$

Lossy

$$\Gamma(\ell) = \Gamma e^{-2j\beta\ell} e^{-2\alpha\ell} = \Gamma e^{-2\gamma\ell}.$$

The input impedance *Zin* at a distance from the load is

$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = Z_0 \frac{Z_L + Z_0 \tanh \gamma\ell}{Z_0 + Z_L \tanh \gamma\ell}.$$

the power delivered to the input of the terminated line at $z = -l$ is

$$P_{in} = \frac{1}{2}\mathrm{Re}\{V(-\ell)I^*(-\ell)\} = \frac{|V_o^+|^2}{2Z_0}(e^{2\alpha\ell} - |\Gamma|^2 e^{-2\alpha\ell})$$
$$= \frac{|V_o^+|^2}{2Z_0}(1 - |\Gamma(\ell)|^2)e^{2\alpha\ell},$$

The power actually delivered to the load is

$$P_L = \frac{1}{2}\mathrm{Re}\{V(0)I^*(0)\} = \frac{|V_o^+|^2}{2Z_0}(1 - |\Gamma|^2).$$

the power lost in the line:

$$P_{loss} = P_{in} - P_L = \frac{|V_o^+|^2}{2Z_0}[(e^{2\alpha\ell} - 1) + |\Gamma|^2(1 - e^{-2\alpha\ell})].$$

the power loss of the incident wave

the power loss of the reflected wave

Homework

2.16 The transmission line circuit in the accompanying figure has $V_g = 15$ V rms, $Z_g = 75\ \Omega$, $Z_0 = 75\ \Omega$, $Z_L = 60 - j40\ \Omega$, and $\ell = 0.7\lambda$. Compute the power delivered to the load using three different techniques:

(a) Find Γ and compute

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}(1 - |\Gamma|^2);$$

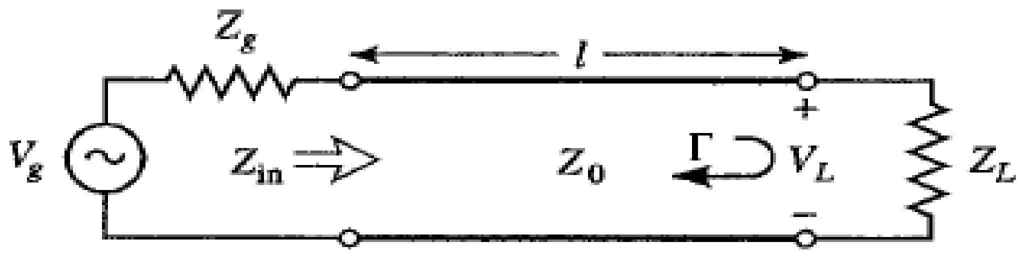
(b) find Z_{in} and compute

$$P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 \mathrm{Re}\{Z_{in}\};$$

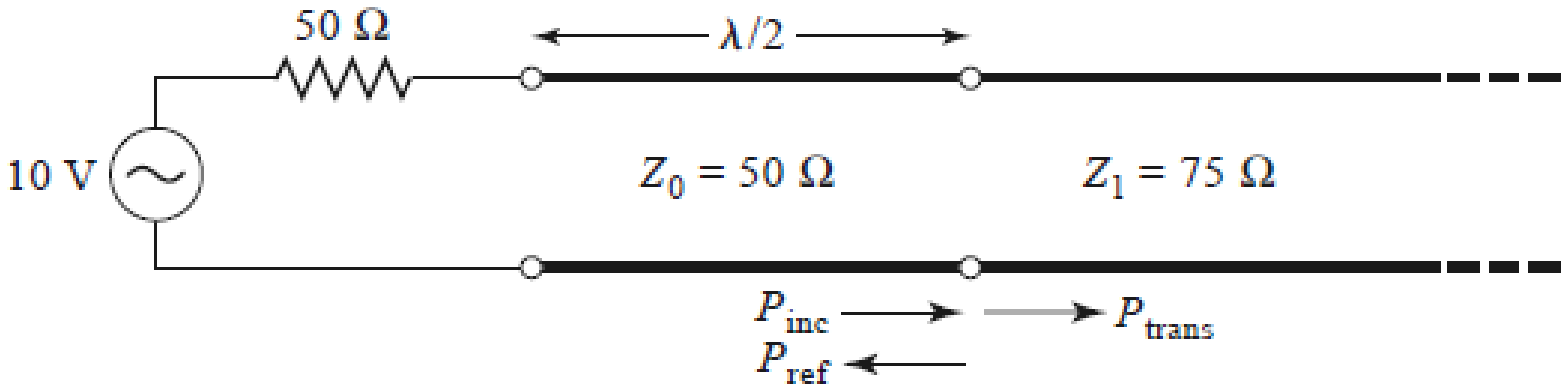
(c) find V_L and compute

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 \mathrm{Re}\{Z_L\}.$$

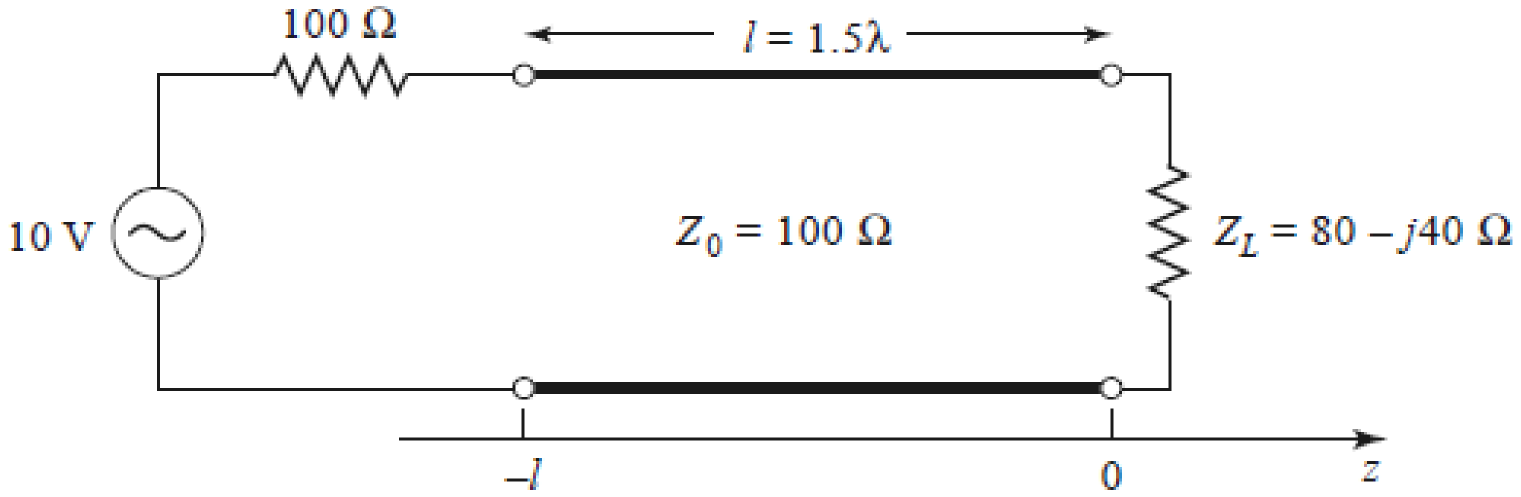
Discuss the rationale for each of these methods. Which of these methods can be used if the line is not lossless?



2.18 Consider the transmission line circuit shown in the accompanying figure. Compute the incident power, the reflected power, and the power transmitted into the infinite 75 Ω line. Show that power conservation is satisfied.



2.19 A generator is connected to a transmission line as shown in the accompanying figure. Find the voltage as a function of z along the transmission line. Plot the magnitude of this voltage for $-\ell \leq z \leq 0$.



2.20 Use the Smith chart to find the following quantities for the transmission line circuit shown in the accompanying figure:

(a) The SWR on the line.

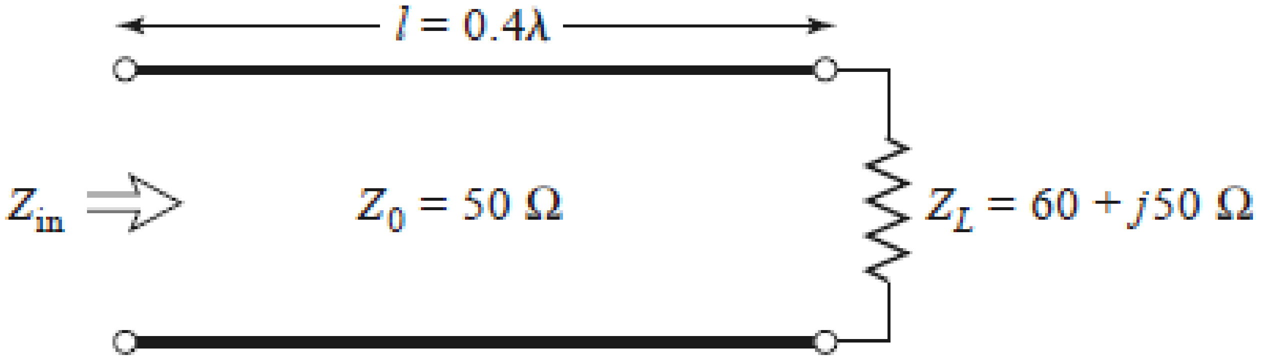
(b) The reflection coefficient at the load.

(c) The load admittance.

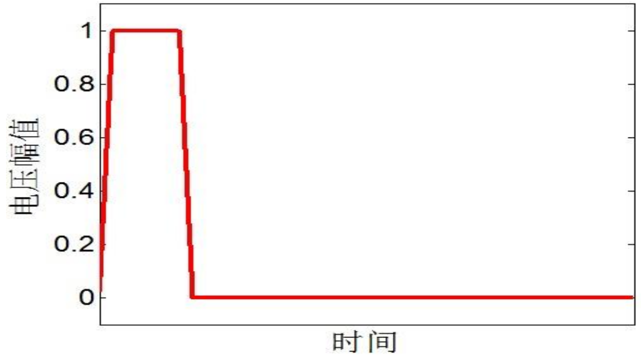
(d) The input impedance of the line.

(e) The distance from the load to the first voltage minimum

(f) The distance from the load to the first voltage maximum

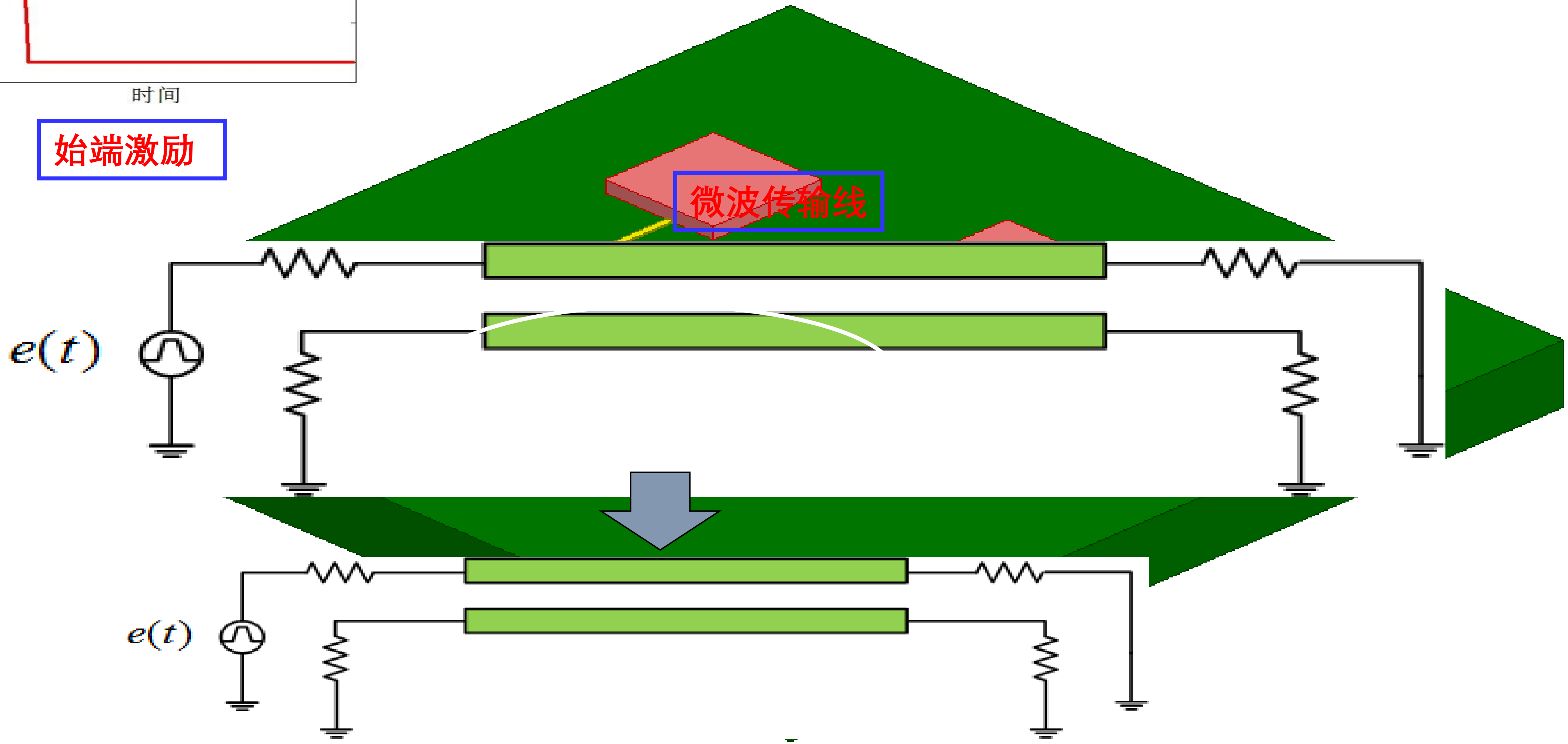


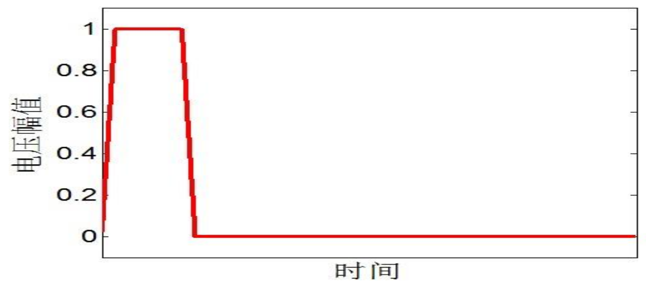
互连的信号完整性问题



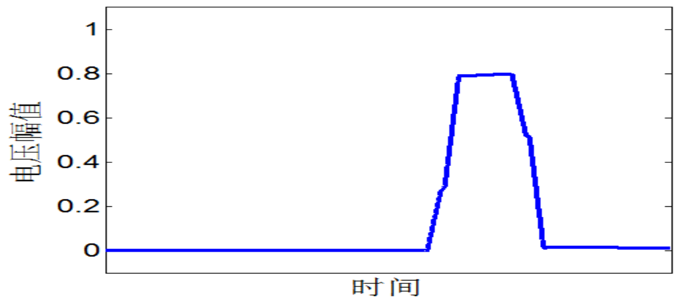
始端激励

- 随着工作速度提高，信号频谱进入微波波段，互连长度与波长与相当，显示微波传输线效应。





信号产生时延、衰减、畸变、反射、串扰等微波传输线效应，信号完整性受到破坏。

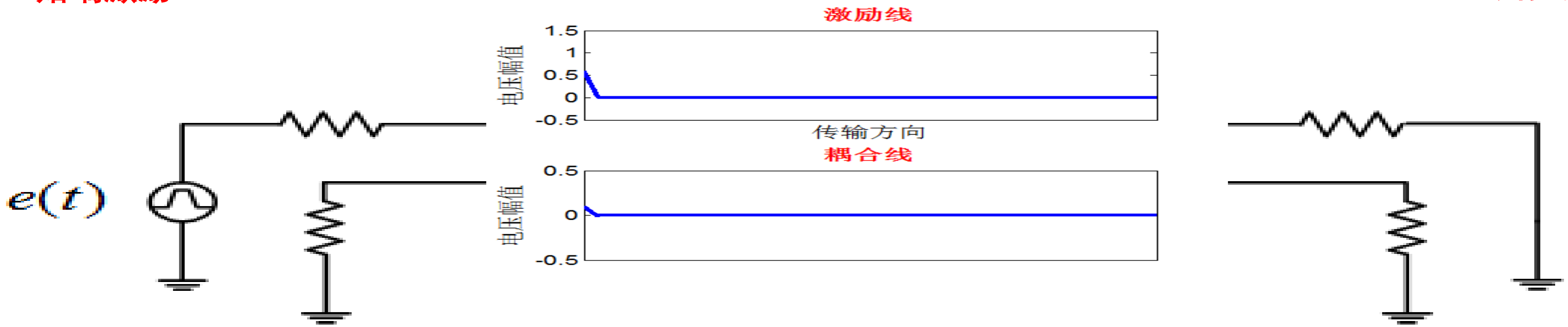


时延

畸变

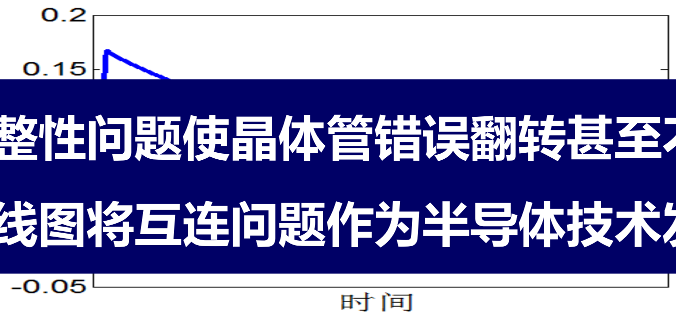
始端激励

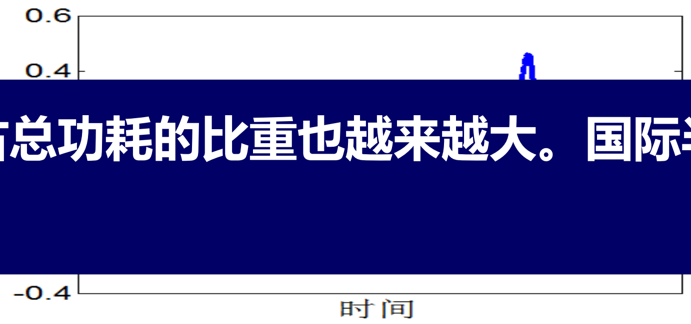
终端响应



近端串扰

远端串扰





信号完整性问题使晶体管错误翻转甚至不能工作！互连功耗占总功耗的比重也越来越大。国际半导体发展路线图将互连问题作为半导体技术发展亟待解决的问题。